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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Notes on the Anatomy of Macrozamia heteromera, Moore. By AGNES ROBERTSON, B.Sc. (Lond.), Newnham College. (Communicated by Mr Seward.)

[*Received* 14 August 1902.]

Introduction.

Ten species of the Australian genus *Macrozamia* are natives of New South Wales, and one of these, *M. heteromera*, described in 1884 by Moore[1], is of interest as possessing, both in its ordinary form and in its two varieties *glauca* and *tenuifolia*, a type of frond unusual among Cycads. The pinnae are very narrow, reminding us in this respect of *Encephalartos Ghellinkii*, and most of them are forked in various ways (Fig. 4). It occurred to Mr Seward, who has referred to and figured this peculiarity of vegetative structure in his *Wealden Flora*[2], that the anatomy might perhaps be expected to shew some features differing from that of the rest of the genus. He was able to obtain a specimen through the kindness of Mr Maiden, Director of the Sydney Botanic Gardens, who informs us that "as a rule the plant grows in a stiff black clay amongst large stones (basalt); occasionally in sand-ridges (sand overlying clay)." The specimen, which on its arrival in England had been placed in methylated spirit, was handed to me for examination.

The diameter of the stem was about 12·5 cms. in its thickest part. The upper half was covered with leaf-bases, but lower down these had disappeared, and the position of the old leaves was only marked by tangentially elongated ridges. The appearance of the

[1] *Trans. Roy. Soc. New South Wales*, 1884, p. 115.
[2] *The Wealden Flora*, Part II. p. 5, and Plate XIII. figs. 1 and 2.

outer part of the stem is thus to some extent intermediate between that of *Cycas* and *Zamia*.

Worsdell has dealt with the anatomy of three species of this genus, and figured a fourth. It may be well for purposes of comparison with our species to quote very shortly the main results which he has arrived at.

In *M. Fraseri* Miq.[1], he mentions as the most noteworthy characters exhibited by the anatomy of the stem:—

(i) The well-developed *medullary system of bundles*.

(ii) The *anomalous zones of secondary thickening* which, to the number of two or three, surround the normal cylinder.

(iii) The presence of a *tertiary cambium*, which, arising in isolated places in the parenchyma either between the normal and the first anomalous ring, or between two anomalous rings, forms wood and bast with inverted orientation, such that the xylem is usually directly opposite that of one of the segments of an anomalous ring.

(iv) The irregularity and displacements of many portions of the vascular tissue, owing to the great expansion of the parenchyma and the medullary rays between the various segments composing the ring.

Worsdell figures a transverse section of *M. Moorei*, F. Muell.[2], possessing the normal vascular ring, one complete anomalous ring, one outside this which is incomplete, and some isolated bundles belonging to a third anomalous ring.

In *M. Denisonii*, F. Muell.[2], of which he examined a plant having a stem diameter of about 1 inch, Worsdell found no medullary bundles. Nor did the plant at this stage possess extrafascicular rings, but in the region of the primary node it had extrafascicular concentric strands, regarded by Worsdell as the first beginnings and foundation of later-formed extrafascicular rings.

Worsdell has also investigated two seedlings of *M. spiralis*, Miq.[3], which shew no extrafascicular zones, and no medullary system of vascular bundles on the stem. He adds, "It is true that I am describing very young plants of this species; but I feel sure that they are not so young but that if these characters were really possessed by the plant, they would already have given some indication of their presence."

It should be noted that though the plant whose anatomy is about to be described is now raised to the rank of a separate species under the name of *Macrozamia heteromera*, both Bentham and Mueller regarded it as an inland variety of the coast species *M. spiralis*.

[1] *Annals of Botany*, Vol. x. 1896, p. 601.
[2] *Trans. Linn. Soc.* Vol. vi. Part ii. 1901, p. 112.
[3] *Journ. Linn. Soc.* Vol. xxxiii. 1898, p. 445.

Anatomy of M. heteromera.

(i) *The Apical Region.*

The apex is sunk in a small pit. Transverse sections taken just below the apex shew a structure to which Mettenius's[1] description of the apical region of *Cycas revoluta* applies almost word for word. "The transverse section of a young stem of *Cycas revoluta* shews at a small distance from the growing point which is sunk in the stem apex, on the boundary between the pith and the cortex, whose cells are filled with starch grains, a ring-shaped stripe of delicate, radially arranged, meristematic cells, the cambial layer."

(ii) *The Pith.*

Near the apex the pith contains mucilage canals, running in every direction (especially radially) and not accompanied by any vascular tissue. Between the levels where the vascular cylinder is respectively 2·5 and 2·7 cms. in diameter the first traces of vascular bundles make their appearance in the pith. At the lower of these two levels, *i.e.* where the stem diameter excluding the leaf bases is 6·5 cms., some of the canals have strands containing lignified xylem beside them, whilst others are merely accompanied by a little meristematic tissue, representing the future bundles. The phloem seems always to be turned towards the canal and the xylem in the other direction. The medullary system of canals and vascular tissue thus agrees in origin and arrangement with that described by Worsdell (*l. c.*) in the case of *Macrozamia Fraseri*. Close to the base of the stem the medullary bundles become more strongly developed and more complicated in their arrangement.

(iii) *The Vascular Tissue.*

Sections taken near the top of the stem shew a single ring of vascular bundles of the normal Cycadean type. The tracheids are pitted, and I have been unable to detect any with spiral thickenings, except occasionally in the very youngest part of the cylinder. At the level at which the normal vascular ring is rather less than 5·8 cms. in diameter, a second ring is seen arising outside it in the form of small separate bundles. There is no clearly differentiated pericycle in the stem, but as these extra bundles arise immediately outside the phloem of the normal ring, their origin is probably the same as that of the supernumerary bundles of *Cycas*, which Constantin and Morot[2] describe as "pericyclic." Lower down in

[1] "Beiträge zur Anatomie der Cycadeen," *Abhandl. Königl. Sächs. Gesellsch. der Wiss.* v. 1861, p. 573.

[2] *Bull. de la Soc. Bot. de France*, XXXII. 1885, p. 173.

the stem these bundles form a fairly complete ring, but it is never very regular, and at the level where the diameter of the inner ring is about 7 cms., the outer ring though well-marked on one side of the stem is only slightly developed on the other.

At the base of the stem the number of concentric rings of vascular tissue increases, and the different zones are seen in longitudinal section to anastomose together (Fig. 6). There appear, at the extreme base, to be as many as four extra rings, but the whole structure is irregular and difficult to make out. As Constantin and Morot[1] have pointed out for *Cycas Siamensis, Encephalartos Alteinsteinii* and *Ceratozamia Mexicana*, the existence of the supernumerary bundle-systems and their early appearance at the base of the stem, are connected, as in the case of secondarily thickened monocotyledons, with the need of obtaining an adequate vascular supply for the adventitious roots which are destined to make up for the insufficient development of the tap root.

I have not been able to find any tertiary inverted strands between the vascular rings, such as those described by Worsdell in *Macrozamia Fraseri*.

(iv) *The Cortex and Leaf-traces.*

Transverse sections through the cortex near the apex shew numerous leaf-traces cut transversely, as in this region they run nearly vertically to join the central cylinder lower down. Above the level of differentiation of the central cylinder, I noticed a single horse-shoe-shaped bundle in the cortex among the normal leaf-traces. This was the nearest approach I have seen in any part of the stem to a concentric bundle. In radial sections somewhat lower down, but still in the younger part of the stem, the leaf-traces can be followed running obliquely upwards from the central cylinder, right to the bases of the leaves. It seems that the characteristic Cycadean girdles, of which traces can be seen in transverse sections in the form of short horizontal strands of spiral tracheids, run so close to the leaf bases as to be cut off by periderm at a very early stage. In the older part of the stem the horizontal radially-running portions of the traces dip down as they approach the central cylinder, and passing through the outer ring join the inner ring. In one case a trace was observed to fork in the cortex, and the two branches united a little further out, forming a loop. Occasionally when a trace is followed outwards from the central cylinder it can be seen to turn when it gets near the periphery and take a course parallel to the surface

[1] "Comparative Anatomy of certain species of Encephalartos," *Trans. Linn. Soc.* Vol. v. Part xiv. 1900, p. 447.

of the stem. I have not been able to follow any girdle for more than about 2 cms., as they all seem to be cut off by periderm formation. We may recall the case of *Dioon edule*, in which Mettenius[1] figures the girdles lying very close to the surface of the stem; also that of the two stems of *Stangeria paradoxa* described by Solms-Laubach[2], one of which shewed no leaf-trace girdles, so that he was at first inclined to think that the plant possessed the simple leaf-traces with which we are familiar in Bennettiteae, but the second specimen shewed a normal development of the girdles. He ascribes the loss of the girdles in the first example to bark formation. Worsdell[3] has examined an adult plant of the same species and finds that in this, "No girdle-leaf-trace bundles are seen, although these may have existed at an earlier stage of the plant's life and been subsequently cut off by the advancing periderm."

(v) *The Mucilage Canals.*

One of the most striking features in the cortex and pith of the plant is the abundance of the large branched mucilage canals so characteristic of Cycads. In the young cortex narrow bands of crushed tissue occur between the mucilage canals. This gives an appearance recalling that of the "Augenstructur" of crushed rocks, in which the uncrushed fragments assume an eye-like form with their long axis perpendicular to the direction of maximum pressure. Each canal with the layers of parenchyma immediately surrounding it appears to have increased in size, and the result has been a zone of crushing along the neutral line where the territories of two adjacent canals meet.

The extraordinary abundance of mucilage in the stem seems to have been a feature common to recent and fossil Cycadean plants. Carruthers[4], describing a Mesozoic genus, says, "From the absence of structure in all the specimens yet discovered of *Bucklandia* there is no evidence as to the presence of canals for the secretion of the mucilaginous juice which is so abundant in the living *Cycadeae*. But our specimen of *B. anomala* preserves on its surface the casts of large drops of this substance, that had exuded and hardened, so as to mould the rock around them; and so rich was this fossil in the gum, that, in this specimen, the interspaces between the scales are filled up with it." Vetters[5], in 1884, suggested that the gum canals serve as water reservoirs, their secretions sucking up the excess of water in the rainy season,

[1] *Loc. cit.* Taf. III. 1.
[2] *Bot. Zeit.* 1890, p. 180.
[3] *Journ. Linn. Soc.* Vol. XXXIII. 1898, p. 452.
[4] *Trans. Linn. Soc.* Vol. XXVI. 1870, p. 685.
[5] *Die Blattstiele der Cycadeen,* Leipzig, 1884, p. 11.

and giving it out to the surrounding tissues in times of scarcity. In describing *Encephalartos Caffer*, Worsdell[1] puts forward the following elaboration of Vetters' theory:—"It appears that these plants, which grow in clefts of rocks in the hilly country of South Africa, have, at a certain season of the year, to endure a dry season, during which their tap-root entirely dies away, along with, presumably, the foliage. During this period the stem, swollen as it is with its massive parenchymatous tissues and protected externally by the thick and coriaceous leaf-bases, probably acts as a water-reservoir by means of the network of mucilage canals penetrating every part of the organ, the mucilaginous secretion serving to retain every particle of moisture previously absorbed by the root, and thus to preserve the stem from desiccation." Worsdell points out in connection with this theory that the *roots* are entirely devoid of mucilage canals.

It has occurred to me that it is just possible that this great accumulation of mucilage may serve another purpose, besides being an adaptation to a xerophytic mode of life. Mucilages seem in general to have the same percentage composition as cellulose, and the gap between typical cellulose and mucilage is bridged by the "hemicelluloses" which often form the reserve material in seeds. Cases are known in which gummy substances serve as a store of food, for example the mucilage dissolved in the cells of Orchid-tubers and the rhizomes of *Symphytum*[2], and according to Haberlandt[3], the "Schleim" of the endosperms of various Leguminosae. It is generally assumed that the mucilaginous substances stored in the canals of Cycads, and other similar receptacles, take no further part in the metabolism of the plant; but is it not possible that they form a reserve of food material which may be drawn upon in case of special need? The development of the large reproductive organs of a Cycad is a very rapid process compared with the slow vegetative growth of the same plant, and must mean a great drain upon its resources. If the accepted view is correct, the plant has locked up irretrievably an abundant store of organic substance in its mucilage canals, and in times of need it might starve within sight of plenty. The curious fact that the bundles which arise in the pith in connection with the mucilage canals seem invariably to orientate themselves without regard to anything but the position of the canals, may perhaps receive its explanation if the canals are really storehouses of food material; for the *phloem* is invariably turned towards the accompanying duct. On the other hand the possibility of an entirely opposite explanation must be remembered,

[1] *Trans. Linn. Soc.* Vol. v. 1900, Part xiv. p. 448.
[2] Pfeffer's *Physiology of Plants*, Vol. i. Trans. by Ewart, 1900, p. 478.
[3] *Physiologische Pflanzen-Anatomie*, 1896, p. 36.

namely that the phloem conducts food to the cells surrounding the duct, so as to enable them to secrete mucilage. These two explanations are not, however, mutually exclusive, and the phloem of the pith bundles may serve to fetch or to carry according to the needs of the plant. In a paper on the gum of Cycads published in 1839 Morren[1] gives some reasons for supposing that the substance is not stationary, but has an ascending motion in the stem, at least at the time when the young leaves are developing. His curiosity was first aroused by noticing the great length of the gum "boudins" which gradually exuded from cut petioles. He performed a number of experiments, of which the following seems to me the most suggestive. On a *Cycas revoluta* in full vegetation he made cuts in the rachis, perpendicular to the axis, 2 inches apart. He removed half the thickness of the petiole between these cuts, and thus made a kind of canal in which he might intercept the gum in its passage. He found that the gum always came from the *stem side*, and never from the *leaf side* of the cut. His final conclusion is:—"La gomme est, dans les Cycadées, une matière formée au détriment de la fécule de la tige, et montant par une progression ascendante de la tige dans la feuille."

(vi) *The Roots.*

At the bottom of the stem there were numerous bases of dead adventitious roots, and part of one root in good condition. The latter was diarch, and had a slender plate of primary xylem, consisting, as seen in transverse section, of two protoxylem groups connected by scattered metaxylem tracheids. A large fan of secondary xylem arose on each side of the primary xylem. These fans, at least in part of the root, were unsymmetrically developed, so that a deep bay of parenchymatous tissue was left opposite one protoxylem, and a shallow bay opposite the other. This recalls the want of symmetry mentioned by Brongniart[2] in the lateral roots of *Cycas circinalis*, and ascribed by him to mutual pressure. A band of secondary phloem occurs on the outer side of each of the two groups of secondary xylem, and a little of what appears to be crushed primary phloem is recognisable. The conjunctive tissue and parenchyma of the stele are extraordinarily rich in calcium oxalate crystals and cells containing tannin and gum.

Some of the parenchymatous cells shew peculiar brown contents. These are generally more or less contracted away from the walls of the cells and appear to contain a number of colourless spheres. These vary from tiny spots which only look like pin

[1] *Bull. Acad. Roy. des Sci. Brux.* Tom. VI. Part II. p. 135.
[2] *Ann. Sci. Nat.* Ser. III. Vol. V. 1846, p. 21.

pricks in the brown substance, to comparatively large spheres, whose mutual contact reduces the brown substance to a mere reticulum. It is interesting to meet with such appearances in the cells of a recent Cycad, as they closely resemble some that have been described and figured by Seward[1] in the ground tissue of the petiole of *Cycadeoidea gigantea.* The author points out that these frothy and granular cell contents bear a deceptive resemblance to the spores or cells of an endoparasitic organism, and mentions similar occurrences in recent plants, referring especially to the vacuolated cell contents found by Professor Marshall Ward[2] in the parenchymatous tissues of some of the Rubiaceae of Ceylon.

The pericycle and the tissue between the primary and secondary phloem contains a great many cells whose extremely thick, beautifully stratified, lignified walls are traversed by numerous branching pits. (Fig. 3.)

The many-layered periderm of thin-walled cells, which at this stage forms the outermost layer of the root, seems to arise in the pericycle. The phellogen gives rise to very little tissue internal to itself.

Another more slender root, of which only the base remained, was found to be triarch.

(vii) *Anomalous Periderm of Root and Stem.*

As the references to this subject in the literature are rather scanty and scattered, it may be well here to bring together some account of the results hitherto obtained.

In fossil Cycadaceous plants the formation of anomalous periderms has attracted more attention than in their modern representatives. Solms-Laubach[3] in a paper published in 1892, points out that in the case of a section of *Bennettites Peachianus* figured by Carruthers[4], the structures in the pith which were at one time compared to the "star-rings" of some Medulloseae, are really cork formations. He shews that some medullary periderm can also be recognised in *Bennettites Gibsonianus,* and in the pith of a specimen of *Cycadeoidea Mantelli,* Solms, from the Isle of Wight, of which he was able to examine a small fragment, innumerable little peridermal rings were found, each with a gum canal in the centre. *Cycadeoidea Ferrettiana* and *C. Capelliniana* offered a more abundant rhytidome formation in the interior of the trunk. The transverse section of the pith of the first shewed

[1] *Q. J. Geol. Soc.* Vol. LIII. 1897, p. 26.

[2] *Fossil Plants,* Seward, 1898, fig. 41 D, p. 214.

[3] Solms-Laubach and Capellini, "I Tronchi di Bennettiteae dei Musei Italiani," *Mem. d. Real. Acc. d. Sci. d. Instit. d. Bologna*, Ser. V. Tom. II. 1892.

[4] *Trans. Linn. Soc.* Vol. XXVI. 1870, Plate LXII. fig. 2.

a number of irregular crooked lines, meeting each other at acute angles, and having the characters of periderm, but apparently crossing and surrounding healthy tissue. Formations of the same sort were seen in the secondary phloem and cortex. *C. Capelliniana* shewed similar anomalous cork, but less well developed. A figure is given of a transverse section of this species with a line of periderm running approximately parallel to the stem surface, partly in the cortex and partly in the phloem, and also a high power drawing of part of the pith of *Cycadeoidea Cocchiana*, Car., shewing a band of periderm. The latter was described by Caruel under the name of *Raumeria Cocchiana*, but he did not recognise the nature of this formation, only speaking of it as a zone of tissue in the pith whose cells were arranged in rows. Caruel's specimen was re-examined by Solms-Laubach, and the cutting of fresh sections revealed the existence of a cylindrical closed periderm.

In describing *Medullosa anglica* Scott[1] points out that from the first the periderm formation is very irregular, and that in one of the specimens where it is little developed "it already appears here and there in the form of loops or rings enclosing leaf-trace bundles, or portions of sclerenchyma."

As regards recent Cycads, Scott[2] speaks of the occurrence of anomalous periderm as "an abnormal condition which is often met with." But although from this it would appear to be familiar to workers in this field, I have only been able to find it described in one case, that of a *Stangeria paradoxa* stem investigated by Solms-Laubach[3]. Here a closed sheath of periderm of an irregular "sackform" occurred, which at one level extended so as to coincide with the stem surface, and diminished in diameter as it was traced up or down the stem. The trunk was rotten at the base, and one of the two growing points was dead, and the author suggests that the periderm formation was for the purpose of arresting the invasion of decay into the sound part of the stem.

Anomalous periderm formation is illustrated both in the stem and roots of the specimen of *Macrozamia heteromera* which forms the subject of these notes. If sections are cut of the remains of the adventitious roots whose bases are still attached to the stem, they are found, as would naturally be expected, to be the seat of considerable cork formation. The tissues are in a bad state of preservation, so that the structure is not altogether easy to make out. A second periderm, apparently also pericyclic in origin, has arisen inside the first in at least one of the roots, and has invaded

[1] *Phil. Trans. Roy. Soc.* Vol. 191, B, 1898, p. 97.

[2] *Studies in Fossil Botany*, 1900, p. 452.

[3] "Die Sprossfolge der Stangeria und die übrigen Cycadeen," *Bot. Zeit.* 1890, p. 214.

the central tissues. In the loose parenchymatous secondary xylem a number of structures occur, which at first sight appear in transverse section like concentric bundles; closer examination however shews that these are merely small islands of secondary xylem excised by the formation of an anomalous periderm. Such islands were seen in the remains of two roots of very different sizes.

In the stem anomalous periderm is also met with, especially in the lower part. On cut surfaces small oval, circular, or quite irregular areas, generally with a lighter coloured border, and a dark or mucilaginous interior, are seen in the cortex, in and between the vascular zones, and in the pith; those in the pith seemed to be confined to the peripheral part. (Fig. 6.) They are mostly 2 or 3 mms. across, but some are 0·5 to 1 cm., and one at the bottom of the pith is about 2 cms. long by 4 mms. wide, as seen on one of the cut surfaces formed by dividing the trunk in half longitudinally. When examined microscopically the pale border is found to be a band of cork consisting of very narrow thin-walled cells. A variable amount of phelloderm is cut off to the exterior. Scattered in the phelloderm and the parenchyma immediately outside it are a large number of cells, containing what appears to be an orange gum. Inside the periderm the walls and contents of the cells have commonly degenerated into a gummy substance, and the centre of the patch is sometimes occupied by a mass of starch grains. All the tissue inside the peridermal sheath is generally more or less in a state of disintegration and tends to fall out when sections are cut.

The only explanation yet given of these curious peridermal formations is that expressed by Scott[1] when he says, "It is in the excision of effete tissues that we must seek the significance of these anomalous developments."

(viii) *The Leaves.*

Close to the apex of the stem a very young leaf was found, whose length, including the petiole, was less than 1 cm. The size of the rudimentary pinnae could clearly be seen to diminish from above downwards, so that their development is basipetal, as already described by Bower[2] for the case of *Macrozamia Miquelii*. In another leaf, 3 cms. long, the pinnae which at this stage shew the double forking characteristic of the mature leaf, were lying flat against one another, and there was no sign of circinnate vernation. The bases of the leaves are covered with hairs of the type

[1] "Medullosa anglica," *Phil. Trans. Roy. Soc.* Vol. 191, B, 1899, p. 97.
[2] "The Comparative Morphology of the Leaf in the Vascular Cryptogams and Gymnosperms," *Phil. Trans. Roy. Soc.* 1884, p. 592.

so characteristic of recent Cycads. These consist of a short, thick-walled, basal cell, and a very long, thinner-walled, terminal cell, the latter containing a single nucleus (Fig. 1). These filaments have a golden sheen and are sinuous, and adjacent ones take on parallel curves; the result is that leaf-bases covered with them look as if they were clothed with "crimped" hair. In transverse sections of the petiole of the leaf 3 cms. long these hairs are seen in their young state. The epidermis consists of columnar cells with rich contents and large nuclei. The hairs at this stage are only 151 to 200 micro-millimetres long, while when fully developed their length may be from 0·5 to 1 cm. or even more. The nuclei of the terminal cells of these young hairs are very large and conspicuous and occupy the whole breadth of the cell. In nearly all cases the nucleus is situated about half-way between the base and apex of this cell. At this young stage the hypodermal masses of sclerenchyma so conspicuous in the mature petiole are represented by groups of thin-walled cells, each of which is almost filled by its nucleus. They give the impression of having recently arisen by division. The vascular bundles are represented by strands of meristematic cells in which a certain amount of differentiation has taken place; in each bundle a little group of lignified elements (2 to 5, generally 4) occurs about in the middle of the strand, separated by several layers of cells from the periphery of the bundle on the one hand, and from the arc of phloem on the other. The latter is distinguished from the surrounding tissue by the large size and comparative emptiness of its cells. A transverse section of a pinna 2·8 mm. long from this leaf shews that the lamina at this stage consists entirely of undifferentiated thin-walled tissue, highly meristematic, and with large nuclei staining vividly with gentian violet. The position of the future vascular strands is just indicated by the manner of grouping of the cells.

The mature rachis shews a general similarity in structure to that of *Dioon edule.* Groups of small lignified cells occur under the epidermis. The ground tissue consists of cells which are round in transverse section and rectangular in longitudinal section, and of very various proportions, some being about twice as long as broad, whilst in others the breadth is the greatest dimension (Fig. 5). These cells present the striking peculiarity of being lignified and reticulately thickened or pitted. In transverse section they are seen to be separated by triangular intercellular spaces, and the parts of the cell wall bordering on the lacunae are unlignified. In describing the petioles of the different Cycads Vetters[1] mentions *Macrozamia* as particularly characterised by

[1] *Die Blattstiele der Cycadeen*, Leipzig, 1884, p. 10.

thick-walled lignified cells. The phloem is a thin-walled homogeneous tissue, and is thus markedly distinguished from that of the stem with its noticeable fibres. There is a good deal of centripetal xylem and a little centrifugal, the protoxylem is equally in contact with both. This is generally the case, too, in the pinna, but here it occasionally happens that the whole or a part of the centrifugal xylem is slightly separated from the protoxylem. It may be worth while to recall in this connection that, according to Scott[1], in the foliar organs of Cycads it is more usual to find the protoxylem elements in contact with the centripetal part of the wood, but that there are numerous exceptions to this rule. He mentions *Macrozamia heteromera*, amongst others, as a case in which the protoxylem of the pinna is equally in contact with both parts of the wood.

According to de Bary[2] mucilage canals do not enter the pinnae of Cycads except in *Dioon*, *Encephalartos*, and *Stangeria*; in *Encephalartos* they alternate with the veins. This forms an anatomical distinction between *Encephalartos* and *Macrozamia*, two genera which Worsdell[3] regards as shewing such close resemblance as to make it probable that they will at some future time be united into one. The distinction, however, is not so absolute as might appear from de Bary's statement, for as a rule in *Macrozamia heteromera* three bundles enter the pinna accompanied by a mucilage duct, which dies out before the leaflet divides into two, so that it only runs in the latter while it has the appearance of a branch of the rachis, and before it has really quite attained its flattened leaf-like character. Vetters[4] says that in *Macrozamia* a single bundle is given off into the pinna which divides in the base; but in our species the division appears to take place *before* the bundle enters the leaflet at all. Kraus[5] describes the peculiar pit-connections of the mesophyll of Cycadean leaves, and figures separate cells to shew the structure; de Bary[6] mentions that the pitted areas turn brown with Schultze's solution and deep red with solution of anilin. *Macrozamia heteromera* exhibits these pit-connections remarkably well. The spongy mesophyll cells are only in contact by comparatively small roundish areas, which are lignified and pitted (Fig. 2). By comparison of transverse and tangential sections of the leaf it is easy to see that these cells lie with their greatest diameter in a direction at right angles to the long axis of the leaf, and so form connections stretching between

[1] "The Anatomical Characters presented by the Peduncle of Cycadaceae," *Annals of Botany*, Vol. XI. 1897, p. 404.
[2] *Comparative Anatomy of Phanerogams and Ferns*, English Trans. 1884, p. 441.
[3] *Trans. Linn. Soc.* Vol. v. Part XIV. 1900, p. 452.
[4] *Loc. cit.* p. 13.
[5] "Ueber den Bau der Cycadeenfiedern," *Prings. Jahrb.* IV. p. 326.
[6] *Loc. cit.* p. 117.

the veins. It seems to me that in the existence of this tissue we have a confirmation of Worsdell's[1] view that the "accessory transfusion tissue" of the leaves of *Cycas* is a modification of the mesophyll tissue of the leaf, and not, as Lignier[2] supposes, the remnant of a lateral system of dichotomizing bundles such as that found in the leaf of *Stangeria*. Worsdell describes this tissue in *Cycas* as follows:—"Stretching amidst similarly elongated cells in the central region of the mesophyll of the leaf, from the midrib to the margin, is a band of conspicuous tracheides, narrow and elongated in shape and with bordered pits on their walls. These bordered pits are mostly of a rather rudimentary type, with wide slit-like pores, but in many of the tracheides nearest the vascular bundle they are more typical in appearance......They form a loose complex of tissue extending along the whole length of the leaf in a plane parallel to its two surfaces." In the mesophyll cells of *Macrozamia heteromera* lying between the upper and lower palisade layers, we may recognise a conducting tissue which, though far less well differentiated than the "accessory transfusion tissue" of *Cycas*, has accomplished something in the direction of such differentiation.

Summary.

The main points in the anatomy of *Macrozamia heteromera* brought out by this investigation are:—

1. The occurrence throughout a large part of the stem of a single extra zone of vascular tissue outside the normal one, and the existence of an increased number of such zones at the extreme base.

2. The occurrence of a well-developed system of medullary bundles.

3. The occurrence in both roots and stem of anomalous periderm formations.

In conclusion I wish to express my indebtedness to Mr Seward for providing the material, and for the advice and help which he has kindly given me throughout my work.

[1] "Transfusion Tissue," *Trans. Linn. Soc.* Vol. v. Part VIII. 1897, p. 308.
[2] *Bull. de la Soc. Linn. de Normandie*, Sér. IV. Tom. VI. fasc. 1.

DESCRIPTION OF PLATE ILLUSTRATING MISS A. ROBERTSON'S PAPER ON *MACROZAMIA HETEROMERA*.

FIG. 1. Hair from leaf-base. *e*, surface of epidermis; *n*, nucleus. (× 20 *circa.*)

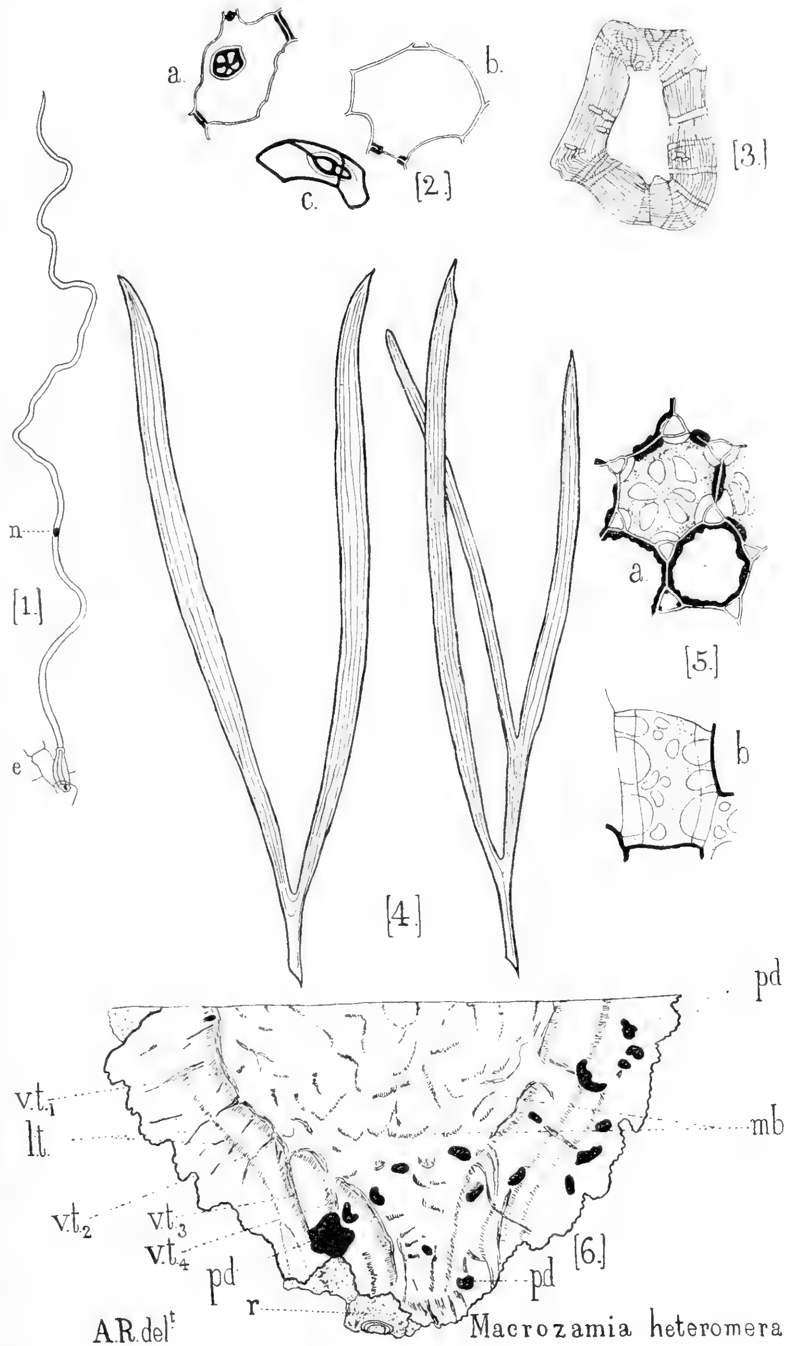
FIG. 2 *a*, *b*, *c*. Mesophyll cells of leaf to shew lignified and pitted connecting areas in surface and section; *c*, also shews the lignified framework which is exhibited by some of the cells. Drawn from transverse sections of pinnae stained with gentian violet and eosin in oil of cloves. (× 250 *circa.*)

FIG. 3. A fibre with thick, stratified, pitted walls, from the transverse section of a root, where it occurred between the primary and secondary phloem. (× 250 *circa.*)

FIG. 4. Two pinnae of the most characteristic forms (nat. size). (Simple and twice-forked pinnae also occur.)

FIG. 5 *a*, *b*. Lignified and pitted cells of ground tissue of leaf rachis; *a*, in transverse section, *b*, in longitudinal section. (× 250 *circa.*)

FIG. 6. Diagrammatic sketch of the surface exposed by cutting the base of the stem in half longitudinally. ($\frac{2}{3}$ nat. diam. *circa.*) *m. b.*, anastomosing system of medullary bundles and mucilage canals; $v.\ t_1$, $v.\ t_2$, $v.\ t_3$, and $v.\ t_4$, concentric vascular zones; *pd.*, patches of tissue excised by anomalous periderm, 18 of which are shewn; *l.t.*, leaf traces which by the growth of the stem have been gradually forced to point downwards.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Notes on the genus Liparis. By J. J. LISTER, M.A., St John's College.

[*Read* 27 October 1902.]

The degeneration of the alimentary system in two at least of the British representatives of this family of Moths, *Porthesia auriflua* and *Psilura monarcha*, was referred to, the maxillae being vestigial in character, not accurately opposable to form a proboscis, and shorter than the labial palps.

Sexual dimorphism, though not attaining the high degree found in other members of the family as in *Hypogymna dispar*, and especially in *Orgyia* (the Vapourers), is yet well marked in *P. auriflua*, secondary sexual characters being found in the "expansion" of the wings, colour, and the larger size of antennae and eyes in the male sex. A paired longitudinal groove on the sides of the 4th abdominal segment of the male is probably homologous with the much longer groove found in the males of many *Noctuae*, notably in *Xylophasia polyodon*, extending from the 2nd to the 4th abdominal segment and lodging a remarkable tuft of hairs, which springs from the front end of the groove and is protrusible;—an organ which appears to have been generally overlooked by entomologists.

In illustration of the high perceptive powers of the male of *auriflua* an instance was given in which three assembled in the neighbourhood of a female of this species, which had been dead for three days, within five minutes after the laying open of the store box in which the insect was pinned.

When fertilisation of the eggs has been accomplished, the insects rapidly die. Attention was drawn to the difference between the conspicuous satiny white colouring of the three species *Porthesia chrysorrhœa, P. auriflua* and *Liparis salicis* and the quiet buffs, browns and blacks of the other members of the family, conforming closely with their environment; and it was pointed out that there is a considerable body of evidence showing that the conspicuous species are noxious to other animals, both in the larval and adult state, by reason of the urticating properties of the hairs.

Further experiments on radio-activity from rain. By C. T. R. WILSON, M.A., F.R.S., Sidney Sussex College.

[*Read* 27 October 1902.]

In a paper read before this Society on May 5th, 1902, experiments were described which showed that a vessel, in which freshly fallen rain has been evaporated to dryness, shows radio-active properties lasting for a few hours only.

Many samples of freshly fallen rain have since that date been tested both here and at Peebles, and all have shown this effect. The magnitude of the effect obtained from a given quantity of rain has nearly always been of the same order, whether the rain has consisted of large or small drops, and whether it has been collected by day or by night, at the beginning of a shower or after some hours of continuous heavy rainfall. Once however during a thunderstorm an abnormally large effect was obtained.

The radio-activity is obtained equally well, whether the rain is boiled down in platinum or porcelain vessels. It is not destroyed by heating the vessel to dull redness; in this as in other points it resembles the induced radio-activity obtained on negatively charged wires.

A radio-active precipitate may be obtained from freshly-fallen rain by adding a little barium chloride and precipitating the barium with sulphuric acid; the barium sulphate is temporarily radio-active. A more convenient method, suggested to me by Dr A. Scott, is to add a small quantity of alum to the rain water and then precipitate the aluminium as hydrate by means of ammonia. The hydrate thus precipitated from a given volume of rain gives even larger effects than would have been obtained by evaporation of the rain, while the filtrate when boiled down to dryness now shows no trace of radio-activity. From 190 c.c. of rain a precipitate was in this way obtained sufficiently radio-active to increase the ionisation within the testing vessel to about 100 times its normal value; to enter the vessel the rays had to penetrate aluminium about ·00032 cm. in thickness. The intensity of the radio-activity falls to about $\frac{1}{4}$ of its initial value in an hour, like that obtained by evaporation. Similar precipitates formed in tap-water, or in rain-water that has stood for 24 hours, are quite inactive.

A case of extreme visceral dislocation: with remarks on the functional interpretation of the agminated glands of the intestine. By E. BARCLAY-SMITH, M.D., King's College; Senior Demonstrator in Anatomy.

[*Read* 27 October 1902.]

The case under notice is remarkable for the enormous displacement of some of the larger and more important abdominal viscera as a result of a distension of the proximal segment of the large intestine.

The ascending colon and the right end of the transverse colon were involved in an enormously distended coil which had insinuated itself upwards on the right side of the abdominal cavity, gaining an extensive area of contact with the diaphragm.

The viscera which had suffered the greatest displacement as the result of this colic enlargement were the right kidney, the liver, and the stomach.

The right kidney was displaced upwards. The displacement was not a very serious one, but was sufficient to place the right kidney at a level considerably higher than that of the left, and to impose an upward obliquity on the right renal vessels, *i.e.* an obliquity which is the converse of the normal disposition of these vessels.

The liver had undergone what at first sight seemed an almost complete transposition. When viewed from in front, a considerable portion of the right lobe was disposed to the left of the mesial line, and the fundus of the gall bladder had acquired a relation to the left infracostal margin, nearly complemental to its usual position on the right side of the body. A closer examination however elucidated the fact, that the hepatic displacement was not due so much to a direct transposition, as to an irregular torsion of such a kind as to effect a very considerable displacement leftwards of the fore-part of the liver, while the hinder-part had suffered in this respect to a very much lesser degree. In other words, the liver had undergone a considerable amount of distortion, the fore-part having felt the effect of the colic distension to a very much greater extent than the hinder.

The stomach was almost completely on the left side of the vertebral column, and this in spite of the fact that it exhibited a

considerable amount of abnormal enlargement. The pylorus was situated well on the left side of the vertebral column, and the proximal part of the duodenum was disposed almost vertically with a slight inclination leftwards, by which it attained its usual relations to the displaced gall bladder.

Such in brief were the main features of the case which has provided material for the following reflexions and speculations.

The renal displacement probably points to the history of the case. A ballooning of the first part of the large gut would exert a direct pressure on the kidney from below; the cæcum therefore was conceivably the first part to be affected, and the distension then gradually and progressively involved the successive segments of the gut.

The enormous displacement and severe distortion which the liver had undergone would at first sight seem incredible if the weight, bulk, and resistance of this organ are taken into account. But the plasticity of the solid viscera, and the extent to which they can be moulded by the hollow viscera are points which have been specially emphasised as the result of investigation by the method of hardening the viscera *in situ*, and have been brilliantly elucidated by His and Cunningham. The hepatic displacement and distortion may therefore be regarded as an exaggerated condition of the normal influence of the hepatic bend of the colon on the liver; room was not to be denied to the distending tube, and the liver retreated along the direct line of pressure.

Perhaps the greatest interest is attached to the position occupied by the stomach, a viscus which had also become abnormally enlarged, and which in the ordinary course of events would have overlapped its boundaries rightwards. The colon and stomach both being hollow viscera, the battle for increased accommodation would be fought apparently at equal odds. The result, however, whereby the stomach succumbed to the colic aggression and retreated towards its own side, only finding additional space by encroaching downwards, may be explained by assuming that the gastric distension was periodic, a condition of dilatation alternating with that of collapse, while the colon presumably was in a state of more or less constant dilatation, and gradually thrust its neighbour aside by more prolonged if not by more forcible pressure.

The colic distension exhibited by this case was obviously gaseous, the gut not being loaded with fæcal material, and as such must be regarded as a simple exaggeration of a condition which is almost universally met with in the dissecting and post mortem rooms. This condition is a more or less well marked dilatation of the proximal part of the large gut as compared with the distal. The cæcum, the ascending colon, and frequently the

commencement of the transverse colon are usually much more conspicuous objects when the abdominal cavity is opened than the descending colon, which is more often than not contracted up to a cord-like consistency. The cause of this distension is fairly obvious; gas is the distending agent, and the gas is the result of decomposition affecting the intestinal contents. Further, there can be little doubt that this gaseous decomposition is due to bacteria. A definite segment of the gut therefore presents definite anatomical features which are the result of microbic activity. It is obvious that there must be some peculiar conditions whereby the microbic activity with the resulting gaseous decomposition should preferentially take place in one segment of the gut on which it leaves such an indelible mark.

These peculiar conditions are open to the conceptions that this segment of the gut usually contains material suitable for bacterial decomposition, that by some means the stay of this material is prolonged and that the bacterial activity is to a certain extent localised, or at any rate, that indefinite bacterial invasion of the proximal part of the gut is prohibited.

The first condition is fulfilled by the nature of the intestinal contents which pass from the small intestine into the large through the ileocolic valve, and it is conceivable that if this material has not undergone sufficient change under the influence of the digestive fluids present in the small intestine, *i.e.* is in a more or less undigested condition, it will afford a more suitable medium for microbic activity, will favour a microbic increase and at the same time will provide a more abundant and a more suitable material for undergoing gaseous decomposition, thus bringing about conditions favouring undue distension of this part of the gut.

A stay of certain duration is essential in order that time may be afforded for the production of this gaseous decomposition. That there is a tendency to accumulation in the cæcum, whether in man or in any other animal provided with this appendage to the gut[1], is a well established fact, but as far as I know, no completely satisfactory explanation as to its causation has been adduced. The key to the matter is probably to be found in the results of a remarkable series of experiments made by W. B. Cannon, who directly studied the movements of the intestine in the cat by means of the Röntgen rays, the outlines of the intestine being brought into view by mixing subnitrate of bismuth with the food[2]. Cannon found that the movements of the proximal part of the large gut,

[1] In connection with this vide an interesting paper entitled "On some remarkable Digestive Adaptations in Diprodont Marsupials" by Dr Einar Lönnberg. *Proceedings of the Zoological Society of London*, 1902, Vol. I., Part I., p. 12.

[2] W. B. Cannon, "The movements of the intestines studied by means of the Röntgen rays," *The American Journal of Physiology*, Vol. VI., No. V., p. 251.

viz. of the ascending and transverse colons, were usually of a character quite distinct from those involving the small intestine and the distal part of the large gut. These movements were of the nature of an anti-peristalsis which exhibited a certain rhythm, and was remarkably constant. The effect of these anti-peristaltic waves was to cause an accumulation of the intestinal contents in the proximal part of the large intestine and a prolonged arrest of their progress.

This mechanism seems to be brought into play when the contents of the small intestine pass into the large, and become more pronounced as the accumulation increases. He states that the ileocolic valve is to be regarded as normally competent, and consequently the anti-peristaltic contraction does not bring about an escape of the contents of the large intestine into the small. Though Cannon seems to prove fairly conclusively that this anti-peristaltic wave is a constant and normal feature of the intestinal mechanism, he makes no suggestion as to its physiological meaning.

It probably, however, throws valuable light on the interpretation of the ileocolic valve and the mechanism whereby a well developed cæcum is of functional importance as an appendage to the gut. It has always been a problem why an enormously long cæcal appendage, such as is found in a typical herbivorous mammal, should be found distended with semi-digested contents, why the cæcal tube should apparently prove the path of least resistance, and why the intestinal contents should not take preferentially the continuous route *viâ* the large gut.

It is certainly open to surmise that the anti-peristaltic wave is a pronounced feature in the intestinal mechanism; that its object is to fill the cæcal diverticulum and that the ileocolic valve is an arrangement whereby the anti-peristalsis will not bring about regurgitation into the small intestine.

One feature of Cannon's results is at first sight remarkable, viz. that he should have found such clear evidence of this anti-peristaltic wave in the cat, an animal carnivorous in diet and with but slight cæcal development. This however may be a fulfilment of the general law that physiological function once established, it proves remarkably stable; in this respect presenting a great contrast to anatomical features which may be trimmed to an indefinite extent to meet the necessities of the race.

These premises being granted, the adaptation of an ileocolic valve, and the interpretation of a cæcal diverticulum now become intelligible.

The differentiation of a so-called large or distal from a small or proximal gut is established at a very early evolutionary stage, and this differentiation is usually in the form of a valvular arrangement.

In most fishes the large gut is short and straight, a typical rectum, which is marked off from the rest of the intestine by a valve; this valve with but few exceptions being circular in arrangement. The first part this valve is called upon to play is obviously to prevent regurgitation when the rectum becomes involved by a contraction which results in the expelling of its contents.

In the herbivorous reptile, the valve recedes from the anal orifice owing to the elongation of the large gut, and the presence of a cæcal diverticulum becomes established. The character of the diet is obviously the influence which accounts for these modifications, and as undigested material in the form of grass, leaves, &c. is often found in the colon, digestive changes, to a considerable extent at any rate, must take place in this part of the gut.

Of course it is possible that the mucous membrane lining this part of the gut is capable of furnishing a secretion which can deal with this vegetable material; but judging from the resistance displayed by vegetable material largely composed of cellulose and chlorophyll to the influence of the digestive fluids ordinarily furnished by the alimentary canal, this is hardly probable. Further, from the constancy with which micro-organisms are found in the large intestine as compared with the small, it is legitimate to surmise that they are to be found in the large intestine in these animals, and there play an important rôle in digestive processes. If this be true, a herbivorous reptile is incapable of furnishing digestive fluids which can satisfactorily deal with a considerable part of its vegetable diet, and has to rely, partially at all events, on the results brought about by parasitic micro-organisms.

It, as will be afterwards pointed out, seems to be an economical advantage that the activity of these micro-organisms should be confined as far as possible to the large gut, therefore an elongation of this part of the intestine in order to provide an increased absorptive surface for the results of the microbic digestion, and a cæcal appendage at its proximal end whereby a more or less stationary accumulation of the intestinal contents undergoing digestive processes is assured, become physiological necessities.

In order that the cæcum may be filled and the stay of its contents be prolonged, it is probable that the anti-peristaltic wave becomes simultaneously established. This anti-peristaltic wave is probably a simple modification of the rectal contraction whose rôle is the simple one of discharge in the fish. The presence of the circular valve is evidence that the contractile wave makes itself felt in opposite directions in such an animal. With the elongation of the rectum to provide a large intestine, it is conceivable that the contractile wave, to a certain extent, retains its primitive features, viz. starting in the middle of this

segment of the gut, it travels in opposite directions. The stimulus for such contractions may be provided by the entrance of the undigested material into the large gut from the small. After this material has undergone sufficient digestive changes, it may exert such an influence on the intestinal wall that contractions of another character make their appearance, contractions whereby the intestinal contents are propelled onwards and ultimately evacuated.

In birds the large intestine is remarkably short, a modification which would scarcely be expected. A cæcum which is in most cases double is however a constant feature, and it is most marked in the vegetable feeders. The reason for an abbreviated large intestine is not very obvious. It is probably a modification, which, together with the shortening of the whole intestinal canal met with in the bird, is correlated with the development of those accessory organs which are found at the proximal end of the digestive tube, and play such an important mechanical part in preparing the food for the action of the digestive fluids. Further, even in the vegetable feeders, the food is not of such an indigestible character as that of the typical herbivor whose diet consists largely of grasses, leaves, &c. Seeds forming the staple diet of such birds, once the cellulose coat is got rid of, and the substance of the seed particulated by the mechanical apparatus, a material is presented with which the normal digestive fluids are capable of dealing. Probably then bacterial digestion plays only a subsidiary part in such cases. In some birds, however, an apparatus for special bacterial digestion is a necessity owing to the special character of the food. The grouse is a case in point:—a yard long cæcum filled with homogeneous pultaceous matter, without any trace of the heather buds which are abundant in the ordinary tract of the intestine, affords an extremely suggestive example[1].

The arrangements in the typical mammalian herbivora are too well known to require notice, but it is in these animals that the most extreme forms of cæcal development and elongation of the large gut are to be met with, and that bacteria do play an important and essential part in the digestive processes of the herbivorous mammal is an accepted fact.

From this brief survey it is legitimate to conclude that the development of the gut on the distal side of the ileocolic valve, whether in the form of a diverticulum or in the extension of absorptive surface, is so universally correlated with the character of the diet that they must be special adaptations for digestion and absorption.

[1] Owen, *Comparative Anatomy of Vertebrates*, Vol. II., p. 171.

That profound changes do take place in the contents of the cæcum is a well-known fact, but little seems to be known as to the character of these changes and the agents by which they are brought about. As the large intestine seems to be the special home of the intestinal bacteria, it is not too much to assume that they are the principal digestive agents rather than a special intestinal secretion of which there seems to be no practical evidence.

If bacteria play an important rôle in the digestive processes of the herbivor, they have established a right to dwell as parasitic inhabitants of the intestine, and there seems at first sight no reason why they should not increase and multiply indefinitely, and why the whole length of the intestine should not become filled with a seething mass of bacterial colonies.

Obviously it is a physiological necessity, in the first place, that bacterial activity should be limited, and in the second place that it should be localised.

If the bacteria became too numerous and too active, the breaking-down processes to which the food is subjected as the result of their agency would become too profound, and no useful purpose would be filled by their presence. Further, if there was indefinite bacterial invasion of the small intestine, they would seize upon and convert to their own use the material which was the result of the normal digestive secretions. Being thus robbed of valuable digested material, the animal might starve to death, owing to the avidity of its guests.

The means whereby bacterial activity is limited and localised in the gut are probably two-fold. It is a well-known fact that certain of the digestive fluids are inimical to bacterial life, bile being a conspicuous example; further certain decomposition products resulting from the digestion of food, such as the fatty acids, must also have an important part to play in this respect. The bactericidal influence of the intestinal contents, however, cannot be the only means: if such were the case, the appearance of micro-organisms in the intestinal tube would be a gradual and a varying one; whereas they do not make their appearance in any considerable quantity till the ileocolic valve is passed.

The special restraining apparatus is probably furnished by the collections of adenoid tissue which are such remarkable features in certain parts of the intestine.

Superficial masses of adenoid tissue typified by the tonsils, have long been recognised as protective arrangements against bacterial invasion, the tonsillar apparatus being regarded as more especially protective to the air tube[1].

Agminated patches of adenoid tissue make their appearance

[1] Gulland, "Functions of Tonsils," *Edinburgh Medical Journal*, 1891.

very early in the evolutionary history of the intestine, being found in the reptiles[1].

It is a significant fact that in animals with well developed cæca, agminated masses of adenoid tissue, or adenoid tissue diffused throughout the mucosa, are nearly always a conspicuous feature of this part of the gut, being present to a very much greater extent than in any other part of the tube.

It is highly probable that these aggregations of adenoid tissue are specially concerned in restraining bacterial activity; that they are local collections of leucocytes whose especial rôle is phagocytic, and that there is a constant war to the death between them and the bacteria infesting the cæcum.

It is a fact, pregnant with meaning, that the adenoid tissue is usually most marked at the cæcal apex[2], viz. at the spot where the stay of the contents will be most prolonged.

The cœcal accumulations of adenoid tissue are then probably efficient in restraining undue bacterial increase, but they will afford little or no hindrance to the invasion of the small gut.

The ileocolic valve, though competent to hinder regurgitation, would obviously furnish no effectual bar to the passage of micro-organisms.

But again, the protective adenoid tissue makes its appearance in the form of those remarkable collections known as the agminated glands or Peyer's patches, which are such conspicuous features at the distal end of the small intestine, and concerning whose functional interpretation no suggestion, as far as I know, has yet been offered.

If there is any truth in the suggestions offered in this paper, they have an important practical application as regards the large intestine as found in man.

From the nature of his diet a reliance on extrinsic digestive aid such as is furnished by bacteria is no longer a physiological necessity. The statement is perhaps a bold one, but I am convinced that the large intestine is practically a useless encumbrance to him. From his herbivorous ancestors he has inherited not only a bacteria infested colon but a tendency to a cæcal diverticulum. Further, one great penalty he has had to pay for assuming the erect posture is to add yet another factor, viz. gravity, to favour the stay of the intestinal contents at the proximal end of the large gut. He has done his best, as it were, to adapt himself to these untoward circumstances. Though probably the tendency to cæcal diverticulation is yet strong within him, this appendage to the gut has dwindled as rapidly as possible to a vermiform appendix with

[1] Gegenbaur, *Vergleichende Anatomie der Wirbelthiere*, 1901, Vol. II., p. 415.
[2] Berry, *Journal of Anatomy and Physiology*, Vol. XXXV., p. 83.

a lumen so small as to practically forbid the entry of intestinal contents; further, its position has been shifted to such an extent as to occupy what may be regarded under the circumstances as an ideal situation, where it may feel the distending effect of the anti-peristaltic wave combined with gravity to the least extent possible.

The adenoid tissue in the intestinal wall has often been regarded as something vicious, something which is largely responsible for the many intestinal troubles to which man is a victim. This idea, I am certain, will be found to be absolutely erroneous. The adenoid accumulations in the intestinal wall are probably of the greatest protective value, and the special accumulation which is such a remarkable feature of the appendix vermiformis must be regarded simply as one of self-defence.

The fact that, when once the defensive energy of the adenoid tissue is strained to the breaking point either from an irresistible onslaught or from the weakening of its vital resistance, these accumulations prove weak spots in the armour of the first line of the body defence, does not invalidate the argument as to their essentially protective nature.

Notes on a Vibration Magnetometer and on the Ball-ended Magnets of Robison. By G. F. C. SEARLE, M.A., Peterhouse, University Lecturer in Physics, and Demonstrator in Experimental Physics.

[*Read* 10 *November* 1902.]

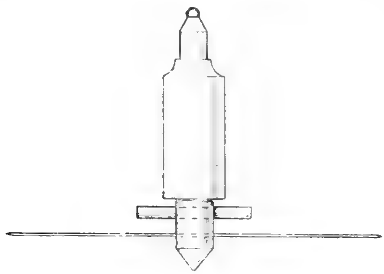
§ 1. The comparison of the horizontal components of magnetic fields by the method of vibrations forms one of the experiments in most elementary courses of practical physics, and presents no difficulty when each of the magnetic fields is so nearly uniform that a vibrating magnet several centimetres in length may be used. But when the fields are far from being uniform the vibrating magnet must be quite short. The magnet must also be slender, for unless its length be at least 10 times its diameter, the magnetic moment varies appreciably when the field varies, even though the maximum field is not greater than 3 or 4 times the earth's horizontal force.

But a simple magnet 1·50 cm. long and ·15 cm. in diameter has some disadvantages. Its small length makes it difficult to observe small arcs of vibration, though with care some accuracy in timing can be reached. Thus for such a magnet vibrating in the earth's magnetic field I found for the time of 50 vibrations 71·1, 71·4, 71·0, 71·5, 71·1, mean 71·22 secs., the initial arc being 10° on either side of the zero. Further, when such a magnet is suspended in a non-uniform field, it is drawn aside so that the suspending fibre is no longer vertical. But the resultant horizontal force exerted on the magnet changes as the angle between the magnet and the direction of the horizontal magnetic force varies, and hence, if the time of vibration of the magnet about a vertical axis be comparable with the time of the pendulous vibration of the magnet about the point from which the suspending fibre is hung, the vibration about the vertical axis will give rise to a pendulous vibration. In fact it is easy to arrange matters so that the motion of the magnet is alternately a pure vibration about a vertical axis and a pure pendulous oscillation. But, however interesting this motion may be to a spectator, it is useless as a means of determining the strength of the magnetic field.

§ 2. In arranging the experiments for the examination in Practical Physics in Part I. of the Natural Sciences Tripos, 1899, I endeavoured to provide the Candidates with vibration magneto-

meters free from the defects just mentioned, and, after several attempts, I designed the instruments described in this paper. The magnetometers have been in use in my class at the Cavendish Laboratory, Cambridge, since 1899, and have proved very efficient in the hands of my students. A description of them may therefore, perhaps, be welcome to other teachers.

The vibrating system is shewn drawn to scale in the figure.



The magnet is a cylinder, 1·50 cm. in length and ·14 cm. in diameter, of the "silver steel" used for making small tools, the steel being rendered "glass hard" by quenching at a bright red heat. The magnet is magnetised between the pole-pieces of a small electromagnet and its magnetic moment is about 3 C.G.S. units. In order to make a great increase in the time of vibration the magnet is attached to a brass plumb bob. The main part of the bob is 1·6 cm. in length and ·8 cm. in diameter, the total mass of the system being about 8·5 grms. The lower part of the bob is turned down to a smaller diameter, and into this part are fitted the magnet and a pointer of aluminium wire 5 cm. in length; the lower end terminates in a sharp point. The upper end of the bob is fitted with a small eye, made by soldering a loop of fine wire into a small hole drilled along the axis of the bob; to this eye the suspending fibre is attached.

If the system be carefully constructed and suspended, the centre of the magnet will lie very accurately in the vertical line through the point of the bob. This circumstance renders the magnetometer very suitable for experiments in which the distances are marked off on a sheet of paper fastened to a drawing-board, since the centre of the magnet can be placed very exactly over any given point. (See § 8 below.)

The bob is some 50 times as heavy as the magnet itself, and thus the system is not perceptibly drawn aside in a non-uniform field, unless the space-rate of variation of the field is very great. The system therefore vibrates quite steadily about a vertical axis without developing any pendulous motion.

The magnetometer is suspended by a fibre of unspun silk 10 to 15 cm. in length from a small stand furnished with a simple torsion head.

§ 3. By placing the magnet in a small coil and noting how the deflexion of a mirror magnetometer varied with the current flowing in the coil, I found that up to a field of 4·10 gauss the change of magnetic moment due to induced magnetisation was proportional to the applied magnetic force, and that it amounted to 1·11 per cent. per gauss. I also found that up to 4·10 gauss the magnetic moment was not permanently changed by the application of the magnetic force. Thus the change of magnetic moment due to the earth's horizontal force (·18 gauss) is ·20 per cent. These results agree roughly with those obtained by Mr T. Gray (A. Gray, *Absolute Measurements in El. and Mag.*, vol. II. part I. p. 89).

Thus, in comparing a field of about ·9 ($= 5 \times \cdot 18$) gauss with the earth's field, the error due to neglecting the induced magnetisation only amounts to $4 \times \cdot 20$ or ·80 per cent.

If the magnetic moment of the magnet when placed in a field of zero strength be M_0, then when it hangs in a field H its moment is $M_0(1 + \lambda H)$ where $\lambda = \cdot 0111$ in the experiment just described. Hence if T_E be the time of vibration under the earth's force H_E, we find that the magnetic moment in a field which gives the time T is approximately $M_0(1 + \lambda H_E T_E^2/T^2)$.

The aluminium pointer allows the time of vibration to be accurately observed even though the arc is quite small. Thus for 30 vibrations in the earth's field, the initial arc being 10° on either side of the zero, I found 189·6, 189·7, 190·0, 189·7 secs., the greatest divergence from the mean (189·8) being only about $\frac{1}{10}$th per cent.

§ 4. In accurate work a small correction must be made for the torsion of the suspending fibre. The magnetic north and south line is first marked out on a drawing-board by aid of a long, pivoted magnet, and the torsion head from which the bob is hung is so adjusted that the magnet and the aluminium pointer are parallel to the N—S line, when the system is at rest. The magnet is next turned through one complete revolution and is then set free, and is brought to rest. The magnet will no longer be parallel to the N—S line but will make an angle of ϕ *radians* with it; this angle can be measured by a small protractor. If μ be the torsional couple per radian of twist, M the magnetic moment of the magnet, and H_E the earth's horizontal force, then

$$\mu = MH_E \frac{\sin\phi}{2\pi - \phi} = fMH_E, \text{ say.}$$

Hence the restoring couple due to a small angle θ, when the magnet hangs in a field of strength H, is

$$(MH + \mu)\,\theta \quad \text{or} \quad M(H + fH_E)\,\theta,$$

so that the time of vibration is given by

$$T = 2\pi\left\{\frac{K}{M(H + fH_E)}\right\}^{\frac{1}{2}}.$$

Thus, to compare two fields, we have

$$\frac{H_1 + fH_E}{H_2 + fH_E} = \frac{T_2^2}{T_1^2}$$

provided M may be treated as constant. When $H_E = {\cdot}18$ gauss and ϕ is $5°$, $fH_E = {\cdot}00254$ gauss.

§ 5. The second part of this paper, now to follow, is written with the hope of reviving the use of the ball-ended magnets devised by John Robison, LL.D., F.R.S. Edin., Professor of Natural Philosophy at Edinburgh, from 1774 to 1805[1].

Robison gives a brief account of his new form of magnet in an article on "Magnetism" which he contributed to the Supplement to the third edition of the *Encyclopaedia Britannica*. The Supplement was published at Edinburgh in 1801. Robison says (p. 116):—"We got some magnets made, consisting of two balls connected by a slender rod. By a very particular mode of impregnation, we gave them a pretty good magnetism; and the force of each pole seemed to reside almost in the centre of the ball. This was our object in giving them this shape. It reduced the examination both of the attractive and of the directive power to a very easy computation. The result was, that the force of each pole varied in the inverse duplicate ratio of the distance. The error of this hypothesis in no case amounted to $\frac{1}{15}$th of the whole. In computing for the phenomena of the directive power the irregu-

[1] From the "biographical account" of John Robison, read before the Royal Society of Edinburgh by his successor, Professor John Playfair (*Phil. Trans. Roy. Soc. Edin.* Vol. VII. pp. 495—539), we learn that Robison was born near Glasgow in 1739. He was educated at the Grammar School of Glasgow, and after entering the University of Glasgow as a student of Humanity in 1750, he took his degree in Arts in April 1756. From 1759 to 1761 he served in the Royal Navy, and saw active service at the taking of Quebec. On the election of Dr Black to the Professorship of Chemistry at Edinburgh in 1766, Robison succeeded him as Lecturer on Chemistry at Glasgow, holding this post till 1769. In 1770 he went to Russia to assist the Empress in "placing her marine on the best footing," and in 1772 was appointed to "the mathematical chair attached to the Imperial Sea Cadet Corps of Nobles, at Cronstadt," being then proficient in the Russian language. He returned to Scotland in 1774 to succeed Dr Russell as Professor of Natural Philosophy at Edinburgh, and he held this post till his death in Jan. 1805.

larities and deviations from this ratio were much smaller." Robison again refers to the magnets on pp. 121 and 123 and mentions that "these experiments [with ball-ended magnets] were first made in a rough way in 1769 and 1770."

§ 6. Before I knew that ball-ended magnets had been devised by Robison I was led to the same design[1]. I argued that if a sufficiently easy inlet and outlet were provided by which the magnetic induction might flow into and out of the ends of the magnet, it would be content to flow in and out there, and would consequently not enter or leave the magnet at intermediate points. There would then be no free magnetism except at the ends of the magnet. I conceived that, if two spheres of iron or steel were fitted to the ends of a steel rod, the spheres, if their diameters are large compared with that of the rod, would provide the necessary easy inlet and outlet for the magnetic induction. I also hoped that the flow of induction would be nearly uniform over the surface of each sphere. In this case the magnet would act like an ideal magnet with its poles at the centres of the spheres.

The expectations which I had formed have been closely verified in practice, the students at the Cavendish Laboratory obtaining very definite results with the Robison magnets. The magnets are made by drilling a hole in each sphere and then driving it tightly on to the end of a thin steel rod. We have generally used the steel balls supplied by the Auto-Machinery Co., Coventry, softening them before drilling the holes. I have also tried balls turned out of mild steel of high permeability and I think that they give slightly better "poles" than the bought balls, which are of hard steel. But the advantage is very slight. We do not generally harden the rods; rods of "silver steel" as purchased, or knitting needles, retain their magnetisation quite well enough for most purposes. When put together, the system is magnetised in a long cylindrical coil by a powerful current. To concentrate the magnetism upon the balls as much as possible, a rod of mild steel is placed in the coil at each end of the system, a spherical hollow being turned out of the end of the rod in contact with the ball. To obtain a good magnetic contact, the radius of the hollow is made equal to that of the spheres. A somewhat similar plan is described by Sir W. Snow Harris in his *Magnetism*, ed. 2, 1872, p. 109.

§ 7. The magnets produced in this way behave very nearly as if they had ideal poles at the centres of the spheres. As an illustration, I give some results obtained in May, 1899. A magnet formed of a steel knitting needle ·15 cm. in diameter, with balls

[1] Natural Sciences Tripos, 1899, Part I., Practical Physics, question 16.

$\frac{5}{16}$ inch (·79 cm.) in diameter, their centres being 10·41 cm. apart, was placed in the "end on" position near a tangent magnetometer. Denoting the distances (in centimetres) of the centres of the two balls from the centre of the magnetometer by r_1 and r_2, the results were as follows:

r_1	8·68	10·68	12·68	14·68
r_2	19·09	21·09	23·09	25·09
$\tan\theta$	·8079	·5008	·3365	·2379
$\tan\theta\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)^{-1}$	77·4	77·6	77·4	77·8

The last line gives the values of m/H_E, calculated on the assumption that the magnet acts as if it had two poles of strength m, situated at the centres of the balls. The close agreement between the numbers in the last line shews that the assumption is very nearly exact.

Magnets such as this, having a pole-strength of from 10 to 15 C.G.S. units, are very useful for experiments in which lines of force are traced out on a drawing-board by the aid of a search compass.

An alternative method of testing these magnets is to fit a small coil to the rod and to slide it from point to point along the rod, and to note the "throws" of a ballistic galvanometer connected in series with the coil. Such a test on a magnet made of a hardened steel wire ·19 cm. in diameter and about 60 cm. in length, fitted with mild steel spheres 1·12 cm. in diameter, shewed that nine-tenths of the magnetic induction entered and left the magnet by the spheres, and that two-thirds of the remaining tenth entered and left the wire within 2 cm. of the spheres. A further experiment, made by jerking a small coil away from various parts of the surface of one of the balls, shewed that the flow of induction from the surface was uniform to about 25 per cent.[1]

§ 8. By using one of the vibration magnetometers described in § 2, in conjunction with a Robison magnet, students are able to test the law of the inverse square by Coulomb's vibration method with some accuracy.

The experiment is arranged in the following manner:—On a

[1] A qualitative test of the ball-ended magnets was demonstrated to the Society by optically projecting on a screen the forms taken up by iron filings near a plain bar magnet and near a magnet formed of an exactly similar bar fitted with balls. The lines of force indicated very clearly that the Robison magnet had by far the better "poles."

sheet of paper, fixed to a drawing-board held firm by *lead* weights, the magnetic north and south line OA is drawn and distances of 10, 12, 14... cm. from O are marked off along OA. The magnetometer is then adjusted so that the point of its bob is accurately above O, and the time of vibration under the earth's force is determined. The Robison magnet is then placed with its lower pole on one of the points marked off on OA, and is held, with its axis vertical, by means of a clip. If the magnet be placed to the south of the magnetometer, its north pole should rest on the paper, so that the magnetic force due to the magnet may be in the same direction as the earth's force. Two or three observations of the time of 30 vibrations are then made for each position of the magnet. If F be the horizontal component of the magnetic force due to the magnet, the total horizontal force is $F + H_E$, and this is compared with H_E by the method of § 4. If the centre of the lower sphere be on the same level as the magnetometer magnet, and if the distance between the centres of the spheres of the Robison magnet be h, then, when the lower pole is at a distance r from the magnetometer, the pole strength is given by

$$m = r^2 F \left\{1 - \frac{r^3}{(r^2 + h^2)^{\frac{3}{2}}}\right\}^{-1}.$$

For the following detailed results of an experiment of this kind I am indebted to Miss J. M. W. Slater.

$H_E = \cdot 176$ gauss. $T_E = 6\cdot 127$ secs. $h = 30\cdot 5$ cm.

r ...	10	12	14	16	20	24 cm.
T ...	3·892	4·335	4·695	4·965	5·360	5·600 secs.
F ...	·261	·176	·124	·092	·055	·035 gauss.
m ...	26·9	26·6	26·3	26·2	26·4	26·4 C.G.S.

The close agreement among these values of m shews that with careful observations this form of Coulomb's experiment leads to a very satisfactory confirmation of the law of the inverse square.

The magnet used in these experiments had a rod of "silver steel" ·23 cm. in diameter and was fitted with steel spheres $\frac{1}{2}$ inch (1·27 cm.) in diameter. I find that magnets of this size are convenient for this experiment.

In conclusion I have to thank Mr W. G. Pye and Mr F. Lincoln, the instrument makers at the Cavendish Laboratory, for their care in the construction of the vibration magnetometers and of the Robison magnets, as well as for some useful suggestions.

On Cavitation in Liquids, and its Occurrence in Lubrication. By S. SKINNER, M.A., Christ's College.

[*Read* 10 November 1902.]

If water is run into the space between two lenses, arranged so as to shew Newton's rings, and if one of the lenses is rolled on the other a crescent-shaped cavity is developed when the velocity of rolling exceeds a certain critical value. The cavity fills as soon as the rolling ceases. With more viscous liquids, such as lubricating oils or glycerine, the formation of the cavity is more marked. With colourless liquids the production of the cavity is observed by taking advantage of total internal reflection, or by using sodium light and observing the Newtonian rings formed in the cavity. With deeply coloured liquids the effect may be observed by transmitted light. Instantaneous photographs have been obtained of the effects with lenses rolling on planes, cylinders on planes, lenses sliding on planes, and in some other cases. The effects are shewn to agree with Osborne Reynolds' theory of the viscous origin of friction when copiously lubricated surfaces move over one another (*Phil. Trans.* A, 1886). That the maximum negative pressure occurs at some distance from the point of nearest approach is confirmed by these observations, and it appears that the layer of lubricant which separates the surfaces at the point of nearest approach is thinner than the wave-length of sodium light. Cavities of the same character probably occur in all sufficiently lubricated bearings.

[*Note added* 20 *December*, 1902. When the above paper was read before the Society I had only consulted certain reprints of Newton's *Optics*, and I was surprised that there was no mention of the formation of the cavity in the experiment which he made with water between the lenses. I now find that in Observation 11 he did notice the cavitation of the water but he did not go further with the observation. He supposes the cavity to be filled with a subtile medium which passes through the glass when the water creeps in. The following is quoted from the Third Edition, published in 1721, of Newton's *Opticks*, Second Book, part I. page 183.

"Obs. 11. When the Water was between the Glaſſes, if I preſſed the upper Glaſs variouſly at its edges to make the Rings move nimbly from one place to another, a little white Spot would immediately follow the center of them, which upon creeping in of the ambient Water into that place would preſently vaniſh. Its appearance was ſuch as interjacent Air would have cauſed, and it exhibited the ſame Colours. But it was not Air, for where any Bubbles of Air were in the Water they would not vaniſh. The Reflexion muſt have rather been cauſed by a ſubtiler Medium, which could recede through the Glaſſes at the creeping in of the Water."]

The Coral Reefs of Pemba Island and of the East African Mainland. By Cyril Crossland, B.A., Clare College.

[*Read* 10 November 1902.]

The following observations form a continuation of my paper on the Coral Reefs of Zanzibar[1]. It is only necessary, therefore, to draw comparisons between these reefs and those of Zanzibar.

Pemba Island[2].

Physical conditions.

The tides rise 12 ft. at spring, 8 ft. at neap, being thus considerably less than those of Zanzibar. The northerly current is slightly less strong, varying from one to three knots in the S.W. monsoon up to two knots in the N.E. Tidal currents are of course very strong in the long bays of the west coast.

The seasons are the same as in Zanzibar, but the rainfall is much heavier. This fact and the abundance of soil has made Pemba the Emerald Isle of the Arabs, to whom it is known as El Huthera, The Green.

Structure of the Island.

Pemba lies between latitudes 4° 50′ S. and 5° 30′ S., the line of longitude 39° 45′ E. almost bisecting it longitudinally. It is thus about 40 miles long with an extreme breadth between the reefs of the west coast and those of the east of sixteen miles.

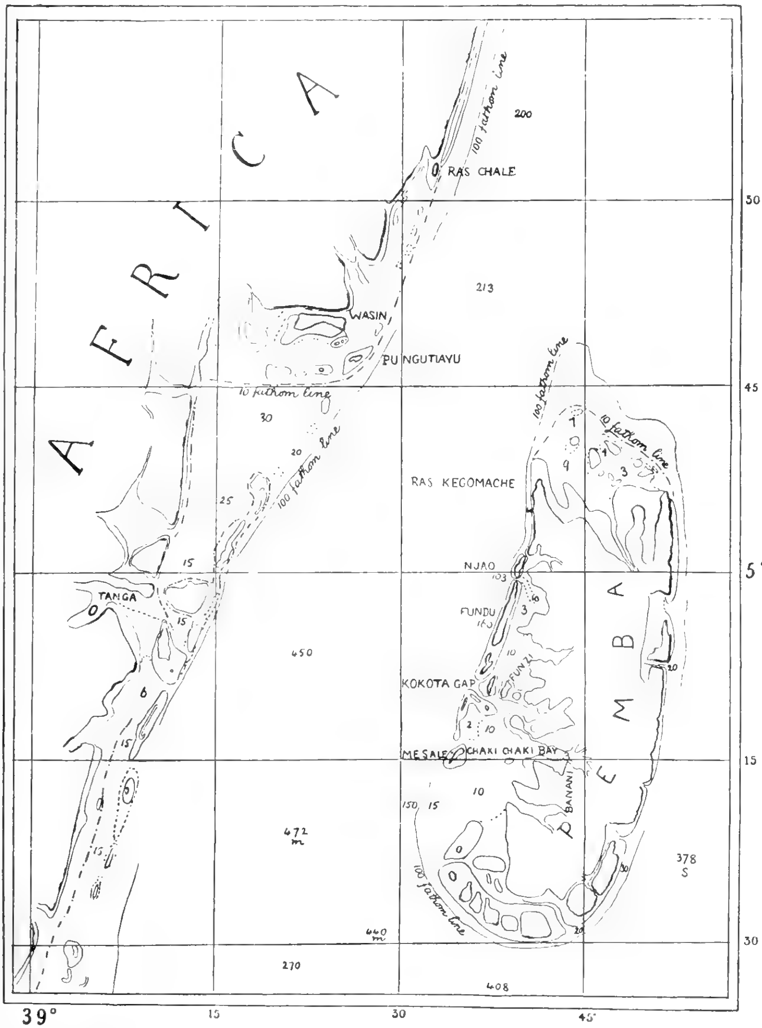
The results of wave and current erosion on the original mass of rock and sand are much the same as in the case of Zanzibar, viz. a regular and comparatively slight erosion of the more uniformly hard rock of the east coast and a deeper and very irregular irruption into the west, which is formed of materials of varying hardness.

The materials of the island are the same as those of Zanzibar,

[1] *Camb. Phil. Proc.* xi. 493.

[2] For my opportunity of visiting Pemba I am indebted to the Zanzibar Government, especially to Mr Bomangi, Minister of Public Works, whom I accompanied on a visit to the lighthouse in course of erection on Ras Kegomache, the northernmost point of the Island. I owe my exploration of Chaki Chaki Bay, Mesale Islet, and the East coast to the hospitality of the Missionaries of the Society of Friends of Banani.

but the coral rock is confined more to the edges of the island than in the latter case, appearing on the surface only as a narrow belt along the east coast, and forming the outermost islets of the west



Immediately within this barrier the surface is of coherent sand, beneath which is an extremely soft sandstone[1] lying above the coral

[1] This contains rounded pebbles of quartz smaller and less coloured than those occurring in Zanzibar.

rock. For instance, Mesale, Kokota, and Fundu Islets are all of coral rock, while Funzi and the long promontories of the main island are of sand with a limestone foundation. This difference of material is rendered conspicuous not only by the differing shapes of the islands, but by the characters of the vegetations they support, the outer flat coral islets, *e.g.* Kokota, being clothed with dense bush, the higher rounded sandy islets, *e.g.* Funzi, having open spaces of grass and usually wild palm trees[1]. As one proceeds up Chaki Chaki Bay the level of the coral rock gets lower and lower, disappearing below low tide level just beyond Banani.

Pemba and Zanzibar are thus somewhat similar and similarly situated figures, with this difference, that the channel between Pemba and the mainland averages a depth of 400 fathoms and is entirely free from shoals, other than the mainland barrier referred to below. The hundred-fathom line closely encloses the island at a distance of from two-thirds to one mile on all sides except the north, where its distance from the land is from seven to eleven and a half miles, or from the outlying reefs from three to seven and a half.

Pemba therefore is an independent formation, in contrast to Zanzibar, which is an enlarged portion of the barrier reef of the mainland coast.

The Reef of the East Coast.

The east coast differs from that of Zanzibar in the rarity of sand beaches, the lines of cliff being almost continuous.

The reef differs in

(1) its narrowness, its breadth varying from one-sixth or less up to two-thirds of a mile, as against one or two miles in the case of Zanzibar;

(2) there is no boat channel;

(3) the reef edge is covered with organisms of genera absent from the corresponding position in Zanzibar.

The characters of the surface rock of the reef, and the gravel and coarse sand matted together by the roots of *Zostera*, the slope of the outer edge, and so on, are exactly as seen in Zanzibar. The boat channel, however, is rudimentary, being represented by numerous pools a foot or two deep. The straight outline of the edge and the uninterrupted surface of the outer slope are alone sufficient proof that the reef did not originate through the growth of organisms *in situ*. The stunted corals, *Alcyonaria* (including *Tubipora*) and abundant foliaceous and encrusting nullipores, and

[1] A similar difference is noted by Gardiner as rendering conspicuous the distinction between the limestone and volcanic islands of the Fiji Group. On Funafuti, Rotuma, and Fiji, *Proc. Camb. Phil. Soc.* Vol. IX, Pt VIII.

the seaweeds *Halimeda* and *Caulerpa*, which entirely cover the surface of the outer slope, do no more than protect the underlying rock. This protection must be, however, quite complete.

All these differences indicate that this reef is at an earlier stage of formation than is that of Zanzibar. Whether younger in point of time, or merely because erosion is retarded by the greater hardness or more perfect homogeneity of the Pemba rock, is unknown. The latter explanation is suggested by the almost continuous line of cliffs which forms the east coast-line.

In any case, as erosion proceeds, the state of things found in Zanzibar will tend to be reproduced. As the reef widens by the cutting down of the land, and by the formation of a boat channel, a large amount of *débris* will come to be carried out by each tide. The pollution of the water by this and by organic impurities produced in consequence of the exposure of a broad reef to the sun at low tide, will kill off all the organisms of the reef edge except those which we find existing under these conditions in Zanzibar.

This latter flora can hardly be so completely protective as are the corals and nullipores of Pemba. It seems probable if the edge of the Zanzibar reef had depended for protection upon its present flora during the whole of its existence that it might not have so well resisted the eroding force to which it has been exposed.

In writing of the reefs of Zanzibar I cited the Bermudas as an instance of results of erosion simulating the results of the growth of reefs. The west coast of Pemba affords a much more direct comparison, the bays included within the islands and reefs of the Mesale-Kokota-Fundu-Njao barrier being entirely homologous with the lagoon of the Bermuda Pseud-atoll. The reduction of these islands to reefs (a process already far advanced between Mesale and Kokota) and the enlarging of the bays, converting the present barrier reef into the edge of an atoll, may in time cause the map of Pemba to bear a strong resemblance to that of the Bermudas.

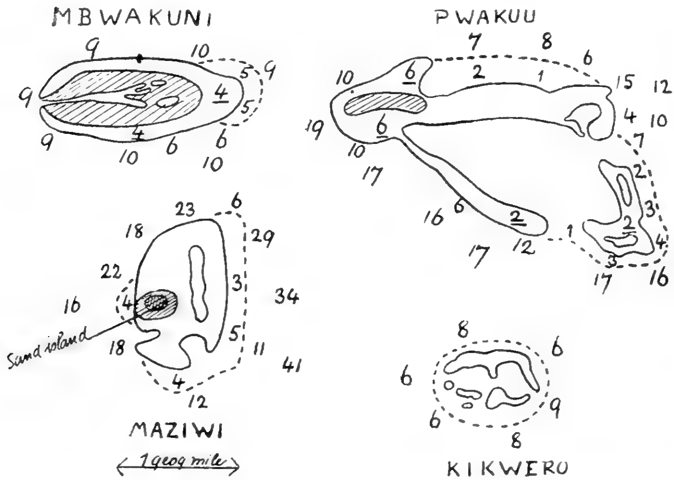
The Mainland Reefs.

These appear to fall into two groups which exist together, one fringing, the other forming a barrier. Both are usually somewhat irregular. The latter appears as a series of more or less elongated reefs of old coral rock formed by the partial or complete cutting down of islands to below tide level as described before in the case of reefs and islands near Zanzibar.

In the Zanzibar channel the barrier system is not traceable, though distinct in the Pemba channel. This anomaly is

explained by the fact that Zanzibar is itself a part of the barrier system, whereas Pemba is independent of the mainland formations.

The forms assumed by portions of the barrier are interesting. In several cases an approximation to an atoll is found, as in the following sketches of reefs from Zanzibar channel. The fact that



solution alone has formed these rings from originally plane areas supports strongly Gardiner's theory that it is to solution that atolls owe their lagoons.

In two places, viz. to the north of Mombasa, and at Chale point, near Wasin, the barrier for many miles becomes nearly as straight, narrow and uninterrupted as an artificial breakwater, enclosing an equally regular channel two or three fathoms deep at low tide and about one-third of a mile wide. I had no opportunity of visiting Chale, but I examined the exactly similar reef situated immediately to the north of Mombasa harbour. Its surface, alike on both sides, is very regularly rounded, loose stones and hummocks continuous with the rock beneath occurring rarely. Coral is entirely absent, and nullipores are almost so, the surface being covered by brown filamentous Phaeophyceae. The reef is thus, as in the other cases examined, formed entirely of dead rock, and probably at one time it extended above the level of high tide. The channel contains muddy water with blocks of living *Porites* at long intervals. The bottom is largely covered by a marine phanerogam which usually grows on rock.

Recent Coral Growths.

Cases of possible embryo reefs have been given in my account of Zanzibar. Two areas of luxuriantly growing coral remain to be described, viz. the west side of Ras[1] Kegomache, the northernmost point of Pemba, and the islets of Pungutiachi and Pungutiayu to the S.W. of Wasin.

Ras Kegomache, like all the west coast of Pemba, is extremely steep, 40 fathoms being found at a distance of a quarter of a mile. The steamer anchored very near to the reef edge in 15 fathoms. Even at this depth the bottom is dimly visible, in marked contrast to Zanzibar, where the impurity of the water hides everything below about five fathoms. The reef edge is a steep bank of growing coral. Its surface, level with the water at low spring tides, is one-third of a mile broad, without any channel, and composed for the greater part of recently dead and still growing corals, old rock appearing only near shore. A little to the south this coral bank encloses holes several fathoms deep, floored with white sand, looking blue through the clear water, or it breaks up into isolated mounds. The state of things is exactly as that described by Dana for coral growths in the sheltered water inside reefs. A typical coral reef is a very different thing to a mere mass of corals lying in mounds as here, where neither formation of reef conglomerate nor reef edge with its characteristic fissures occurs.

The rock flats of Pungutiayu and Pungutiachi Islets, which are exposed to the open ocean, are at a level of a few feet above low tide, their edges sloping down to the surface of its fringe a few yards wide of recent coral which is at this level. The edge of the reef is rather irregular, fissures appearing as the breakers recede, and pinnacles of coral occur just beyond it.

The importance of the proximity of clear and deep water to coral growth is strikingly exemplified in these two cases. In Zanzibar channel abundant coral growth extends only to a depth of five fathoms or less, which is about the depth to which one can see the bottom dimly. Only at Ras Kegomache is the bottom seen at a depth of fifteen fathoms (the limit of the visible zone about the Pacific atolls[2]), and here the coral extends, in greater luxuriance than I have seen elsewhere in East Africa, to a depth of at least ten fathoms, and in isolated clumps to fifteen, as evidenced by the occasional fouling of the steamer's anchor. Immediately to the north of Ras Kegomache but a few hundred yards away from

[1] Ras, Arabic = Cape or Point.
[2] J. Stanley Gardiner, *loc. cit.*

the deep water and the current everything is different. There is no trace of coral, nothing but weed and dead rock being seen through the clouded water.

The same fact is better illustrated at Pungutiayu, where I carefully examined and compared the depths to which corals extend on the north and south sides of the island. The latter is exposed to the open ocean, the north is separated from the islands of Pungutiachi by a channel two-thirds of a mile wide and nine fathoms deep. On this side the corals end quite definitely at a depth of two fathoms, below which the dredge travelled quite smoothly, bringing up sponges, soft alcyonarians (*e.g.* Xeniidae) and a little weed. The bottom is apparently of smooth rock without loose stones. On the S. and S.E. sides, on the other hand, coral growths extend to depths of from five to seven fathoms. Below this is again smooth rock. Still further out, in twenty fathoms, occur patches of gravel formed of much corroded coral fragments.

From this one concludes that around atoll reefs of the ocean coral growth would not only be much more luxuriant than near a continental area, but would extend to a greater depth.

General Conclusions concerning the whole coast of East Africa.

1. *There are no reefs due to growth in situ.*

This is shewn to be true not only by those places specially examined, but of this region generally, by the state of things seen where coral is growing most vigorously.

2. *There have been great coral reefs in geologically recent times, consisting of a fringing reef and a barrier in deep water, whose upheaval and consequent crystallization forms the rock of the whole coast and the outlying islands and reefs.*

3. *The islands of Pemba and Mnemba are formations independent of the above system.*

4. *That all the forms typical of growing coral reefs are formed by physical agencies operating upon this dead rock* aided by the protection against erosion afforded by organisms (principally algae, calcareous or not) which grow especially where the rock is exposed to the open sea.

Examples. Fringing reefs, east coasts of Zanzibar and Pemba Islands.

Barriers (*a*) off the mainland, broken as in Pemba channel, regular as at Chale and Mombasa; (*b*) off the west coast of Pemba.

Atoll formations in miniature near Zanzibar and in the mainland barrier. See also comparison of Pemba and the Bermudas above.

5. *That the luxuriance of coral growth, and especially the depth to which it extends, depend upon the depth of the adjacent sea and upon the purity of its water.*

6. *That recent growth cannot form typical reefs in these regions under present conditions,* since the surf is not strong enough to throw up sufficient coral fragments above the level of low tide for the formation of reef conglomerate. For the same reason *typical fissured reef-edges are not being formed.*

Note on spontaneous ionization in air at different temperatures and pressures. By J. PATTERSON, B.A., Professor of Physics, Muir Central College, Allahabad.

[*Read* 24 November 1902.]

The object of the experiment was to find if possible the cause of the so-called spontaneous ionization in air.

To measure the spontaneous ionization at different temperatures, the air was contained in an insulated iron cylinder containing about 13 litres, and the rate of leak was measured between the walls of the vessel and an insulated electrode. This electrode was connected to one pair of quadrants of a very delicate electrometer and the rate of leak observed.

The experiments shewed that from the temperature of the room (20° C.) to about 500° C. the current through the gas was constant, the air in the cylinder being at atmospheric pressure throughout the investigation.

To measure the ionization at different pressures the same cylinder and electrode were used. The joints were made air-tight with sealing-wax and the air filtered through glass wool.

The results shewed that down to a pressure of about $\frac{1}{3}$ atmos. the current through the gas was independent of the pressure and that for pressures below 90 mm. of mercury the ionization was proportional to the pressure.

Using the value 6×10^{-10} for e, the charge on an ion, the number of ions produced per c.c. per sec. was about 30.

The results of the experiments indicate that the "spontaneous ionization" is really due to easily absorbed radiation from the walls of the vessel.

Note on the behaviour of a Potassium Amalgam Cathode in a Vacuum Tube. By THEODORE LYMAN, Jefferson Physical Laboratory.

[*Read* 24 November 1902.]

The photo-electric behaviour of sodium or potassium amalgam and of the sodium-potassium alloy is well known, and has been investigated by Elster and Geitel, among others. The present note refers to the action of these same amalgams when used as cathodes in an exhausted discharge tube.

The first experiment was made with an amalgam containing one per cent. of potassium in mercury. The tube had the form of an inverted U. One of the lower arms contained about a centimetre of pure mercury, the other the same quantity of the amalgam under investigation. By means of suitable connecting-wires sealed into the tube, either the mercury or amalgam could be made the cathode. An aluminium plate 8 mm. in diameter was sealed into the bend of the U tube, and served as a permanent anode. The internal diameter of the tube was 1·2 cm., the distance of either cathode to the common anode was 15 cm. The current was supplied by a Wimshurst machine, driven by a small electric motor. The potential difference between anode and cathode was measured by a Kelvin electrostatic voltmeter. The limb of the tube containing the potassium amalgam was covered by several thicknesses of black paper to protect it from external illumination. The pressure in the tube was measured by a McLeod gauge.

The tube was filled with dry air and exhausted step by step. At each step the pressure was read. The difference of potential between the terminals of the tube was also taken, first with the mercury as cathode, second with the amalgam as cathode. The aluminium plate at the bend of the tube remained the anode throughout the experiment.

If V_1 be the potential difference with the mercury cathode and V_2 that with the amalgam cathode, then the observed facts are as follows:—At a pressure of about 1 mm. of mercury, V_1 was equal to V_2; as the pressure decreased V_1 remained nearly equal to V_2 until the region of ·2 to ·3 mm. pressure was reached, here V_1 began to increase much faster than V_2. At a pressure of ·08 mm. V_1 was twice as large as V_2, and at a pressure of ·06 mm. the ratio was 3 to 1. The experiment was repeated many times.

In order to study the phenomenon under conditions of constant current, the Wimshurst was replaced by a number of small storage cells, connected to the tube through a suitable resistance. The

U-tube was replaced by a cylindrical one, of 2·2 cm. internal diameter, and with a distance of 9·1 cm. between the anode and cathode. As before, an aluminium plate served as anode. The tube was held vertical with the cathode downward. A D'Arsonval galvanometer served to measure the current, the potential difference was read by a Kelvin voltmeter as before. The tube was filled with air, and mercury was run in at the bottom to form the cathode; with this arrangement the tube was exhausted step by step, and the potential difference and current were read at each step. The current was kept as nearly constant as possible. After the readings were completed the mercury was removed, the tube was cleaned, and the 1 per cent. potassium amalgam was introduced as cathode. Under these conditions a new set of observations were taken.

From these data a curve for a current of ·26 milli-ampères was drawn.

The results are similar to those obtained in the first experiment. V_1 does not differ greatly from V_2 down to a pressure of ·3 mm., but from here V_1 increases much faster than V_2. At a pressure of ·08 mm. the ratio of V_1 to V_2 is 20 to 7.

Though at first the potassium amalgam cathode was protected from illumination, this precaution proved unnecessary, for a series of experiments showed that exposure of the cathode to diffused light from an external source produced no effect on the result.

An alloy of sodium and potassium obtained by mixing the metals in the proportion of their combining weights gave results similar in character and magnitude to those obtained with the potassium amalgam.

A similar set of experiments were also undertaken with a cathode of 1 per cent. sodium amalgam. The general nature of the results was the same as with potassium, but not to such a marked degree.

The conclusion to be drawn from this investigation is that, with low pressures, the potential difference necessary to send a discharge through air is less when a potassium amalgam is used as cathode than when the cathode is of pure mercury. I am unable to decide whether the result is produced by the action of the light inside the tube upon the amalgam, and is a photo-active effect, or whether the electric force to which the amalgam is subjected is the cause of its action.

This experiment was carried out in the Cavendish Laboratory, and I wish to express my best thanks to Prof. Thomson for much kindly interest and advice during the progress of the work.

On rearing the later stages of Echinoid Larvae. By L. DONCASTER, B.A., King's College.

[*Read* 24 November 1902.]

In working at the development of Echinoids the chief difficulty which all observers have met with has been in rearing the larvae in the later stages, especially at the time immediately preceding the metamorphosis. Echinoids have been reared from the egg to the young urchin by Bury (*Q. J. M. S.*, Vol. XXXVIII. 1895), and Théel (*Act. Soc. Reg. Upsala*, 1892), and more recently by MacBride (*Journ. Mas. Biol. Ass.* VI. 1, 1900), and Grave (*Science*, Vol. XV. No. 378, 1902). Only the last two observers succeeded in obtaining any considerable number of specimens showing the later stages, and in both cases rather complicated methods were adopted. MacBride reared his larvae in very large jars, containing relatively few larvae, and fitted each vessel with a "plunger," in order to keep the water perpetually in motion, and further changed the water daily, and supplied the larvae with fresh water brought from a considerable distance from land. Grave succeeded in rearing the larvae without changing the water, by supplying them with abundant food by means of a species of diatom, but it is always possible that the particular diatom may not be obtainable, and that the culture may thereby be destroyed.

When working this spring at Echinoid hybrids at Naples I made efforts to rear the more advanced stages of the larvae, and to obtain young urchins, with the object of comparing the hybrids with the parent species. Attempts were made with *Sphaerechinus granularis*, *Strongylocentrotus lividus*, and *Echinus microtuberculatus*, and with several hybrids between these species. The larvae were kept in jars of four or five litres, covered to keep out dust, and it was found that they lived healthily until about the 8th day after fertilization when not supplied with fresh water or food of any kind. After that time, however, they became less healthy, ceased to develop further, and finally died off.

In order therefore to rear the larvae further, special methods were resorted to. Some larvae were put in an ordinary 4-litre jar, and others in a similar jar fitted with a plunger, and both were

supplied about five times a week with fresh seawater brought from some distance from the land. Under these circumstances the larvae developed much more successfully, but there was comparatively little difference between those with the plunger and those without. In some cases the plunger was absolutely harmful, and the larvae in the vessels to which it was attached died off very rapidly, but this was no doubt due to some impurity accidentally present, for in other cases the larvae flourished as well with the plunger as without it.

The results obtained differed very greatly according to the species; *Sphaerechinus granularis*, and hybrids with this species, never developed further than the stage reached about the 8th day; although they sometimes lived as long as three weeks. *Strongylocentrotus lividus* and *Echinus microtuberculatus*, on the other hand, and also the hybrids between these, continued to grow and develop. In the first lot, tried in March and April, the cultures were destroyed by using a dirty plunger when they had already reached an advanced stage, but in May and June others were started, in which many larvae survived until they became young urchins, and those which grew most rapidly were simply supplied with fresh water from a distance, without the use of a plunger or of specially large jars. The *Strongylocentrotus* larvae began to metamorphose after 34 days (May 14 to June 17), and the hybrids from *Echinus* eggs by *Strongylocentrotus* sperm reared at the same time produced urchins after 27 days. The *Echinus* larvae on this occasion reached an advanced stage, and then gradually died off, but another attempt yielded several young urchins after 32 days (June 23 to July 25). The cross *Strongylocentrotus* female by *Echinus* male is difficult to obtain, but on one occasion some healthy plutei were produced, which at first grew more rapidly than the converse cross, and afterwards died off.

In the hot weather it is necessary to cool the jars with a slow stream of running water, but no other precautions were taken, and most of the larvae seemed to grow quite healthily. It appears therefore that while with certain species very special precautions are necessary in order to rear the more advanced stages, in others, especially *Strongylocentrotus lividus*, these can be obtained in the spring and early summer with no great difficulty if the larvae are supplied with clean water containing food.

A detailed account of the later plutei and young urchins is outside the scope of this paper, since accurate accounts of their external features have been given by Bury, Théel, and MacBride in the papers referred to above, and no anatomical observations were made in this case. The plutei resembled those figured by MacBride very closely, and the young urchins were not very different from those figured by Bury and Théel. Some of the

Strongylocentrotus urchins lived for a month after the metamorphosis, and during that time the spines increased in length, the skeletal structures became more strongly developed, and five conspicuous Sphaeridia appeared, which were not present at first, but otherwise there was no great change. The hybrid urchins died a few days after the metamorphosis.

There are two points of some interest connected with these observations. The first is that the difficulty in rearing the later larvae of Echinoids varies very greatly according to the species, so that methods which are successful with one may be of no value when applied to others, and it is therefore important to anyone who proposes to work at the development of this group to choose a suitable species. The second point is that it is possible in certain cases to rear hybrid larvae beyond the metamorphosis, and so to obtain hybrid urchins, and this raises the further question of why such hybrids do not occur in nature. *Strongylocentrotus lividus* differs very greatly in the adult condition from *Echinus microtuberculatus,* although the very young urchins are very much alike, and it is possible that at the time when these differences would show themselves the young urchins would die off; and as a matter of fact, in the experiments described they only lived for a few days. Or it is possible that the two species normally shed their sexual cells at different times, but in a place like Naples, where both are exceedingly common, and where both are ripe at the same seasons, this by itself seems hardly a sufficient explanation.

Note on the Resolution of Compound Characters by Cross-breeding. By W. BATESON, M.A., F.R.S., St John's College.

[*Read* 24 November 1902.]

In the recently-published part of his *Mutationstheorie* (Lief. IV. pp. 194—206), Professor de Vries records evidence of a valuable kind respecting certain cases of resolution of compound characters, and offers an interpretation of the phenomena. On consideration it seems doubtful whether this interpretation is not unnecessarily complicated, and it is the object of the present note to suggest a simpler account.

It will be remembered that in a case of simple Mendelian inheritance the offspring of heterozygotes, AB, self-fertilised or bred *inter se*, consist of only three forms, viz. the two homozygous forms AA and BB, representing the pure parental types, and one heterozygous form AB repeating that of the first cross. In such a case we recognize that this result is due to the fact that the gametes of AB are each bearers of either the A or the B character in its entirety. In the constitution of these gametes the characters A and B are alternative or *allelomorphic* to each other.

But already many examples are known in which when two pure forms A and B are crossed, the gametes of the AB's are bearers not simply of characters A and B, but of a number of new characters formed, as we may conceive it, by the resolution of the compound character of one (perhaps both) of the original pure forms. Until lately we had no evidence as to the statistical relations of the several gametes in such a case of resolution. By the work of de Vries and of Tschermak[1] we are provided with some materials for calculating these relations. From the distribution of characters among the offspring in these cases, and from the statistical relations of the several zygotic forms to each other, it is clear that the resolution and redistribution of characters may take place in several different methods, but until the facts of the next generation (grand-children of the hybrid) shall have been

[1] Tschermak, *Ueber die gesetzmässige Gestaltungsweise der Mischlinge*, Ztschr. landw. Versuchswesen in Oesterr., 1902, Tab. II.—V.

more fully studied, we can scarcely appreciate the relation of these methods to each other. In one instance, however, that of de Vries' *Antirrhinum*, the evidence as to this generation (F_3 of my notation) is more complete, and we can attempt to realise what has taken place in that case.

In most of the instances hitherto studied the compound character is a colour-character[1], which is resolved after a cross with a white variety, and such an example is that of de Vries' *Antirrhinum*, which I propose to consider here. In this case a red *Antirrhinum* was crossed with a white, giving a first cross (generation F_1) red like the red parent, white being recessive. F_1 on self-fertilisation gave a somewhat miscellaneous offspring (generation F_2), which could nevertheless be classified with fair accuracy into four forms, which de Vries calls Red, Flesh colour, Delila[2], White. These four forms existed in numbers approaching the ratio 9 : 3 : 3 : 1, if the minor differences are neglected.

This ratio 9 : 3 : 3 : 1 suggests to de Vries that there is here a phenomenon of "Di-hybridisation," a term used to denote the case of a cross between parents differing in *two* pairs of allelomorphic characters, as, for example, between a *yellow round* pea and a *green wrinkled* pea. In this latter case, if we call the heterozygote (F_1) $YRGW$, then, as we know, the offspring of self-fertilisation will be in appearance

$$9\ YR : 3\ GR : 3\ YW : 1\ GW,$$

and in reality

The 9 $YR = 1\ YRYR + 2\ YRYW + 2\ YRGR + 4\ YRGW$
The 3 $GR = 1\ GRGR + 2\ GRGW$
The 3 $YW = 1\ YWYW + 2\ YWGW$
The 1 $GW = 1\ GWGW$.

In interpreting his case of resolution on the analogy of di-hybridisation, de Vries regards the red parent as consisting of a union of flesh colour, F, and Delila, D, corresponding to the two dominant characters, yellow and round, while he conceives the white parent as representing the *two* recessive characters, which he calls W and W', though it is not suggested that W differs from W'.

The first cross he represents as

$$\begin{array}{c} \text{White} \quad \times \text{Red} \\ W + W' \times F + D \\ \text{giving } W + W' + F + D, \text{ the red hybrid.} \end{array}$$

[1] Indications however are not wanting that similar phenomena of resolution may occur in regard to structural characters also.
[2] Pale tube with red lips.

The offspring of self-fertilisation he represents as

1 FD	appearing	Red
1 FW'	,,	Flesh colour
1 WD	,,	Delila
1 WW'	,,	White
2 $FD + FW'$	,,	Red
2 $WD + WW'$	,,	Delila
2 $FD + WD$	,,	Red
2 $FW' + WW'$	,,	Flesh colour
4 $FD + FW' + WD + WW'$	,,	Red.

It was found by experiment moreover that $F \times D$ gives the red again. Other experiments showed that one of the D plants produced only D, and one of the F plants produced only F offspring; that other D plants might give a mixture of D plants and white plants; and that other F plants might give a mixture of F plants and white plants. Finally some red plants gave reds and F plants, while others gave reds and D plants.

In de Vries' account it seems to me that great complication is introduced by the attempt to express the *unresolved* red as $F + D$; and the representation of white by $W + W'$ is still more open to objection, for here at least there can be no reason to suppose *two* characters concerned. In fact the white is only represented as $W + W'$ because it is *crossed* with a compound character.

These difficulties, and others to which they lead, do not yet seem inevitable, and one is tempted to look for a simple account. Assuming the distribution of offspring to follow the ratio $9\,R : 3\,F : 3\,D : 1\,W$, this result would equally be arrived at on the hypothesis that the hybrid of red × white, $R \times W$, produced *equal numbers of gametes of each sex having the characters R, F, D, W, respectively.* The fortuitous union of these would give $9\,R : 3\,F : 3\,D : 1\,W$ as before. The actual composition of the offspring would be as follows:—

The 9 $R = 1\,RR + 2\,RF + 2\,RD + 2\,RW + 2\,FD$

The 3 $F = 1\,FF + 2\,FW$

The 3 $D = 1\,DD + 2\,DW$

The 1 $W = 1\,WW$.

We know from the experiments that R is dominant to both F and D; that F and D are each dominant to W; and that $F \times D$ gives R.

On this hypothesis the serious theoretical difficulties inherent in the other view are avoided. Far more than a mere question of notation is involved. On de Vries' view the gametes must have the four forms FD, FW', WD, WW', so that the white character would be present in three out of the four gametes,

though in differing capacities. From the records of the results thus far there seems no good reason for introducing these difficulties, or for regarding the case as one of di-hybridisation.

In attempting to apply either hypothesis to de Vries' other cases or to Tschermak's results in *Phaseolus*, various complications are encountered, and it is clear that in those phenomena other processes are concerned. Until there is a fuller experimental basis discussion of these cases is scarcely possible. To this subject I hope to return when the experiments on resolution of characters undertaken by Miss Saunders and myself are further advanced. Nevertheless while our conceptions of allelomorphic characters, simple and compound, are in process of formation, it seemed desirable to point out how an unnecessary complication might be avoided.

From all that we know of these cases of resolution it seems fairly certain that the *whole* of a compound character need not be resolved on the first cross, but that the cross-bred may bear some gametes which transmit it in its entirety.

The visible phenomena of cell-division would suggest that the number of gametes bearing the simple character of one parent—here the white—should equal the *sum* of those representing the other parent and bearing the compound character and its various components. On the view here suggested, the number of gametes bearing the simple character is regarded as equal to the number of *only one* of these classes; and in the case under discussion the latter so far seems the more acceptable account.

We must, however, remember that in this case the *simple* character is *recessive*, the compound being dominant; and we naturally inquire if when a *dominant simple* character is crossed with a *recessive compound* character, the result would be the same. Whether the fact has a relation to this matter of dominance or not cannot be said, but in the only case I can adduce of a cross of the latter kind, namely that of a white and a coloured breed of poultry, there seems to be reason for the belief that the white gametes are as numerous as those of the various coloured ones collectively, for in the latter case the ratio 3 light to 1 dark (in generation F_2) seems distinctly indicated as the result of breeding the hybrids *inter se.*

In conclusion, since the resolution of a compound character may be spoken of as an *analysis* leading to a distribution of the components among the gametes, the term *synthesis* should surely be reserved for a recombination that has taken place in such a way that the *gametes* become bearers of the compound character again, as they were in the pure compound form. In the place referred to, de Vries, however, using "analysis" as above, speaks of the union of F and D gametes to form a red *zygote* as a "synthesis." But until we know in some given case that the zygote so produced

breeds true, and forms gametes bearing the synthesised character, the term synthesis seems inappropriate. If we use "analysis" for a distribution of component characters among gametes, "synthesis" should be reserved for a reconstitution of the compound character in the gametes.

In a sense, of course, the union of the gametes in the zygote leads to an apparent synthesis—$F \times D$, for instance, giving a zygote resembling the original R—but to use the term in that sense introduces great obscurity. Further difficulties would arise also from the fact that we already know cases where *various* pairs of dissimilar gametes lead to the formation of the *same* heterozygote form which may exactly reproduce the compound character of a pure type. Clearly there is no true synthesis here; for while the pure type could be resolved into its several components, each heterozygote would contain only the elements brought in by the particular gametes which produced it.

On the variation with the wave-length of the double refraction in strained glass (Second paper). By L. N. G. FILON, B.A., King's College.

[*Received* 20 December 1902.]

1. In a previous paper (*Proc. Camb. Phil. Soc.*, Vol. XI. Part VI.) an account was given of experiments on glass beams with the object of finding out how the double refraction induced by strain varied with the wave-length of the light. The experiments exhibited the variation continuously throughout the spectrum. They showed that, as a rough first approximation, Wertheim's law, that the double refraction exhibits *no dispersion*, is correct; a divergence was however shown to exist, the difference in the refractive indices being smaller in the red than in the violet, even for the comparatively light glass examined (density 2·50).

The accuracy attainable under the given conditions, however, was not sufficient to allow the law of this divergence to be exhibited.

It seemed necessary, therefore, to devise a totally new method of investigation, which should allow of greater precision. The experiments undertaken with this object are described in the following pages.

2. Light coming from a powerful arc lamp E (Fig. 1) was focused

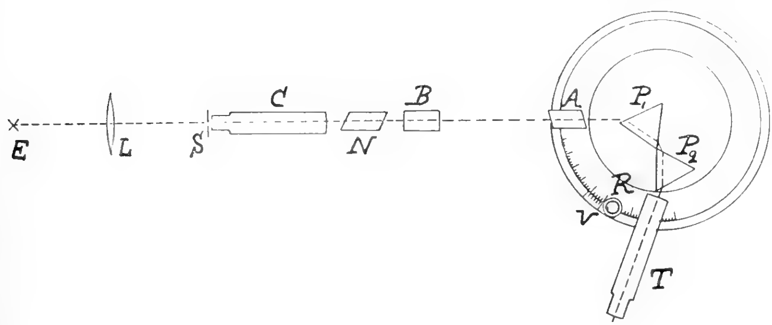


FIG. 1.

by a large lens L on to the slit S of a collimator C. The pencils of parallel rays, after emerging from the collimator, passed through a polarizing Nicol N, whose axes were inclined at 45° to the vertical and horizontal, then through a block of glass B, then through another Nicol A. The rays were then broken up by two prisms

P_1 and P_2, and the spectrum viewed in the telescope T of a spectroscope.

The block B was placed in a straining apparatus (Fig. 2) made

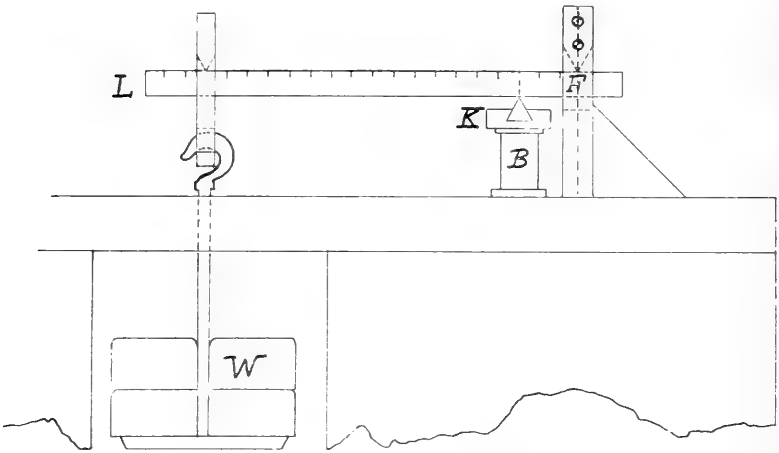


Fig. 2.

for the purpose by Messrs Harling of Moorgate Street. A steel lever L, the top surface of which was graduated, carried a weight W whose distance from the fulcrum F was adjustable, and rested on a knife-edge block K which in turn pressed on the glass block B. The horizontal distance between F and K was constant and equal to 3 cms., the knife edges fitting into fine notches cut into the steel.

The pressure P on the block B was then

$$P = \frac{Wx}{3},$$

x being the distance from F, in cms., of the knife-edge hanger which carried the weight.

The instrument was designed to carry safely a fairly large weight. The limiting values actually used were roughly

$$W = 40 \text{ lbs.}, \quad x = 45 \text{ cms.},$$

so that the pressure on the block was about 600 lbs.

In order to ensure the uniform distribution of this pressure throughout the block, which was too short in proportion to its length to make it safe to assume that the stress was uniform, even over the middle section, sheets of lead were introduced between the block B and the compressing blocks.

Even this, however, did not suffice to make the stress suf-

ficiently homogeneous. A rather delicate preliminary adjustment was therefore necessitated, the glass being shifted slightly several times until, on examining in the ordinary way between crossed Nicols, a fairly large region in the centre of the block showed a uniform tint when the block was loaded, *even when the tint of passage was reached.* The aperture of the pencil which was to be examined was then narrowed by means of a diaphragm so that the rays were restricted to pass through the region of the block where the tint had been found to be uniform. The straining apparatus was then left untouched, save for the putting on of the weights, and these were applied with great care, so as to avoid any jar.

Finally by placing the crossed Nicols horizontal and vertical and then loading the block, it was found that, over the region of uniform tint, the field remained very sensibly dark.

It was assumed therefore that throughout this region, the stress should be taken vertical and, at all events in planes perpendicular to the paths of the rays, uniform. It is easy to see that variations in intensity (but not in direction) of the stress *along* the paths of the rays will not affect the results, the total effect being the sum of the effects of elementary laminae perpendicular to the path of the light, and depending only upon the *mean* stress.

3. When the block B is unloaded, no light is visible in the telescope T if the Nicols A and N be crossed. But when the block B is under vertical pressure, the light breaks up into two pencils, one horizontally and the other vertically polarized, which suffer a difference of retardation in passing through the block B. The light is now no longer quenched and a spectrum appears in the field of T. This spectrum consisted of two continuous bands and a bright line spectrum in between, being in fact the spectra of the two carbons and of the arc.

The difference of retardation of the two rays passing through the block B is (see first paper *loc. cit.* p. 481)

$$r = C\tau T,$$

C being what I have called the stress-optical coefficient, τ the thickness of the glass, T the mean stress.

If a, b, c be the breadth, thickness and height of the block B,

$$\tau = b, \quad T = \frac{P}{ab},$$

$$\therefore r = \frac{CP}{a}.$$

Now when this difference of retardation is equal to an integral multiple of the wave-length, light of that particular wave-length is again quenched.

The spectrum should therefore be crossed by a dark band

corresponding to this particular wave-length. By sufficiently increasing the brightness of the light, this dark band might be made sufficiently sharply defined to allow of accurate measurement.

This was, as a matter of fact, found to be the case: no very definite advantage, however, was gained by increasing very greatly the brightness of the light, because it was not found possible to obtain perfect uniformity and perfect verticality of the stress in the block B, so that some small fraction of the light managed to get through, even when theoretically there should have been absolute extinction. Hence the band was not absolutely black and no increase in definition resulted from increasing the brightness of the light beyond a certain limit.

Nevertheless, well-defined dark bands were obtained and these allowed of being measured with considerable exactness. The spectroscope was provided with a vernier V and a reading lens R by means of which angles could be read up to $\frac{1}{2}'$ of arc. The probable error of the position of a band varied considerably according to the part of the spectrum observed. In the yellow and green parts of the spectrum, where the background was brighter and the lines more sharply defined, the probable error, as deduced from differences of readings, was under $1'$. In the blue and violet, however, where the bands were diffuse and the background faint, the readings were far more uncertain.

The probable error also varied considerably from day to day, as was natural. On four different days the following probable errors were calculated from differences of readings in *all* parts of the spectrum: $1'\cdot49$, $2'\cdot48$, $0'\cdot60$, $3'\cdot15$. The mean of these, $1'\cdot93$, may be taken as a fair representation of the average error. The range of angle being about $8°$, this corresponded to about 4 per 1000 of the range.

The weight could be found easily to the nearest gramme and this gave an accuracy of 1/10,000, or even greater accuracy, the weight used exceeding 10 kilogrammes in each case. Hence in calculating the value of C, the error of 4/1000 of the whole range (λ red $-$ λ violet) would amount to about 10 tenth-metres. This for a wave-length of 5000 would give an error of 1/500 in the wave-length of the light quenched, and since

$$n\lambda = \frac{CP}{a},$$

the error in C would also be 1/500. This is nearly 10 times the accuracy of the observations of the earlier experiments (*Camb. Phil. Proc., loc. cit.*).

4. In the present experiments no special comparison spectrum was used, the spectrum of the arc itself forming a most beautiful natural comparison spectrum, spread out like a luminous

scale alongside the continuous spectra of the carbons, in which the dark bands were observed.

The first step was to identify a large number of well-defined, easily recognisable, lines in the arc spectrum. This was done by comparison with the solar spectrum, from which a number of lines were first of all identified. A curve was then obtained, connecting wave-lengths with degrees read off on the spectroscope circle. By means of this curve the approximate wave-lengths of other lines were obtained and characteristic groups identified from Watts' and Rowland's lists of spectra. In this way a very great number of lines were found and a very accurate curve connecting the spectroscope readings with the wave-lengths deduced.

The spectroscope and all the arrangements described in Fig. 1 were, throughout all this latter part of the work and the experiments which followed, kept fixed. The block B had been put into place and the Nicols had been accurately crossed. The apparatus was then touched only for loading and unloading the straining apparatus; when it was desired to take an observation of the spectrum, the block B was slightly loaded, so as to let the light pass through. Thus the mapping of the spectrum was carried out with all the apparatus under precisely the same conditions as when the bands were observed.

The value of these precautions will become apparent when it is explained that the bright line spectrum was found inconvenient for *direct* reference, as it was not by any means so accurate to estimate the position of a diffuse black band with regard to two lines (which would often be inconveniently placed) in a bright line spectrum below it, as to place a cross-wire upon the middle of the band.

Accordingly, once the comparison spectrum had been mapped, the method adopted was simply to take the circle readings of the black bands and to deduce the corresponding wave-length from the curve of wave-lengths to deviations for the spectroscope. In order, however, to eliminate any small shifts due to temperature &c., which might occur from day to day, the circle readings of two well-defined lines (D and E of the solar spectrum) were taken on each occasion and the difference of the actual readings of these two lines from the readings corresponding to them in the initial diagram (which difference was in all cases small) was treated as an index error.

5. In every case, only the black bands of the *first* order, that is, those corresponding to a relative retardation of one whole wave-length, were observed. We had therefore the formula

$$\lambda = \frac{C}{a} P,$$

where $P = \frac{Wx + w\xi}{3} + w'$,

W = weight and its hanger,

x = distance from the fulcrum of knife-edge supporting weight,

w = weight of lever,

ξ = distance of C. of G. of lever from fulcrum,

w' = weight of knife-edge block.

For the instrument used

$$w' = 97 \text{ gms.}$$
$$w = 977 \text{ gms.}$$
$$\xi = 22{\cdot}75 \text{ cms.}$$

Hence

$$\lambda = \frac{C}{3a}(22{\cdot}52 + Wx),$$

W being in kilogrammes weight.

The block of glass was a piece cut from one of the beams ordered for the earlier experiments.

$$a = 1{\cdot}231 \text{ cms.}$$
$$b = 2{\cdot}91 \text{ cms.}$$
$$c = 3{\cdot}71 \text{ cms.}$$

the density being about 2·50.

The results of the experiments are shown in the tables below:

TABLE I.

July 8th. $x = 40$.

W	$22{\cdot}52 + Wx$	λ in tenth-metres	C
15·215	631·12	4,507	26·37
15·765	653·12	4,655	26·32
16·309	674·88	4,793	26·23
16·853	696·64	4,931	26·14
17·140	708·12	4,997	26·06
18·234	751·88	5,262	25·84
19·184	789·88	5,518	25·80
20·278	833·64	5,783	25·62
21·164	869·08	5,975	25·39
22·258	912·84	6,235	25·22
23·326	955·56	6,475	25·02

TABLE II.

July 9—11th. $x = 40$.

W	$22{\cdot}52 + Wx$	λ in tenth-metres	C
15·325	635·52	4,576	26·59
15·875	657·52	4,721	26·52
16·419	679·28	4,847	26·35
17·250	712·52	5,045	26·15
18·344	756·28	5,316	25·96
19·294	794·28	5,553	25·82
20·388	838·04	5,816	25·63
21·274	873·48	6,026	25·48
22·368	917·24	6,280	25·29
23·436	959·96	6,520	25·08

TABLE III.

July 11th. $x = 45$.

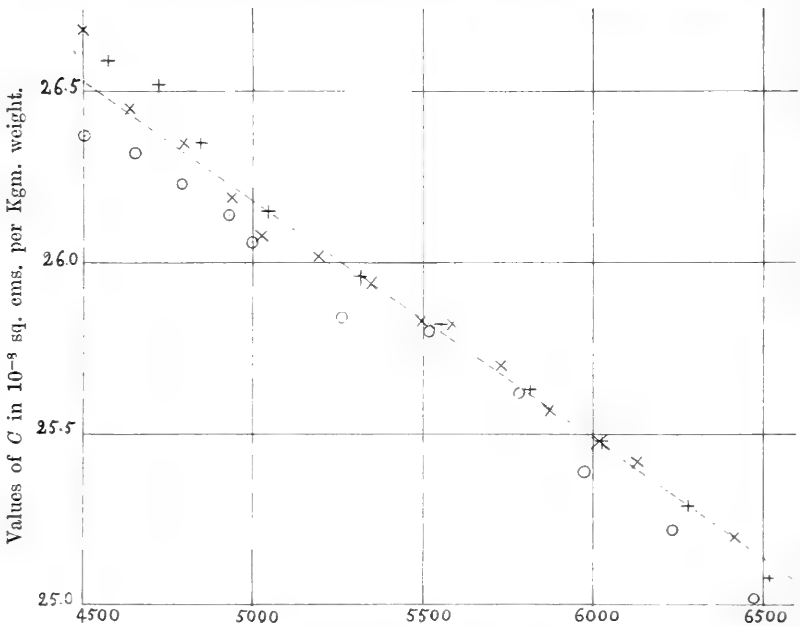
W	$22{\cdot}52 + Wx$	λ in tenth-metres	C
13·342	622·91	4,500	26·68
13·892	647·66	4,639	26·45
14·436	672·14	4,796	26·35
14·980	696·62	4,940	26·19
15·325	712·14	5,029	26·08
15·875	736·89	5,192	26·02
16·419	761·37	5,347	25·94
16·963	785·85	5,497	25·83
17·250	789·77	5,586	25·82
17·800	823·52	5,730	25·70
18·344	948·00	5,871	25·57
18·888	872·48	6,020	25·48
19·294	890·75	6,132	25·42
20·388	939·98	6,415	25·20

where in the above C is expressed in 10^{-8} sq. cms. per kilogramme weight.

6. The above results have been plotted together in Fig. 3, the points belonging to the three different sets of observations

having been differently marked. They all show a steady decrease of the stress-optical coefficient from λ 4500 to λ 6500, the total variation being about 5 per cent. This agrees very well with the results of the previous paper.

The present observations, however, are sufficiently accurate to allow us to form some estimate of the law of this variation. The observations of the last two groups (Tables II. and III.) lie very close indeed upon a straight line, which has been put in dotted in Fig. 3. Considering the smallness of this residual effect the



Wave-Lengths in tenth-metres.

⊙ Observations of Table I.
+ ,, ,, II.
× ,, ,, III.

Fig. 3.

agreement with the straight line is exceedingly good, except at the blue end where, for reasons explained previously, the observations were far more difficult and the points accordingly more irregular.

The observations of the first series (Table I.) lie in most cases considerably off this straight line, although the difference is not, perhaps, so great as the large scale of the diagram would lead one to think. This set of observations would be very fairly well

fitted by a straight line parallel to the dotted line, but differing slightly in position.

The explanation of this is probably that between the first and the second set of observations, which were not taken on the same day, the straining apparatus suffered some slight jar, which must have altered the distribution of stress in the block B, raising or lowering by a small amount the value of the stress in the particular region through which the light passed.

For this reason a complete set of observations was in all cases taken as nearly as possible at the same time, and only observations of the same group are, in strictness, comparable. Nevertheless the results of July 9 and 11 agree so well that I have felt justified in grouping them together.

It may be noticed here that, for the reason mentioned above, although the present is a delicate method for detecting *variations* in the stress-optical coefficient (the stress, whatever the distribution, being always proportional to the total load, so long as the mode of application of the latter is unchanged), it is a very bad one for obtaining its *absolute value.*

For, according as the stress in the middle of the block is greater or less than the mean value calculated from the total load, the calculated coefficient C will be greater or less than its true value.

No very great importance, therefore, should be attached to the absolute values of C given in Tables I.—III. It will be seen, on comparing with the table on p. 491 of the paper quoted (*Camb. Phil. Proc.*, Vol. XI. Pt. VI.), that those values are considerably less (about 10 per cent.) than those found for the two beams used in the Cambridge experiments[1].

This of course may be due to the fact that glasses, outwardly similar, may differ in their stress-optical properties, the glass used in these experiments not having been cut from the identical beams employed in the Cambridge observations.

It appears far more likely, however, that this discrepancy in the absolute value is due to the fact that the stress in the centre of the block was *less* than its mean value, a result obtained theoretically by the author when considering an elastic cylinder compressed between lead sheets, which force the ends to expand ("On the equilibrium of circular cylinders under certain practical systems of load," *Phil. Trans.* A, Vol. 198, pp. 147—233).

If we bear in mind these causes of error, to which the determination of the absolute value is subject, we see that, as far as the relative values are concerned, the observations of Fig. 3 indicate quite a good agreement between the three series.

[1] I regret that, owing to an error in copying, a 10^{-6} appears multiplying the figures in the last column of the table referred to. It should be 10^{-7}.

To the first approximation the variation in C is given by a straight line whose slope downwards is ·00070 per tenth-metre, from which, using the values of C here obtained,

$$C = (29{\cdot}69 - ({\cdot}00070)\,\lambda) \times 10^{-8} \text{ sq. cms. per kgm. wt.}$$

where the absolute term is determined with some uncertainty as stated above.

On examining the points representing the observations still more closely, it will be seen that there are traces of a point of inflexion about λ 5300. The observations come down at first more sharply than they should according to the straight line law, then the rate of decrease is suddenly checked, but recovers itself afterwards. The effect may be seen in all three sets of observations, but is most marked in those of Table I. It appears not unlikely that this effect may be real, and in this case a cubic term should be added to the expression for C.

Altogether it seems that this method of detecting variations in the stress-optical properties of a substance is more delicate and accurate than any which has yet been tried. It should be remembered that Pockels (*Wied. Ann.* 1902, Ser. IV. Vol. VII. p. 745) has examined light glasses of this density and found no sensible dispersion. The author hopes at no distant date to apply this method to the heavier glasses, which undoubtedly show a large dispersion of artificial double refraction, and it is hoped that the results obtained will yield considerable information.

The author wishes to thank Professor Trouton of University College, London, for having allowed him to use the Physical Laboratory of University College for this investigation and also for the uniform kindness with which he has assisted him in every way; and also to acknowledge gratefully his indebtedness to Mr A. W. Porter, Assistant-Professor of Physics at University College, London, for much invaluable assistance.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the invariant factors of a determinant. By H. F. BAKER, D.Sc., St John's College.

[*Read* 19 January 1903.]

This paper gives a proof of the fundamental theorem relating to the reduction of a matrix to its canonical form; for it the writer would claim only that it is complete in itself and strictly elementary. It was written in July 1900 in connexion with a reading of Schlesinger's treatise on linear differential equations and the paper of Ed. Wehr on matrices in the first volume of the *Monatshefte für Mathematik.* The proof in §§ 7, 8 of the paper appears interesting notwithstanding its length; of the result there verified another proof is given by Netto, *Acta Math.* t. XVII.

§ 1. Let a be any square matrix of n rows and columns, and θ, θ'... the different roots of the determinantal equation of the nth order $F(\rho) = |a - \rho| = 0$. Then since the determinant of the matrix $D = a - \theta$ vanishes, the n equations in the n variables $x_1 \ldots x_n$ expressed by

$$D^l x = (a - \theta)^l x = 0,$$

wherein l is any positive integer, have a certain number of linearly independent sets of solutions, in terms of which all other sets of solutions can be linearly expressed.

Let f be the greatest number of sets $x^{(1)}$, $x^{(2)}$, ..., each of n elements, which satisfy the equations

$$D^l x = 0$$

and satisfy no linear relation

$$A_1 D^{l-1} x^{(1)} + A_2 D^{l-1} x^{(2)} + \ldots = 0,$$

wherein $A_1, A_2, \ldots$ are each single quantities. Choose such sets, say $x^{(1)}, \ldots, x^{(f)}$. Then each of the sets

$$y^{(1)} = Dx^{(1)}, \ldots, y^{(f)} = Dx^{(f)},$$

satisfies the equations

$$D^{l-1} y = 0,$$

and there exists no linear relation connecting the sets

$$D^{l-2} y^{(1)}, \ldots, D^{l-2} y^{(f)}.$$

If beside $y^{(1)}, \ldots, y^{(f)}$ there exist other sets of solutions

$$y^{(f+1)}, \ y^{(f+2)}, \ldots, \text{ of } D^{l-1} y = 0,$$

such that there exists no linear relation

$$B_1 D^{l-2} y^{(1)} + \ldots + B_f D^{l-2} y^{(f)} + B_{f+1} D^{l-2} y^{(f+1)} + B_{f+2} D^{l-2} y^{(f+2)} + \ldots = 0,$$

in which each of $B_1 \ldots B_{f+2} \ldots$ is a single quantity, let $g-f$ be the greatest possible number of them, and choose such, say $y^{(f+1)} \ldots y^{(g)}$. Then, supposing the case when $g=f$ to be included, each of the sets

$$z^{(1)} = Dy^{(1)}, \ldots, z^{(g)} = Dy^{(g)}$$

satisfies the equations

$$D^{l-2} z = 0,$$

and there exists no linear relation connecting the sets

$$D^{l-3} z^{(1)}, \ldots, \ D^{l-3} z^{(g)}.$$

If, beside $z^{(1)}, \ldots, z^{(g)}$, there exist other sets of solutions

$$z^{(g+1)}, \ z^{(g+2)}, \ldots, \text{ of } \ D^{l-2} z = 0,$$

such that there exists no linear relation

$$C_1 D^{l-3} z^{(1)} + \ldots + C_g D^{l-3} z^{(g)} + C_{g+1} D^{l-3} z^{(g+1)} + \ldots = 0,$$

we can proceed as before. We shall arrive at length at sets of solutions

$$D^{l-2} x^{(1)}, \ldots, D^{l-2} x^{(f)}, \ D^{l-3} x^{(f+1)}, \ldots, \ D^{l-3} x^{(g)}, \ D^{l-4} z^{(g+1)}, \ldots, \\ Dt^{(p)}, \ u^{(p+1)}, \ldots, \ u^{(q)},$$

of the equations $D^2 u = 0$, which we denote in turn by

$$u^{(1)}, \ldots, u^{(f)}, \ u^{(f+1)}, \ldots, u^{(g)}, \ u^{(g+1)}, \ldots, u^{(p)}, \ u^{(p+1)}, \ldots, u^{(q)},$$

which are such that, beside $u^{(1)}, \ldots, u^{(p)}$, the greatest number of

sets of solutions of $D^2u=0$, say $u^{(p+1)}, \ldots$, for which there exists no relation

$$H_1 Du^{(1)} + \ldots + H_p Du^{(p)} + H_{p+1} Du^{(p+1)} + \ldots = 0,$$

is $q-p$, a set of such $q-p$ being $u^{(p+1)}, \ldots, u^{(q)}$. Then, supposing the case when $q=p$ to be included, each of the sets

$$v^{(1)} = Du^{(1)}, \ldots, v^{(q)} = Du^{(q)}$$

satisfies $Dv=0$, and they are linearly independent. If beside these there exist other solutions $v^{(q+1)}, \ldots$ of $Dv=0$, such that there exists no linear relation connecting $v^{(1)}, \ldots, v^{(q)}, v^{(q+1)}, \ldots$ let $r-q$ be the greatest number of these, and choose such, say $v^{(q+1)}, \ldots, v^{(r)}$. Then $v^{(1)}, \ldots, v^{(r)}$ are linearly independent; possibly $q=r$.

Then we have the following $f+g+\ldots+r$ sets of solutions

of $D^l x = 0$; $x^{(1)}, \ldots, x^{(f)}$,

of $D^{l-1}y=0$; $Dx^{(1)}, \ldots, Dx^{(f)}, y^{(f+1)}, \ldots, y^{(g)}$,

of $D^{l-2}z=0$; $D^2x^{(1)}, \ldots, D^2x^{(f)}, Dy^{(f+1)}, \ldots, Dy^{(g)}, z^{(g+1)}, \ldots$,

....

of $D^2u=0$; $D^{l-2}x^{(1)}, \ldots, D^{l-2}x^{(f)}, D^{l-3}y^{(f+1)}, \ldots, D^{l-3}y^{(g)}$,
$D^{l-4}z^{(g+1)}, \ldots, Dt^{(p)}, u^{(p+1)}, \ldots, u^{(q)}$,

of $Dv=0$; $D^{l-1}x^{(1)}, \ldots, D^{l-1}x^{(f)}, D^{l-2}y^{(f+1)}, \ldots, D^{l-2}y^{(g)}$,
$D^{l-3}z^{(g+1)}, \ldots, D^2t^{(p)}, Du^{(p+1)}, \ldots, Du^{(q)}, v^{(q+1)}, \ldots, v^{(r)}$.

§ 2. In regard to these sets of solutions it is easy to prove (a) that they are linearly independent, (b) that their number is equal to the number of linearly independent sets of solutions of the equations $D^l x = 0$.

As to (a) we notice that if we suppose any linear relation, say (A), to connect the sets, the result of operating on this relation with D^{l-1} is to reduce it to a linear relation connecting

$$D^{l-1}x^{(1)}, \ldots, D^{l-1}x^{(f)},$$

which, by the construction of the sets, can only be true when the coefficients of $x^{(1)}, \ldots, x^{(f)}$ in (A) are each zero. The supposed relation thus reduces to a relation, (B), connecting the sets other than $x^{(1)}, \ldots, x^{(f)}$. By operating on this with D^{l-2} this is again reduced, and so on.

As to (b), let x be any set of solutions of $D^l x = 0$. Since f is the greatest number of sets satisfying $D^l x = 0$, for which a relation of the form

$$A_1 D^{l-1}x^{(1)} + \ldots + A_f D^{l-1} x^{(f)} = 0$$

does not exist, there is a relation

$$A_1 D^{l-1} x^{(1)} + \ldots + A_f D^{l-1} x^{(f)} + A D^{l-1} x = 0,$$

in which A is not zero and may be taken unity; we have thus a set of solutions

$$y = x + A_1 x^{(1)} + \ldots + A_f x^{(f)}$$

of the equations $D^{l-1} y = 0$. By the definition of the number g there exists a relation

$$B_1 D^{l-2} y^{(1)} + \ldots + B_g D^{l-2} y^{(g)} + B D^{l-2} y = 0$$

in which B is not zero and may be taken unity. Hence the set

$$z = y + B_1 y^{(1)} + \ldots + B_g y^{(g)}$$

satisfies the equations $D^{l-2} z = 0$. Proceeding thus we obtain a set of solutions

$$v = u + H_1 u^{(1)} + \ldots + H_q u^{(q)}$$

of the equations $Dv = 0$; thence by the definition of the number r there exists a relation

$$K_1 v^{(1)} + \ldots + K_r v^{(r)} + v = 0$$

which expresses v, and hence u, and so on, and hence finally expresses x as a linear function of the $f + g + \ldots + r$ sets put down in § 1, so that $f + \ldots + r$ is the number of linearly independent sets of solutions of $D^l x = 0$.

In precisely the same way it can be shewn that every solution of $D^{l-1} y = 0$ is a linear function of the $g + \ldots + q + r$ sets other than $x^{(1)}, \ldots, x^{(f)}$; and so on; and finally that every set of solutions of $Dv = 0$ is linearly expressible by the r sets occurring in the last line of our scheme. Thus the rank of the matrix D is $n - r$, of D^2 is $n - (r + q)$, and so on; and finally the rank of D^l is

$$n - (f + \ldots + r).$$

Retaining the notation by which all the sets in the first line are denoted by the letter x, all in the second line by the letter $y, \ldots,$ all in the last line by the letter v we clearly have, arranging the columns of the scheme as rows

$$\left.\begin{array}{llll} Dv^{(1)} = 0, & Du^{(1)} = v^{(1)}, & \ldots, & Dx^{(1)} = y^{(1)} \\ \ldots\ldots\ldots\ldots & & & \\ Dv^{(f)} = 0, & Du^{(f)} = v^{(f)}, & \ldots, & Dx^{(f)} = y^{(f)} \end{array}\right\} f \text{ rows, } l \text{ columns}$$

$$\left.\begin{array}{llll} Dv^{(f+1)} = 0, & Du^{(f+1)} = v^{(f+1)}, & \ldots, & Dy^{(f+1)} = z^{(f+1)} \\ \ldots\ldots\ldots\ldots & & & \\ Dv^{(g)} = 0, & Du^{(g)} = v^{(g)}, & \ldots, & Dy^{(g)} = z^{(g)} \end{array}\right\} g - f \text{ rows, } l - 1 \text{ columns,}$$

$\ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots$

$$\left.\begin{array}{ll} Dv^{(p+1)}=0, & Du^{(p+1)}=v^{(p+1)} \\ \dots\dots\dots\dots\dots & \\ Dv^{(q)}\;\;=0, & Du^{(q)}\;\;=v^{(q)} \end{array}\right\} q-p \text{ rows, 2 columns,}$$

$$\left.\begin{array}{l} Dv^{(q+1)}=0 \\ \dots\dots\dots\dots \\ Dv^{(r)}\;\;=0 \end{array}\right\} r-q \text{ rows, 1 column,}$$

wherein any of the quantities f, $g-f, \dots, q-p$, $r-q$ may be zero.

§ 3. Denote $f+\dots+r$ by λ and these sets of solutions corresponding to the factor $(\rho-\theta)^l$ of

$$F(\rho)=|a-\rho|=(-1)^n(\rho-\theta)^l(\rho-\theta')^{l'}\dots$$

by $w^{(1)}, \dots, w^{(\lambda)}$. Corresponding to the other factors $(\rho-\theta')^{l'}, \dots$ we shall similarly obtain lots of sets of solutions

$$w'^{(1)}, \dots, w'^{(\lambda')}, w''^{(1)}, \dots.$$

These $\lambda+\lambda'+\dots$ sets are linearly independent; for the result of operating on any supposed linear equation

$$P_1 w^{(1)}+\dots+P_\lambda w^{(\lambda)}+P_1' w'^{(1)}+\dots+P'_{\lambda'} w'^{(\lambda')}+P_1'' w''^{(1)}+\dots=0$$

with the matrix

$$\Phi(a)=(a-\theta')^{l'}(a-\theta'')^{l''}\dots$$

is to give

$$\Phi(a)\left[P_1 w^{(1)}+\dots+P_\lambda w^{(\lambda)}\right]=0;$$

we have however

$$(a-\theta)^l\left[P_1 w^{(1)}+\dots+P_\lambda w^{(\lambda)}\right]=0$$

and polynomials in the matrix a, $\Phi_1(a)$, $\Psi(a)$, can be found to make

$$\Phi_1(a)\,\Phi(a)+\Psi(a)\,(a-\theta)^l=1,$$

and therefore also

$$P_1 w^{(1)}+\dots+P_\lambda w^{(\lambda)}=0,$$

contrary to the proved linear independence of $w^{(1)}, \dots, w^{(\lambda)}$.

If then we can prove that the numbers λ, $\lambda', \dots$ are respectively at least as great as l, $l', \dots$, we shall have at least $l+l'+\dots, =n$, linearly independent sets of solutions arising altogether; since of sets of n elements at most n are linearly independent we shall therefore have just n. Thus the proof that the numbers λ, $\lambda', \dots$ are respectively at least as great as l, $l', \dots$ involves that $\lambda=l$, $\lambda'=l', \dots$.

§ 4. To prove that the number λ of linearly independent sets of solutions of the equations $D^l x = 0$ is at least l, under the hypothesis that $(\rho - \theta)^l$ is the highest power of $\rho - \theta$ in the determinant $|a - \rho|$, we may proceed as follows.

Denote $D^l = (a - \theta)^l$ by c; the matrix of n rows and λ columns of which any column is formed by the constituents of one of the λ sets of solutions of $cx = 0$, by ξ; take another matrix η of n rows and $n - \lambda$ columns arbitrarily so that the matrix $\sigma = (\xi, \eta)$, of n rows and columns, is of non-vanishing determinant. Every solution x of $cx = 0$ can be written $x = \xi t$, where t is a row of λ elements; and every matrix ξ', such as ξ, formed of λ columns constituted by linearly independent sets of solutions, can be written $\xi' = \xi\tau$, where τ is a matrix of λ rows and columns. In particular, since from $c\xi = 0$ follows $ac\xi = ca\xi = 0$, there is a matrix β of λ rows and columns such that $a\xi = \xi\beta$, which, if the polynomial of order λ in ρ, given by $|\beta - \rho|$, be denoted by $H(\rho)$, satisfies the equation $H(\beta) = 0$. Also the matrix $\sigma^{-1}a\sigma$ is of the form

$$\sigma^{-1}a\sigma = \begin{pmatrix} \beta & \gamma \\ 0 & \kappa \end{pmatrix},$$

where γ, κ are matrices of $(n - \lambda)$ columns and respectively λ and $n - \lambda$ rows; for the condition for this is

$$a(\xi, \eta) = (\xi, \eta)\begin{pmatrix} \beta & \gamma \\ 0 & \kappa \end{pmatrix},$$

namely $a\xi = \xi\beta$ together with

$$a\eta = \sigma\begin{pmatrix} \gamma \\ \kappa \end{pmatrix}, \quad \sigma^{-1}a\eta = \begin{pmatrix} \gamma \\ \kappa \end{pmatrix}$$

which determines γ and κ; and gives also

$$ca\eta = (c\xi, c\eta)\begin{pmatrix} \gamma \\ \kappa \end{pmatrix} = (0, c\eta)\begin{pmatrix} \gamma \\ \kappa \end{pmatrix} = c\eta\kappa;$$

and we have

$$|a - \rho| = |\beta - \rho|\,|\kappa - \rho|.$$

Let $(\rho - \theta)^r$ be the highest power of $\rho - \theta$ dividing $|\beta - \rho|$ or $H(\rho)$, and hence dividing $|a - \rho|$, so that $r \leqq l$, $r \leqq \lambda$; we can find two integral polynomials $A(\rho)$, $B(\rho)$ such that

$$(\rho - \theta)^r = A(\rho)(\rho - \theta)^l + B(\rho)H(\rho);$$

if then $x = \xi t$ be a set of solutions of $cx = 0$, or $(a - \theta)^l x = 0$, since from $a\xi = \xi\beta$ we infer $a^m\xi = \xi\beta^m$ and hence $H(a)\xi = \xi H(\beta) = 0$, we have

$$(a - \theta)^r x = A(a)(a - \theta)^l x + B(a)H(a)\xi t = 0$$

or the set also satisfies $(a-\theta)^r x=0$. From this it follows that if z be a set of solutions of $(a-\theta)^{l+1} z=0$ or $c(a-\theta)z=0$, then $(a-\theta)^{r+1}z=0$.

Suppose now if possible that $\lambda<l$; then $\rho-\theta$, which occurs in $|a-\rho|$ to power l, must divide $|\kappa-\rho|$; and hence a set, u, of $n-\lambda$ quantities can be found so that $\kappa u=\theta u$; then, by $ca\eta=c\eta\kappa$, we have $ca\eta u=c\eta\theta u$, or $(a-\theta)^{l+1}\eta u=0$, and hence $(a-\theta)^{r+1}\eta u=0$, and therefore, as $r \overline{\gtrless} \lambda<l$, also $(a-\theta)^l \eta u=0$, or $c\eta u=0$, shewing that ηu is capable of a form ξt, where t is a set of λ quantities; but the equation $(\xi, \eta)(t, -u)=0$ or $\sigma(t, -u)=0$ is contrary to the hypothesis that σ is of non-vanishing determinant.

§ 5. Hence by § 3 we can form a matrix M of non-vanishing determinant of n rows and columns of which the first l columns are formed by the constituents respectively of the sets $w^{(1)}, \ldots, w^{(l)}$, arising for $D^l x=0$, the following l' columns are formed by the constituents respectively of the sets $w'^{(1)}, \ldots, w'^{(l')}$, arising for $(a-\theta')^{l'} x=0$, and so on. Since equations

$$Dw^{(i)}=0, \quad Dw^{(i+1)}=w^{(i)}$$

are the same as

$$aw^{(i)}=\theta w^{(i)}, \quad aw^{(i+1)}=w^{(i)}+\theta w^{(i+1)},$$

etc., what we have proved is equivalent to

$$aM=MA,$$

where A is a matrix having elements only in the diagonal containing the places $(1, 1), \ldots, (n, n)$ and in the line parallel thereto and next to it on the right which contains the places $(1, 2)$, $(2, 3)$, $(3, 4)$, ... The elements in the diagonal of A consist of θ coming l times, followed by θ' coming l' times, and so on; the elements in the other line consist of sequences of unities separated by zeros, the number and position of the zeros being determined by the scheme in § 2. Since therein some of the numbers f, $g-f$, ... may be zero, it is desirable to rewrite it, with the omission of zeros, say as follows:

$$\left.\begin{array}{l} D\xi_1^{(1)}=0,\ D\xi_2^{(1)}=\xi_1^{(1)},\ \ldots,\ D\xi_\alpha^{(1)}=\xi_{\alpha-1}^{(1)} \\ \ldots\ldots\ldots\ldots\ldots \\ D\xi_1^{(\kappa)}=0,\ D\xi_2^{(\kappa)}=\xi_1^{(\kappa)},\ \ldots,\ D\xi_\alpha^{(\kappa)}=\xi_{\alpha-1}^{(\kappa)} \end{array}\right\} \kappa \text{ rows, } \alpha \text{ columns,}$$

$$\left.\begin{array}{l} D\eta_1^{(1)}=0,\ D\eta_2^{(1)}=\eta_1^{(1)},\ \ldots,\ D\eta_\beta^{(1)}=\eta_{\beta-1}^{(1)} \\ \ldots\ldots\ldots\ldots\ldots \\ D\eta_1^{(\mu)}=0,\ D\eta_2^{(\mu)}=\eta_1^{(\mu)},\ \ldots,\ D\eta_\beta^{(\mu)}=\eta_{\beta-1}^{(\mu)} \end{array}\right\} \mu \text{ rows, } \beta \text{ columns,}$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\left.\begin{array}{l} D\zeta_1^{(1)}=0,\ D\zeta_2^{(1)}=\zeta_1^{(1)},\ \ldots,\ D\zeta_\gamma^{(1)}=\zeta_{\gamma-1}^{(1)} \\ \ldots\ldots\ldots\ldots\ldots \\ D\zeta_1^{(\sigma)}=0,\ D\zeta_2^{(\sigma)}=\zeta_1^{(\sigma)},\ \ldots,\ D\zeta_\gamma^{(\sigma)}=\zeta_{\gamma-1}^{(\sigma)} \end{array}\right\} \sigma \text{ rows, } \gamma \text{ columns,}$$

where $\quad l \geqq \alpha > \beta > \ldots > \gamma > 0,\ \kappa > 0,\ \mu > 0, \ldots,\ \sigma > 0,$

$$\kappa\alpha + \mu\beta + \ldots + \sigma\gamma = l.$$

With this notation, the matrix A in the equation

$$M^{-1}aM = A,$$

will have the first $\alpha - 1$ elements (1, 2), (2, 3) ... each unity, then the element $(\alpha, \alpha+1)$ a zero, then the next $\alpha - 1$ elements $(\alpha+1, \alpha+2)$, ... unities followed by a zero, and so on.

Further this equation gives, if ρ be an arbitrary single quantity,

$$M^{-1}(a-\rho)M = A - \rho$$

and therefore $|a-\rho| = |A-\rho|$. It is however the case that the power of a factor $\rho - \theta$ which is common to all minors of the determinant $|a-\rho|$ of s rows and columns is the same as the power common to all minors of $|A-\rho|$ of the same order; for suppose these powers to be respectively p_s and P_s; then from

$$A - \rho = M^{-1}(a-\rho)M, \quad a - \rho = M(A-\rho)M^{-1},$$

since the determinant of a minor, of the sth order, of a product of two matrices is a linear aggregate of products of determinants of minors, also of the sth order, from the component matrices, it follows, respectively, that

$$P_s \geqq p_s, \quad p_s \geqq P_s.$$

§ 6. Now supposing as before $(\rho-\theta)^l$ to be the highest power of a factor $\rho - \theta$ of the determinant $|a-\rho|$, let $(\rho-\theta)^{l_1}$ be the highest power of the same factor dividing all minors of this determinant of $(n-1)$ rows and columns, $(\rho-\theta)^{l_2}$ the highest power dividing all minors of $(n-2)$ rows and columns, and so on, and put

$$\epsilon_1 = l - l_1,\ \epsilon_2 = l_1 - l_2,\ \ldots,\ \epsilon_r = l_{r-1},\ \epsilon_{r+1} = 0 = \epsilon_{r+2} = \ldots$$

so that $n - r$ is the rank of the matrix $a - \rho$ corresponding to $\rho - \theta$; then

$$(\rho-\theta)^l = (\rho-\theta)^{\epsilon_1}(\rho-\theta)^{\epsilon_2}\ldots(\rho-\theta)^{\epsilon_r}$$

and $(\rho-\theta)^{\epsilon_1}$, $(\rho-\theta)^{\epsilon_2}$ ... are called the invariant factors of the matrix $a - \rho$ or of the determinant $|a-\rho|$ corresponding to the root θ.

We desire to shew now that, in the notation of § 5, the first κ of the series of exponents

$$\epsilon_1,\ \epsilon_2,\ \epsilon_3,\ \ldots$$

are each equal to α, the succeeding μ of them each equal to β, and so on, and finally the last σ of them each equal to γ. It will then follow from the preceding work that

$$\epsilon_1 \gtreqless \epsilon_2 \gtreqless \epsilon_3 \gtreqless \ldots.$$

As we have remarked, the invariant factors are the same for the matrix $A-\rho$ as for the matrix $a-\rho$; it is then sufficient to prove the result for $A-\rho$.

§ 7. For this purpose let the elements of $A-\rho$ be denoted by b_{ij} and the matrix itself by b; let ϕ be an arbitrary matrix of n rows and columns of non-vanishing determinant.

The expansion of the determinant of the matrix $b+\frac{1}{t}\phi$ in powers of $\frac{1}{t}$ being

$$|b|+\frac{1}{t}\,\delta\,|b|+\frac{1}{t^2}\,\delta^2\,|b|+\ldots,$$

where
$$\delta=\sum_{i,j}\sum\phi_{ij}\frac{\partial}{\partial b_{ij}},$$

we see that the first term will divide by $(\rho-\theta)^l$; the second term, being a linear aggregate of first minors of $|b|$, each multiplied by an element of ϕ, will divide by $(\rho-\theta)^{l_1}$; similarly the third will divide by $(\rho-\theta)^{l_2}$, and so on; if, as before, $l_{r-1}>0$, $l_r=l_{r+1}=\ldots=0$, the term in $\left(\frac{1}{t}\right)^r$, being

$$\frac{1}{r!}\left(\Sigma\Sigma\,\phi_{i,j}\frac{\partial}{\partial b_{ij}}\right)^r|b|,$$

and therefore an aggregate of determinants of minors of $|b|$ of $n-r$ rows and columns, will contain one term at least not divisible by $\rho-\theta$, and therefore will not divide by $\rho-\theta$; while the term in $\left(\frac{1}{t}\right)^{r-1}$ will divide by $(\rho-\theta)^{l_{r-1}}$.

Thus if $\rho-\theta$ be denoted by Θ, we have an equation of the form

$$|\phi+tb|=t^n\Theta^l H+t^{n-1}\Theta^{l-\epsilon_1}H_1+\ldots+t^{n-r+1}\Theta^{l-\epsilon_1-\ldots-\epsilon_{r-1}}H_{r-1}+\psi,$$

where $H, H_1, \ldots, H_{r-1}$ are integral polynomials in ρ, and ψ is an aggregate of terms not divisible by Θ; on the other hand

$$|\phi + tb| = |\phi| + t\left(\Sigma\Sigma b_{ij}\frac{\partial}{\partial\phi_{ij}}\right)|\phi| + t^2\left(\Sigma\Sigma b_{ij}\frac{\partial}{\partial\phi_{ij}}\right)^2|\phi| + \ldots;$$

hence we see that the determination of the numbers $l, \epsilon_1, \epsilon_2, \ldots$ is equivalent to the determination of the powers of $\Theta = \rho - \theta$ which enter into the successive quantities

$$\left(\Sigma\Sigma b_{ij}\frac{\partial}{\partial\phi_{ij}}\right)^h|\phi|.$$

Such powers arise in virtue of the fact that every term in the expansion of this quantity which involves a factor

$$\frac{\partial^2}{\partial\phi_{ij}\partial\phi_{i_1j_1}}|\phi|$$

wherein either $i = i_1$ or $j = j_1$, vanishes identically; and if we neglect the possibility of the existence of linear relations connecting minors of the same order of the arbitrary matrix ϕ, they arise for no other reason. Taking account of the special form of $|b|$, our problem is then the same as the following:—

Let, as before, $\Theta = \rho - \theta$ and

$$W_a = \Theta(e_1 + e_2 + \ldots + e_a) + f_2 + f_3 + \ldots + f_a$$

wherein $e_1 \ldots e_a, f_2 \ldots f_a$ are indeterminates obeying the fundamental laws of algebra and such, in addition, that

$$0 = e_1^2 = e_2^2 = \ldots = e_a^2 = f_2^2 = \ldots = f_a^2 = e_2 f_2 = \ldots$$
$$= e_a f_a = e_1 f_2 = e_2 f_3 = \ldots = e_{a-1} f_a,$$

namely their squares, and the products of an e with an f in which either the suffixes are the same or that of the f is greater by one than that of the e, are all zero. Take κ such symbols

$$W_a^{(1)},\ W_a^{(2)},\ \ldots,\ W_a^{(\kappa)},$$

the $e_1 \ldots e_a, f_2 \ldots f_a$ occurring in $W_a^{(i)}$ not being the same as those in $W_a^{(j)}$, and there being no equations such as $e_s^{(i)} f_s^{(j)} = 0$. Put

$$P_{a,\kappa} = W_a^{(1)} + \ldots + W_a^{(\kappa)}.$$

Similarly form, with the same Θ, a sum of μ symbols W_β, each involving β symbols e and $\beta - 1$ symbols f, say

$$P_{\beta,\mu} = W_\beta^{(1)} + \ldots + W_\beta^{(\mu)};$$

the symbols e, f which enter in any $W_\beta^{(i)}$ are subject to rules such as those above, but the $\kappa + \mu$ symbols W contain different sets of such e and f. And so on.

Finally, as in the notation of § 5, form, with the same Θ, a symbol

$$P_{\gamma,\sigma} = W_\gamma^{(1)} + \ldots + W_\gamma^{(\sigma)}.$$

Let
$$Q = P_{\alpha,\kappa} + P_{\beta,\mu} + \ldots + P_{\gamma,\sigma}.$$

Then, for the other factors $\Theta' = \rho - \theta'$, $\Theta'' = \rho - \theta''$, ... of the determinant $|b|$, form corresponding symbols Q', Q'', ...

We are to investigate the powers of Θ which enter as factors in the various powers

$$(Q + Q' + Q'' + \ldots)^s.$$

§ 8. To do this we distinguish the powers of any such symbol as *useless, significant* or *vanishing* powers, according as they are divisible by no power of Θ, by such a power, or vanish entirely.

Then firstly for

$$W_m = \Theta\,(e_1 + \ldots + e_m) + f_2 + \ldots + f_m$$

we find that W_m, W_m^2, ..., W_m^{m-1} are useless powers, that W_m^m is a significant power and divides by Θ^m, and that

$$W_m^{m+1},\; W_m^{m+2},\; \ldots$$

are all vanishing powers. For example, taking $m = 3$,

$$W_3 = \Theta\,(e_1 + e_2 + e_3) + f_2 + f_3,$$

gives
$$W_3^2 = 2\Theta^2\,(e_2e_3 + e_3e_1 + e_1e_2) + 2\Theta\,(e_1f_3 + e_3f_2) + 2f_2f_3,$$
$$W_3^3 = 6\Theta^3\, e_1e_2e_3,$$
$$W_3^4 = 0,$$
....

Thus, taking

$$P_{\alpha,\kappa} = W_\alpha^{(1)} + W_\alpha^{(2)} + \ldots + W_\alpha^{(\kappa)},$$

a power of $P_{\alpha,\kappa}$ cannot be significant if its expansion contains such a term as

$$\frac{(r_1 + \ldots + r_\kappa)!}{r_1! \ldots r_\kappa!}\,(W_\alpha^{(1)})^{r_1}\,(W_\alpha^{(2)})^{r_2} \ldots (W_\alpha^{(\kappa)})^{r_\kappa},$$

where everyone of the exponents $r_1, \ldots, r_\kappa$ is less than α; therefore its lowest significant power is not less than $\kappa(\alpha - 1) + 1$. Conversely if we take this power one at least of the exponents $r_1, \ldots, r_\kappa$ in any term of the expansion is as great as α. Thus this is the lowest significant power. The highest significant power is clearly $\kappa\alpha$, and all higher powers are vanishing powers. We easily

see in fact that $P_{\alpha\kappa}^{\kappa(\alpha-1)+1}$ divides by Θ^{α}, that $P_{\alpha\kappa}^{\kappa(\alpha-1)+2}$ divides by $\Theta^{2\alpha}$, and so on, and finally that $P_{\alpha\kappa}^{\kappa\alpha}$ divides by $\Theta^{\kappa\alpha}$.

Now consider

$$Q = P_{\alpha\kappa} + P_{\beta\mu} + \ldots + P_{\gamma\sigma};$$

if we take a power for which there is one term of its expansion

$$\frac{(r+s+\ldots+t)!}{r!\,s!\ldots t!} P_{\alpha\kappa}^{r} P_{\beta\mu}^{s} \ldots P_{\gamma\sigma}^{t},$$

in which every exponent $r, s \ldots t$ is less than a significant exponent, this power is not a significant one; thus the lowest possible significant exponent is

$$\kappa(\alpha-1) + \mu(\beta-1) + \ldots + \sigma(\gamma-1) + 1,$$

or

$$l - (\kappa + \mu + \ldots + \sigma) + 1,$$

and conversely this power divides by the lowest of the powers $\Theta^{\alpha}, \Theta^{\beta}, \ldots \Theta^{\gamma}$, that is by Θ^{γ}. The highest significant exponent is similarly

$$\kappa\alpha + \mu\beta + \ldots + \sigma\gamma, = l.$$

We have in fact, as is immediately clear on consideration, the following statement of the powers of Θ by which the various powers divide

$Q^{l-(\kappa+\mu+\ldots+\sigma)+1}$ by Θ^{γ}, ..., $Q^{l-(\kappa+\mu)+1}$ by $\Theta^{\sigma\gamma+\ldots+\beta}$,
$Q^{l-\kappa+1}$ by $\Theta^{\sigma\gamma+\ldots+\mu\beta+\alpha}$

$Q^{l-(\kappa+\mu+\ldots+\sigma)+2}$ by $\Theta^{2\gamma}$, ..., $Q^{l-(\kappa+\mu)+2}$ by $\Theta^{\sigma\gamma+\ldots+2\beta}$,
$Q^{l-\kappa+2}$ by $\Theta^{\sigma\gamma+\ldots+\mu\beta+2\alpha}$

....

$Q^{l-(\kappa+\mu+\ldots+\sigma)+\sigma}$ by $\Theta^{\sigma\gamma}$, ..., $Q^{l-(\kappa+\mu)+\mu}$ by $\Theta^{\sigma\gamma+\ldots+\mu\beta}$,
Q^{l} by $\Theta^{\sigma\gamma+\ldots+\mu\beta+\kappa\alpha}$.

Lastly consider

$$R = Q + Q' + Q'' + \ldots$$

where Q' involves $\Theta' = \rho - \theta'$, and Q'' involves $\Theta'' = \rho - \theta''$, and so on. The lowest significant power must be one such that in every term

$$\frac{(r+r'+r''+\ldots)!}{r!\,r'!\,r''!\ldots} Q^{r}(Q')^{r'}(Q'')^{r''}\ldots$$

of its expansion, that does not vanish, the power r is a significant power for Q. As the lowest vanishing powers for Q', Q'', ... are $l'+1$, $l''+1$, ... we see that for the significant powers of R

$$r + r' + r'' + \ldots$$

must not be less than

$$l-(\kappa+\mu+\ldots+\sigma)+1+l'+l''+\ldots$$

and not greater than

$$l+l'+l''+\ldots.$$

Putting $n=l+l'+l''+\ldots$ we further see immediately that the powers of Θ arising in the various powers of R give rise to a table obtainable from that above, for the powers of Q, by the substitution only of R for Q and n for l.

Comparing this result with the expansion given, § 7, for $|\phi+tb|$ we deduce at once, beside the result

$$l=\sigma\gamma+\ldots+\mu\beta+\kappa\alpha,$$

that

$$l \geqq \alpha=\epsilon_1=\epsilon_2=\ldots=\epsilon_\kappa>\beta=\epsilon_{\kappa+1}=\ldots=\epsilon_{\kappa+\mu}>\ldots>\gamma$$
$$=\epsilon_{\kappa+\ldots+\sigma-\sigma+1}=\ldots=\epsilon_{\kappa+\mu+\ldots+\sigma},$$

while $\epsilon_{\kappa+\mu+\ldots+\sigma+1}\ldots$ are all zero, so that the rank of the matrix $a-\rho$ in regard to $\rho-\theta$ is $n-(\kappa+\ldots+\sigma)$.

§ 9. The result then is that for any root θ we have rows of equations of the form

$$(a-\theta)\,x_1=0,\ (a-\theta)\,x_2=x_1,\ \ldots,\ (a-\theta)\,x_\epsilon=x_{\epsilon-1},$$

there being one such row corresponding to each of the invariant factors associated with that root, and the n sets denoted by x which arise can be chosen to be linearly independent.

On Automorphic Functions and the general theory of Algebraic Curves. By H. W. RICHMOND, M.A., King's College.

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The propositions enunciated in this short abstract are concerned with the theory of algebraic curves in its broadest aspect, which embraces curves of any *genus* (or deficiency) and of any *order*, belonging[1] to space of any *number of dimensions*: they are immediate consequences of the researches of Humbert upon the application of Poincaré's automorphic functions to plane curves. These researches put us in possession of the *parametric equations* of the most general algebraic curve conceivable, i.e. equations in which the homogeneous coordinates of points of the curve are expressed by ratios of uniform functions of a parameter;—rational functions if the genus $p=0$, elliptic σ-functions if $p=1$, holomorphic thetafuchsian functions if $p>1$.

Curves in which the coordinates of points are expressed by functions founded upon the same group of substitutions are called *Curves of the Group.* All rational curves ($p=0$) are considered as belonging to a group, as are also all elliptic curves ($p=1$) whose points have their coordinates expressed by elliptic functions with the same periods. All propositions here deal with curves which belong to a group.

Prop. I. Between any two curves of a group a rational point-to-point correspondence may be established, corresponding points having equal parameters, and the coordinates of any point of either curve being expressible rationally in terms of those of the corresponding point of the other curve.

Prop. II. A curve which belongs to space of d dimensions is defined by the ratios of $d+1$ coordinates (x_0, x_1, x_2, ... x_d) expressed as uniform functions of a parameter, among which no linear syzygy can hold. Any three of these (x_0, x_1, x_2) are connected by a homogeneous relation of a degree normally equal to and never greater than the order of the curve; the ratio of any other coordinate x_r to x_0 may as a rule be expressed in rational terms of x_1/x_0 and x_2/x_0.

[1] A curve 'belongs to' space of d dimensions when it lies wholly in space of d dimensions and does not lie wholly in any space of fewer than d dimensions.

[These theorems express in geometric form the fundamental results of Poincaré stated in Forsyth's *Functions*, § 310; the latter is the extension of Cayley's 'Cone and Monoid' method for space-curves.]

For two reasons the most important curves of a group are the *Cardinal* curve and the *Essential* curves now defined:—

Def. The *Cardinal Curve* of a group of genus p is the curve of order $2p-2$ belonging to space of $p-1$ dimensions defined by the fact that the coordinates of its points are proportional to p linearly independent holomorphic thetafuchsian functions of the first degree.

Def. The *Essential Curves* of a group are curves of order $m=2\mu(p-1)$, where $\mu=2, 3, 4 \ldots$, each belonging to space of $m-p$ dimensions, and defined by the fact that the coordinates of its points are proportional to linearly independent holomorphic thetafuchsian functions of degree μ.

Prop. III. The Cardinal curve and the Essential curves of a group, (i) are the only curves of a group on which geometry can illustrate the properties of the group unaffected by irrelevant matters; and (ii) from them every curve of the group may be derived by projection.

Def. *Special* curves of a group are those whose points have coordinates proportional to holomorphic thetafuchsian functions of the first degree; *non-special* curves of the group are those for which the coordinates cannot be so expressed.

[This necessary distinction is made by Humbert for plane curves: as the second kind of curves is defined by a merely negative property the word 'non-special' is preferred to Humbert's word 'normal.']

Prop. IV. The cardinal curve is the chief of the special curves of a group. All the other special curves of a group may be derived from it by projection, and every curve of the group derived from the cardinal curve by projection is a special curve.

Prop. V. The cardinal curve is the unique curve of the group of order $2p-2$ in space of $p-1$ dimensions. If $p<3$, there is no cardinal curve; if $p=3$ the cardinal curve is a plane quartic; if $p=4$, it is the curve of intersection of a conicoid and a cubic surface in a space of three dimensions; if $p=5$, it lies in space of four dimensions and is the curve common to three surfaces[1] of the second order.

[1] By the word 'surface' is understood a locus defined by a single homogeneous equation among the coordinates of its points; should the equation be of the first degree the locus is called a 'plane,' so that in space of d dimensions a 'plane' is a space of $d-1$ dimensions.

Prop. VI. The cardinal curve of a group of genus p lies upon $\frac{1}{2}(p-2)(p-3)$ surfaces of the second order which are linearly independent. If a space of $p-4$ dimensions be chosen which does not contain any point of the cardinal curve, $p-3$ linearly independent surfaces of degree three pass through the cardinal curve and have the space of $p-4$ dimensions as a double space; these with a surface of degree four having the space of $p-4$ dimensions as a triple space completely define the cardinal curve.

Prop. VII. Special curves of orders ranging from $2p-2$ down to $p+d-1$ inclusive always exist in space of d dimensions, provided that $p>d>1$; but exceptional cases when curves of lower order exist often arise. Humbert shews that a plane curve of order m is special if, and only if, it possesses an adjoint curve of order $m-4$.

Prop. VIII. Every curve of the group of order m may be derived from any Essential curve whose order is equal to or greater than $m+p$ by projection. The Essential curves of a group are not the only curves of the group of order $m=2\mu(p-1)$, $\mu=2, 3\,...$, in space of $m-p$ dimensions: they are the unique examples of a particular kind of curves of this order.

Prop. IX. A non-special curve of order m must lie wholly in a space of $m-p$ or fewer dimensions. A non-special curve in a plane may be of order $p+2$, but not of lower order: a non-special curve belonging to space of d dimensions may be of order $p+d$, but not of lower order.

A non-special plane curve of order m is always the projection of a curve in space of the same order m unless $m=p+2$, and if $m=p+2$ it is not the projection of a curve in space.

A non-special curve of order m in space of d dimensions is the projection of a curve of order m in space of $d+1$ dimensions if $m>p+d$, not if $m=p+d$. It is always the projection of a curve of order $m+1$ in space of $d+1$ dimensions.

Prop. X. The number of intrinsic constants pertaining to the cardinal curve or to any essential curve of a group is equal to the 'number of moduli' of the group $=3p-3$, if $p>1$.

The number of intrinsic constants pertaining to a non-special curve of order m and genus p in space of d dimensions is $4p-3$, if $m=p+d$. Should d be less than $m-p$ the number is

$$(4p-3)+(d+1)(m-p-d).$$

Prop. XI. It is clear from *Prop.* I. that *sets of points* (or point-groups) upon any curve of a group may be represented and

studied upon the cardinal curve of the group. Equivalent sets are cut out from the cardinal curve by surfaces of the same order, having certain common intersections with that curve.

Prop. XII. To any group of R points of multiplicity r, corresponds on the cardinal curve a group of R points which lie in a space of $R - r - 1$ dimensions. For ordinary sets of points this tells us nothing, inasmuch as $R - r = p$, and the cardinal curve lies in space of $p - 1$ dimensions: for special sets of points it leads at once to the Riemann-Roch theorem.

These propositions follow almost at once from Humbert's researches: some of them are known, and will be found in Segre's writings (*Math. Ann.* XXX. p. 203, and elsewhere). But the simplicity of the proofs by automorphic functions is remarkable.

On the rise of a Spinning Top. By E. G. GALLOP, M.A., Gonville and Caius College.

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(*Abstract.*)

In order to explain the way in which the axis of a top in rapid rotation rises until it becomes sensibly vertical, it is necessary to take account of the fact that the lower end of the top is a surface of small extent and not a mere point. On the assumption that the lower end is rounded off into a portion of a sphere, and that the effect of friction between the top and ground can be represented by a single force through the point of contact, it is proved that when the initial spin about the axis of figure exceeds a certain limit, the inclination of the axis to the vertical can never exceed a certain limiting value. This limit depends on the amount of energy possessed by the top, and as energy is dissipated by sliding friction this limiting value diminishes till when the energy reaches a certain value the limiting inclination is reduced to zero and the top is left spinning with its axis vertical. Whether or not the amount of energy dissipated is sufficient to reduce the limiting inclination to zero, will depend on the coefficient of friction and other circumstances, but in any case it is proved that under the conditions assumed the top will never fall to the ground if the initial spin exceeds a certain value. Thus the ultimate fall of the top, which takes place under actual conditions, must be attributed to the resistance of the air and the friction couple which, though probably not producing much serious effect on the motion at first, eventually diminish the spin till it is unable to counterbalance the ordinary effects of gravity.

The method is also applied to explain the way in which a heterogeneous sphere spinning on a horizontal plane tends to raise its centre of mass until it is vertically above the centre of figure.

Some numerical results are added for the case of a top resembling a hollow sphere of diameter 4 cm. mounted on an axis with its centre 4 cm. above the lower end, which is rounded off into a sphere of radius 0·1 cm. It is proved that if this top is started with its axis inclined at 30° to the vertical with a spin of 32 revolutions per second, it would attain the state of steady motion with axis vertical as soon as the amount of energy dissipated by sliding friction amounted to $\frac{1}{250}$th of the original kinetic energy; and that the inclination of the axis could never reach 32° 23′. A smaller angular velocity would, of course, suffice to prevent the axis of a larger top of similar construction from falling to this inclination.

On the Dynamics of the Electric Field. By Professor J. J. THOMSON.

[*Read* 16 February 1903.]

In *Recent Researches in Electricity and Magnetism* it is shown that in each unit of volume of the Electric field there are $BF \sin \theta$ units of mechanical momentum, B being the magnetic induction, F the electric displacement and θ the angle between these vectors: the direction of the momentum is at right angles both to B and F. In this paper some of the consequences of this distribution of momentum are investigated: it is shown that when the field is that due to a charged point e and a magnetic pole m the distribution of momentum has a moment of momentum round the line joining the point and the pole equal to em. This system is thus a spinning top and it is shown that the mechanical forces acting on a moving electrified point or magnetic pole can be determined from the principle that the moment of momentum of the whole system, point and pole and field, remains constant. Several other cases are investigated, among others it is shown that Maxwell's Vector Potential at a point P is the part of the momentum of the field due to the unit electrical charge at P.

It is shown that all the laws relating to the distribution of momentum in the field follow from the view that the lines of electric force carry along with them a portion of the ether through which they pass, the mass of ether entangled with the tubes being per unit volume proportional to the electrostatic energy of the field in that unit volume; the ether thus entangled can slide along the line of electric force, but as far as motion at right angles to the line is concerned, the entangled ether moves with the line of force, the momentum in the electric field is the momentum of the ether gripped by the lines of force. It was suggested that all mechanical momentum and not merely electrical momentum was really momentum of the ether; the molecules of matter contain a number of electrified bodies ('corpuscles'), and the lines of force starting from these corpuscles grip a certain amount of the ether, and consequently the mass of the body is really the mass of the ether gripped by the lines of electric force starting from its corpuscles. The potential energy of the field is on this view the kinetic energy of the turbulently moving ether imprisoned by the lines of force.

Rust-fungi and the "Mycoplasm" Hypothesis. By Professor H. MARSHALL WARD, D.Sc., F.R.S., Sidney Sussex College.

[*Read* 16 February 1903.]

The author gave a brief account, illustrated with lantern slides and microscopic preparations, of that part of his researches into the histology of Rust-fungi which bears upon the recent pronouncement of Eriksson, that certain *corpuscules spéciaux* observable in the cells of the host-plant, are the assumed "mycoplasm" in the act of growing out to form the hyphæ of the fungus.

The author's preparations show clearly that Eriksson's *corpuscules* are true haustoria, put forth by the hyphæ of the fungus into the cells of the host. Every stage in their development is traced, and since the entering germ-tube, after swelling up as an infecting vesicle and tube in the stomatal cavity, is found to put forth one of these haustoria at a very early date, the reversed order of the phenomena assumed by Eriksson cannot be accepted.

On Radio-activity from Snow. By C. T. R. WILSON, M.A., F.R.S., Sidney Sussex College.

[*Read* 16 February 1903.]

An experiment of the same nature as those already made with freshly fallen rain and described before this Society (*Proceedings*, vol. XI. p. 428 and XII. p. 17, 1902) was made with freshly fallen snow at Peebles on Jan. 10th. The snow was melted and 50 c.c. of the water were evaporated to dryness in a porcelain basin. This was then inverted over the thin aluminium roof of the ionisation apparatus used as a detector of radio-activity (described in the first of the above-mentioned papers).

The rate of leak, measured 10 minutes after the drying was complete, was increased from 0·19 division per minute, its normal value, to 3·6 divisions per minute; the ionisation inside the vessel was thus increased by the radiation to about 18 times its normal value. An exactly similar experiment, made with 50 c.c. of rain on August 17, 1902, gave ten minutes after drying a leakage of 3·7 divisions per minute, the leakage due to "spontaneous" ionisation being 0·17 per minute. In both cases the collecting vessel was exposed for about three-quarters of an hour in the open air to catch the rain or snow, and the evaporation was completed 20 minutes after the vessel was brought in.

There is thus no indication of any difference in the intensity of the radio-activity obtained from equal weights of snow and rain.

Note on the Slipperiness of Ice. By S. SKINNER, M.A., Christ's College.

[*Read* 16 February 1903.]

Papers on this subject have been published by J. Joly (*Proc. Roy. Dublin Soc.* v. p. 453, 1886, and *Nature*, March 23, 1899) and by O. Reynolds (*Mem. and Proc. of the Lit. and Phil. Soc. of Manchester*, No. 5, vol. XLIII. 1899). Both authors attribute the slipperiness of ice to the presence of a layer of lubricating water under the body pressing on the ice. The water is produced by the lowering of the freezing point where the pressure is experienced. On this view the object glides on a liquid layer and consequently viscous friction in water takes the place of the rubbing friction between the solids. Joly shews by calculation that the weight of a man concentrated on the blade of a skate is sufficient to lower the freezing point very considerably, and Reynolds arguing from the difficulty of slipping on very cold ice comes to the same conclusion. In this note I wish to point out that sliding on a liquid layer is a condition under which cavitation will occur in the liquid, and that this will aid the slipping.

A paper read last term (*Proc. Camb. Phil. Soc.* XII. 34) was illustrated by an experiment in which a lens, resting on a flat plane with glycerine between the lens and plane, was made to slide, and behind the point of nearest approach a vacuous cavity was seen to form during motion and to disappear as soon as the motion ceased. I find that a similar cavity can be seen when a lens is pressed strongly on ice and pushed along. A cavity of this kind may be formed behind the sliding contact of a hog-back skate, and behind that of a curling-stone.

It was explained in my paper how the cavity facilitates slipping, for if it is not formed the tensions produced behind the point of nearest approach by the motion of the body over the liquid layer resist the forward motion.

On the probable presence in the Sun of the newly discovered gases of the Earth's Atmosphere. By G. D. LIVEING, M.A., Professor of Chemistry.

[*Read* 2 March 1903.]

Soon after the publication of the list of wave-lengths of the rays which Professor Dewar and I found to be emitted by the most volatile gases of the atmosphere, Stassano pointed out that the bright rays of solar protuberances, of which the wave-lengths had been measured by Deslandres and Hale, agreed closely with rays in our list. The wave-lengths need to be determined with very great exactness in order to prove coincidence, and neither set of wave-lengths have, as yet, been measured with such nicety as that. Nevertheless the agreement is sufficient to make it probable that the same gases are concerned in the production of the rays in question.

When, in the *Astrophysical Journal* for June last, Humphreys published a list of wave-lengths of 339 bright rays of the chromosphere and corona, which he had photographed in Sumatra during the total eclipse of May, 1901, I compared this list with our list of the wave-lengths of rays of the most volatile atmospheric gases, and with those of xenon and krypton, which we had subsequently published, and with those of argon. The result was that of the 339 rays in Humphreys' list 209 do not differ in wave-length by more than one unit from rays emitted by gases of the earth's atmosphere. In our published lists the wave-lengths are given to four figures only in Rowland units, those of Humphreys' list are given to tenths of a unit, so that we are far from being able to prove coincidence in any case. Nevertheless it is so very probable that every gas which is in the earth's atmosphere is also in the sun's, and so much more likely that rays appearing in the chromosphere at a height of 9 seconds of arc, or 4000 miles, above the photosphere, should be due to gases of great volatility than to such metals as titanium or strontium, that it is much safer to treat the question as an open one, than to assume that titanium and other metals of high atomic weight are always present in the state of vapour at such

a distance from the sun's surface, while much more volatile metals, such as sodium, are not regularly present at a quarter of that height.

We have been expecting from Mr Baly for a long time past a list of wave-lengths of the gases neon, krypton and xenon determined with great exactness by the use of a grating. The announcement that these measurements were in progress was made more than two years ago, and when the list appears we shall be in a better position to judge of the nearness of the coincidences. Meantime I have remeasured some of the rays of the most volatile gases, still, however, with only a prismatic spectroscope, so as to get, approximately, the fifth figure of the wave-length, and the result has been in many cases to bring my figures into closer agreement with those of Humphreys. The exactness of Humphreys' wave-lengths is quite as important a factor in the settlement of the question as that of mine. He obtained six photographs on celluloid films with a large concave grating of 30 feet focal length, used without any slit. The first had an exposure of 2 seconds immediately after second contact. Two seconds sufficed for a change of film, and then the second film was exposed for 5 seconds, and the third for 15 seconds, the fourth had a lengthened exposure of 2 minutes during the middle of totality, the fifth 15 seconds, and the sixth 8 seconds, ending 3 seconds before third contact. The dispersion of the spectrum was so great that one millimeter on the film corresponded to 3·66 Ångström units, and the range of the spectrum depicted extended from λ 3118 to λ 5204. The latter limit was probably determined by the want of sensitiveness of the films to lower green rays, for it would have been of great interest if the range had included the most characteristic rays of coronium, krypton, and neon. At the upper end the Fraünhofer line *S* is just excluded. This too is to be regretted, as *S* is a triple line of iron very unlikely to be reproduced by any mixture of elements excluding iron. The dispersion is ample for very exact determinations of wave-length if there were no difficulties in the way of exact measurement. Humphreys has met these difficulties as best he could, and I do not propose to discuss them now, but I may perhaps explain one or two of them. The station where the photographs were taken was near the northern limit of totality, so that the duration of totality was less than half what it was in the centre of the shadow. From these durations of totality I have calculated that the points of contact of the limbs of sun and moon were each nearly 63° from the diameter of the sun parallel to the line of the moon's apparent motion; and the instrument was so arranged that the ruling of the grating was parallel to that line, and in consequence the dispersion at right angles to that

line, and the definition affected as little as possible by the moon's motion relative to the sun. During the exposures the tangents at the centres of the small arcs photographed would at first be inclined at about 27° to the direction of the moon's path and would gradually swing round until at mid totality they would be parallel to it, and then on until they were again inclined at about 27° to that direction, but on the opposite side. This motion tends to blur the definition of the arcs, but the effect would be very small on the 1st, 2nd, and 6th films, and not great on the 3rd and 5th, but it would be very great on the 4th film for rays emitted only from parts of the chromosphere at much less distance than seven seconds of arc from the sun's limb. Rays emitted from the chromosphere at heights exceeding 7·3″ would give arcs crossing the direction of dispersion at right angles, and would have well defined edges on the inside, unaffected by the moon's motion. There are however very few of such rays besides H and K and the strong hydrogen and helium rays.

The wave-lengths had to be determined by measuring the distances between the arcs on the films in the direction of dispersion and interpolating between such of the arcs as could be identified as due to rays of which the wave-lengths were well known. Rays answering to this test would be those above mentioned, namely, H and K and the strong rays of hydrogen and helium, and one or two very characteristic groups such as the b group of magnesium and, perhaps, the iron triplet 4046—4071. Indeed on looking through the list I do not see any ray other than these which I could identify with certainty without assuming it to be a reversal of a Fraünhofer line. Humphreys does not say which rays he used as fiducial rays between which he could interpolate. Those which I have just named seem to me too few, and too unequally distributed through the spectrum, to give very trustworthy wave-lengths in all parts of the spectrum. No scale of reference, made with artificial light and the same grating used without a slit, could well be made, or had in fact been attempted.

In his list Humphreys gives, against each chromospheric ray, the Fraünhofer line in Rowland's list which has the wave-length nearest to that found for the chromospheric ray. As there are 13,595 lines in Rowland's list in the range of the spectrum embraced by Humphreys' 339 lines it is not surprising that a Fraünhofer line is to be found for every one of Humphreys' lines, so close that the difference of wave-length is in most cases less than a tenth of a unit and rarely exceeds two-tenths In discussing his results he evidently assumes the rays to be identical, and ascribes them to the metal (if any) indicated by Rowland.

There are however some remarkable exceptions to this. They are the rays of helium, and the ultra violet-rays of hydrogen which do not appear as Fraünhofer lines. He does not hesitate to ascribe 4 rays to helium and 24 others to hydrogen in preference to identifying them with the nearest Fraünhofer lines. And I think he is right in so doing, notwithstanding the fact that several of these rays have never been observed to be emitted by hydrogen, have not been seen in the spectrum of any star, and are believed to be due to hydrogen only because the wave-lengths found for them agree closely with the values found by calculation for the prolongation of the well known series of hydrogen rays.

It is a noteworthy fact, that whereas it is very difficult to get from a tube containing a residue of carefully purified hydrogen any rays of this series above H_ζ, λ 3889, yet when the tubes contain sensible quantities of helium and neon, they readily give many lines of the series. Up to H_π, λ 3691·5, these rays give very long arcs on Humphreys' films and are therefore emitted from the upper layer of the chromosphere. Why the presence of helium and neon along with hydrogen should conduce to the development of this series I cannot guess, but whatever the cause it seems to prevail in the sun. Besides the 4 helium rays above mentioned, other 3 helium rays might well be identified with arcs on Humphreys' films; λ 3613·78 is much nearer in wave-length to Humphreys' ray λ 3613·8 than the scandium ray with which he identifies it; and λ 4143·92 is nearer to λ 4144·0 than the iron ray to which it is ascribed; and Humphreys' ray 3820·7 gives a very long arc on film IV, and is probably the rather strong helium ray λ 3819·75. There are two rays λ 3388 and λ 3456·5 giving arcs very long, broad and hazy, which Humphreys cannot distinguish by measurement from Fraünhofer lines ascribed to titanium, but doubts whether they are really due to that metal. We have found that the most volatile atmospheric gases, probably neon, give rays at about λ 3388·8 and λ 3456·8; the former is rather a strong ray, the latter a weak one. It is possible the solar rays may be identical with them.

That the same gases must be in the atmospheres of the sun and planets appears to me certain. The subject has been discussed by many, but some of them seem to have gone off the track in ascribing the diffusion of these atmospheres to those molecules which on the kinetic theory chance to acquire such a velocity as will carry them beyond the attraction of the planet with which they had been associated. Dr Bryan has calculated that if we suppose the earth's atmosphere to have a definite surface there is practically no probability that any appreciable amount, of even so volatile a gas as hydrogen, can escape with

such velocity from that surface that it will not return, provided it be actuated only by the kinetic energy of the gas and by gravitation. Dr Bryan is careful to point out that other considerations may entirely alter the problem. Now the postulate of a definite surface to the earth's atmosphere and no gas to be encountered outside it cannot be conceded, nor can the motion of the earth in its orbit be left out of account. If we talk of a gas leaving the atmosphere we need define the limit of the atmosphere, and that is not easy. I do not think that it has any definite limit, but we may make an arbitrary one. Shall we say that when the density is reduced to $\frac{1}{10^{12}}$ of what it is at the earth's surface we have reached that limit? At all events I do not think there is any evidence that the density of the residuum of gas in planetary space is less than that. If we take Dr Bryan's figure (derived I think from Lord Kelvin) that the number of molecules in one cubic centimeter of gas at normal pressure and temperature is approximately 10^{21}, and suppose it reduced to $\frac{1}{10^{12}}$ of that, we shall still have 10^9 molecules left in each cubic centimeter, and a molecule running into such a crowd, has as good a chance of being knocked further away as it has of getting back. In fact if we consider a layer of air at this density, one cubic centimeter thick, all round the earth, and remember that the free path is very long, even if we suppose that the molecules all have only the average kinetic energy due to their temperature, many of them will, in a small fraction of a second, have passed far into the layers outside, and it is an equal chance whether the same molecules get back, or others are knocked back instead of them. We do not know at all what the temperature of the residuum of gas in planetary space may be, but it is probably very different from that which a black body would assume if left there. The diffusion of each layer into those above and below it will go on, even at a low temperature, with rapidity; and though it is true that the layer will contain but an infinitesimal fraction of the earth's atmosphere, yet as it is diffused into neighbouring layers in a small fraction of a second, and there are more than 30 millions of seconds in a year, and the solar system has been many millions of years in settling into its present state, there has been time enough for some considerable interchange of the atmospheres of the sun and earth. It is, however, far more probable that the solar system has settled into its present state from a nearly uniform gaseous mass, than that the materials out of which the sun and planets have been aggregated should have been distributed in such a heterogeneous way as to give a marked distinction between the atmospheres. Nor can we suppose that the sun and planets can

have licked up all the gases out of planetary space. For if we could suppose that, and the earth with its atmosphere moving through empty space, the incoherent atmosphere must, I suppose, distribute itself in a long trail throughout the earth's orbit, and this trail would rapidly diffuse into the void space, and the process of diffusion would go on until the amount lost by the earth in its course was counterbalanced by the amount licked up, in fact until gas at a certain average density was distributed throughout the whole of the space in which the earth circulates. That average density will be the same whether we suppose the earth to start with its atmosphere in void space, or to start with a less dense atmosphere in space filled with gas of greater density than that average. Also the distribution of the several gases which have molecules of different masses will ultimately be the same in these two cases. The proportion of gases of smaller molecular mass will increase outwards from the earth's surface as well as from that of the sun, and the interchange between the sun and earth will go on most quickly in the case of the gases of least molecular mass, or in fact those which are most volatile.

On the synthesis of carboxy derivatives of pyridine. By
W. J. SELL, M.A., Christ's College, and F. W. DOOTSON, M.A., Trinity Hall.

[*Read* 2 March 1903.]

The great importance which attaches to the derivatives of pyridine, in consequence of its close connection with alkaloids and allied substances, renders a knowledge of the syntheses of carboxy and other pyridines of considerable value.

The constitution of few of the alkaloids is known, and the problem of their structure may perhaps be more easily approached from the synthetic side.

Although a considerable number of pyridinöid bodies have been obtained by condensation of open-chain substances, remarkably little has been accomplished in this direction from pyridine itself. The three monocarboxy derivatives have already been described, but they have been obtained by methods so unproductive that they must be regarded as little more than museum curiosities.

By the application of well-known processes to those chloropyridines already described, many derivatives have been obtained. The present communication deals with some attempts to replace the chlorine in these compounds by the carboxy group.

The usual methods of procedure for such conversions were not attended with success, such substances as potassium cyanide, zinc ethide, &c., either not reacting at all, or giving unsatisfactory results. Attempts however made with sodiomalonic ester being more successful, $\alpha\beta\beta'\gamma$ tetrachloropyridine was treated in quantity as follows.

Rather more than two molecules of sodiomalonic ester, in alcoholic solution, was treated with one molecule of the tetrachloropyridine mentioned above. The mixture was boiled for about ten hours in a reflux apparatus: at the end of that time the alcohol was distilled off and the residue acidified with sulphuric acid. The separated oil was then washed repeatedly with water, dried *in vacuo*, and fractionated under a pressure of about 28 mm. till a temperature of 150° C. was reached, when the operation was

stopped. The distillate contained a small quantity of the unchanged tetrachloropyridine, but consisted mainly of ethyl malonate, more than half of the quantity employed being thus recovered. In view of this result experiments were tried with a smaller proportion of sodiomalonic ester, when the yield was found to be perceptibly diminished. After some hours the dark-coloured residual liquid remaining in the flask became solid. This was moistened with alcohol, drained at the pump and washed with small quantities of the solvent, until almost colourless crystals remained. On purification from alcohol these separated in colourless massive forms which melted sharply at 63—64° (uncorr.).

This substance on analysis was found to be ethyl trichloropyridylmalonate having the formula

$$C_5HCl_3NCH\,(COOC_2H_5)_2.$$

It is readily soluble in alcohol, ether and the usual organic solvents. It dissolves in dilute solutions of the caustic alkalies, and is reprecipitated on acidifying. If to its solution in aqueous caustic potash a strong solution of this reagent be added, a potassium salt separates in colourless crystals which may be recrystallised from a hot moderately strong solution of the alkali. A potassium determination confirmed the formula

$$C_5HCl_3NCK\,(COOC_2H_5)_2.$$

Now it was found, by a long series of experiments, that the γ position alone, in the tetrachloropyridine used, was replaced under any of the conditions tried. Therefore in the trichloropyridylmalonic ester above described the malonic residue must be regarded as occupying the γ position. This ester, on treatment with 80 per cent. sulphuric acid, is converted into trichloropyridylacetic acid, slowly at 100° C., and more rapidly as the temperature approaches the boiling point. This substance is precipitated from the sulphuric acid solution on dilution, and may be purified by recrystallisation from water. Thus obtained it forms tabular colourless crystals which melt at 144—145° (uncorr.) and which are represented by the formula

$$C_5HCl_3NCH_2COOH.$$

The *silver* salt of the acid is a white granular substance sparingly soluble in boiling water. The *calcium* salt is soluble. The *barium* salt, in presence of ammonia, slowly separates in long needles.

On heating to its melting point, or a higher temperature, trichloropyridylacetic acid decomposes, carbon dioxide being freely evolved. If the operation be carried out *in vacuo*, trichloromethylpyridine distils over. The yield of the latter substance

is quantitative. $\alpha\beta\beta'$ trichloro γ methylpyridine as prepared in this way is practically pure. Recrystallised from alcohol it forms long colourless needles whose melting point is 31—31·5°. It exhibits the same general solubility as the chloropyridines, which it closely resembles in appearance and odour. It is volatile at the ordinary temperature and rapidly disappears when left exposed to the air. Several analyses confirm the formula

$$C_5HCl_3N\,(CH_3).$$

When trichloromethylpyridine is subjected to prolonged boiling with a solution of potassium permanganate it is converted into the corresponding acid. The boiling was conducted in a reflux apparatus, a slight excess of permanganate being employed. The excess of permanganate was removed by alcohol and after filtering off the separated hydrated manganese dioxide, the colourless filtrate was evaporated to small bulk and acidified with sulphuric acid. When allowed to remain in a desiccator the solution deposited large colourless crystals of constant melting point 188—189° C. Analyses agree with the formula

$$C_5HCl_3NCOOH.$$

This trichloroisonicotinic acid is very soluble in hot water and the ordinary organic solvents but only sparingly so in cold water.

Experiments illustrating new reactions for the identification of Urea and of Primary Amines. By H. J. H. FENTON, M.A., F.R.S.

[*Read* 2 March 1903.]

These experiments illustrated the application of a certain new derivative of methyl-furfural as a reagent for the identification of certain organic nitrogen compounds. Urea and mon-alkyl substituted ureas give a beautiful blue colour with the reagent; this colour is so intense that it is possible to detect less than $\frac{1}{100}$ milligram of urea by the method. The individual substances are further distinguished by the absorption spectra.

The reagent may also be employed as a distinguishing test for primary amines with which it produces an intense green colour.

A rapid method of estimating sugar. By T. B. WOOD, M.A., Reader in Agriculture, and R. A. BERRY.

[*Read* 2 March 1903.]

The work described in the following paper necessitated a large number of determinations of sugar in the juice of swedes. The mixture of sugars in swedes is such that polarimetric estimation is not available. A rapid and fairly accurate titrimetric method had therefore to be devised. After many trials the following method was found to be very satisfactory. The juice, after being clarified with basic lead acetate, treated with dilute acid to invert the cane sugar, and neutralized, was diluted until it contained between ·5 and 1·0 per cent. of reducing sugar. The copper solution used was made by dissolving 23·5 gms. of copper sulphate, 250 gms. of potassium carbonate, and 100 gms. of potassium bicarbonate in 1 litre. This solution was first suggested by Soldaini, *Gaz. Chim. Ital.* 6. 322, and Ost (*Ber. d. d. Chem. Ges.* 24. 14. 3003) has pointed out its many advantages over Fehling's solution and worked out a table giving the relation of sugar present to copper reduced for varying strengths of solution. The advantages are that it keeps much better than Fehling's solution, that it has much less action on cane and similar non-reducing sugars and other substances likely to be present in plant juices, and that a given weight of sugar reduces nearly twice as much copper as it does from Fehling's solution.

The only disadvantage is the necessity for rather long boiling to get constant results.

To 50 c.c. of the copper solution already boiling, 10 c.c. of the sugar solution are added, and the boiling kept up for 10 minutes. The cuprous oxide is then filtered off on asbestos felt in a Gooch's crucible and washed with boiling water. It is then transferred to a wide-mouthed stoppered bottle already filled with carbon dioxide. The last traces are washed through the Gooch with 25 c.c. of $2\frac{1}{2}$ % ferric sulphate solution in 25 per cent. sulphuric acid, and the stopper put in the bottle. A few seconds' shaking at once causes the cuprous oxide to dissolve, reducing an equivalent amount of ferric sulphate to ferrous. This is then titrated with standard solution of potassium permanganate of such strength that 1 c.c. oxidises the ferrous salt formed by the

reducing action of an amount of cuprous oxide containing ·01 gm. metallic copper. The actual strength is just over 5 gms. of potassium permanganate per litre; it is standardised by weighing the copper obtained by reducing the cuprous oxide from 10 c.c. of ·5 per cent. glucose solution, and titrating duplicates. The method is rapid, accurate, and very easy to carry out, and enabled the authors to determine the sugars in about 150 samples of swedes in a very short time. The use of permanganate in this way was suggested in the Annual Report of the U.S. Dept. of Agr. 1879, but has not been used to any extent on account of the trouble of dissolving the cuprous oxide in the ferric sulphate. This is entirely overcome by proceeding as above, and the method should now be a very convenient one.

Selection of seed by chemical methods. By T. B. Wood, M.A., Reader in Agriculture, and R. A. Berry.

[*Read* 2 March 1903.]

The selection of plants for seed-bearing is a subject which has received abundant attention, but except in a few important cases the chemical composition of the mother plants has been almost entirely neglected. Selectors have succeeded in a marked degree in improving such external characters as shape, size, colour, and cropping power, as is evidenced by the excellence of these characters in the common varieties of agricultural plants, but chemical selection has been used only in such cases as sugar-beet and wheat. The chemical selection of sugar-beet is carried out by determining the sugar in a very large number of individual roots, and growing seed only from those containing an exceptionally high percentage of sugar together with a low percentage of objectionable impurities. Such work has been carried out by many continental seedsmen, and by many of the leading sugar manufacturers, and has resulted in more than doubling the sugar content of the beet in about 50 years.

Improved varieties of wheat have also been brought out by Hays (*Univ. of Minnesota, Expt. Stn. Bulletin* 62, 1899) as the result of selecting seeds from among the produce of plants bearing seed which was found to produce flour of good colour and containing a high percentage of gluten of good quality.

Hays' system of working was in the first place the selection of varieties which suited his district as shown by their yielding well

for several years. From these approved varieties a large number of plants were selected, their seeds collected separately, and examined chemically. Seeds from the superior plants were then sown, and their seeds examined as before. After six years of such selection results were obtained which appear to be of great promise, as is shown in the annexed table.

TABLE I.

	Yield per acre	Grade	Gluten		Bakers' tests		
			%	Quality	Rise per gm. Gluten	Time of rise	Water absorbed by 100 gm. flour
Parent variety: Power's Fife	23·6	86·0	14·0	86	58·9	177	81
New variety: Minn. 149	25·6	91·3	13·5	90	66·8	222	84
Parent variety: Hayne's Blue Stem	24·6	87·0	13·4	85	59·5	196	79
New variety: Minn. 169	28·5	86·3	12·5	80	68·5	170	84

Chemical selection has evidently resulted in considerable improvement in quality, and at the same time the yield has been increased, while the appearance of the grain (grade) is fully maintained.

The method has been tried in England only in two cases. Collins (*Durham College of Science Report*, 1900, 1901) describes an application of the method to swedes, but has not yet published results showing the composition of the progeny of his chemically selected mother plants, and Messrs Carter have brought out new varieties of mangels and swedes, which they call "1901," and state to have been produced by growing seed from mother roots of exceptionally high specific gravity. This method of selection was one of the first methods tried for sugar-beet selection, and has long ago been abandoned as unreliable. The percentage of dry matter in Carter's 1901 mangel as shown in the following tables is about the same as, and the percentage of sugar rather lower than, in the Golden Tankards and Globes which it most nearly resembles, so that high specific gravity does not appear to be a safe character

7—2

to select for. It probably indicates rather absence of hollows in the root than high content of dry matter and sugar.

That systematic chemical selection is capable of improving the quality of farm crops is shown by the results of the method when applied to sugar-beet and wheat. That there is need of improvement in quality of English crops is shown by the increasing preference shown year by year by millers and maltsters for foreign wheats and barleys. Work on these crops has already been in progress for several years, and the authors, in view of the importance of root crops in East Anglian farming, the ease with which seed can be grown from mother roots which have been sampled and analysed, and the success which followed the application of chemical methods to sugar-beet selection, have therefore turned their attention to the improvement of the commonly grown root crops, and to mangels in particular, as most nearly allied to sugar-beet, and most likely therefore to be capable of similar improvement.

The plan of the work was to grow as many as possible of the best known varieties side by side at several stations, and to compare their yield and composition, with a view to deciding on the best varieties for improvement. Having decided on the best varieties, a number of picked roots of these were to be analysed, and the best kept for growing seed.

The roots were grown at the following stations:

University Farm. Mangels, 16 varieties.
Mr Garrett Taylor's Farm, Whitlingham, Norfolk.
Mangels 8 varieties.
Swedes 13 varieties.
Mr B. B. Sapwell's Farm, Cawston, Norfolk.
Mangels 8 varieties.
Swedes 13 varieties.
Major White's Farm, Saxlingham, Norfolk.
Mangels 8 varieties.
Swedes 13 varieties.

The authors take this opportunity of thanking Messrs Garrett Taylor, and Sapwell, and Major White for their hearty cooperation in the work.

The sampling of roots for analysis is a subject requiring careful consideration. The method used by the sugar-beet workers is to take a diagonal core or boring through the root along the line AB in figure 1, and this method was used for swedes by Collins (*Durham College of Science Report*, 1901), and found successful.

The advantages of the method are that it allows samples to

be taken from large numbers of roots, and that roots sampled in this way are still capable of being used as mothers for seed bearing. The authors adopted the method, but found that duplicate samples taken from 50 roots of the same variety grown side by side gave results differing far more than could be accounted for by errors in analysis.

This might be due, either to such great individual variation in the roots that 50 roots did not give an average sample, or to a defect in the actual method of sampling. The following figures show that a mixed sample taken from 50 roots should give concordant results. The percentage of dry matter was determined separately in cores taken from 150 to 200 individual roots of each variety, and entered in the order in which the roots were sampled. These percentages were then averaged in consecutive tens, twenties, and fifties.

The above figures show that mixed samples of 50 roots give concordant results, and apparently therefore indicate with sufficient accuracy the composition of the roots on the plot from which they were taken. It is clear also that when the mixed samples are taken from only 10, or even from 20 roots, they give no approach to an accurate indication of the composition of the crop, and this probably explains the discrepancies in the results of many experimenters.

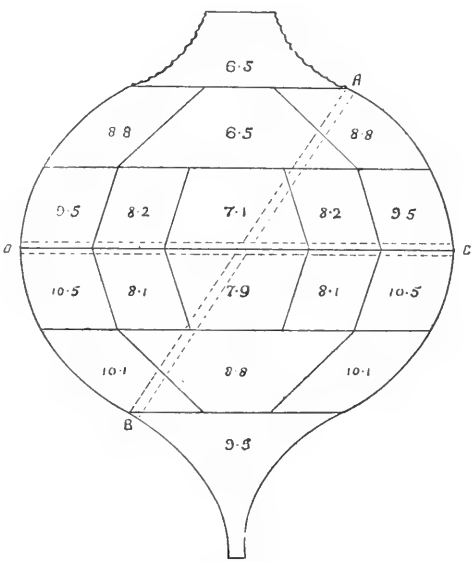


Fig. 1.

TABLE II.

Averages of consecutive tens

	Sutton's Golden Globe		Webb's Golden King		Carter's 1901	
	14·96	14·79	15·81	14·80	15·41	14·04
	13·97	14·39	15·49	15·17	14·02	13·60
	15·10	15·07	15·12	14·61	15·08	14·59
	14·71	13·56	14·78	15·62	13·76	15·01
	14·79	13·56	15·37	15·87	14·33	15·08
	15·17	15·13	15·20		14·42	14·61
	15·31	14·26	14·75		15·28	
	14·35	14·90	15·38		14·29	
	15·31	15·40	15·59		13·95	
	14·29	14·90	15·50		15·08	
Mean...	14·70		15·27		14 53	
Greatest errors...	+ ·70,	− 1·14	+ ·60,	− ·66	+ ·88,	− ·93

Averages of consecutive twenties

	Sutton's Golden Globe		Webb's Golden King		Carter's 1901	
	14·47	14·59	15·65	14·98	14·71	13·82
	14·91	14·32	14·95	15·12	14·42	14·80
	14·98	14·35	15·28		14·47	14·84
	14·83	14·58	15·06		14·79	
	14·75	15·15	15·54		14·51	
Mean...	14·70		15·27		14·53	
Greatest errors...	+ ·45,	− ·38	+ ·38,	− ·32	+ ·31,	− ·71

Averages of consecutive fifties

	Sutton's Golden Globe		Webb's Golden King		Carter's 1901	
	14·71		15·31		14·52	
	14·89		15·38		14·60	
	14·47		15·21		14·49	
	14·92					
Mean...	14·70		15·27		14·53	
Greatest errors...	+ ·22,	− ·23	+ ·11,	− ·06	+ ·07,	− ·04

If the lack of concord between the duplicates is not due to the samples having been taken from too small a number of roots, it must be due to some defect in the method of taking the samples. The most obvious variation likely to occur would be an error in the angle at which the boring was taken, and inspection of fig. 1 which shows the distribution of sugar in a mangel as determined from the examination of a number of roots, makes it clear that such an error would easily account for great variation, since a slight alteration in the inclination of the line AB, along which the boring was taken, would make a very great difference in the composition of the core.

The authors have therefore taken their cores horizontally through the greatest diameter of the root, *i.e.* along the line CD. This position is much more definite and easy to fix, and the variation in composition in this region is much less marked.

Since adopting this precaution it has been possible to obtain much more concordant results.

In sampling the varieties duplicate lots of 50 cores were taken from each variety, very large and very small roots being avoided.

Each bundle of 50 cores was cut across transversely. One half was dried at 65° C. to determine the percentage of dry matter. The other half was pulped, and the juice expressed for the determination of the percentage of sugar. In the mangel the sugar was determined by the polarimeter, after clarification with basic lead acetate, and the result checked by polarising again after inversion by Clerget's method. It was found in every case that the sugar present consisted almost entirely of cane sugar, a small amount, usually under ·5 per cent., of a dextro-rotatory sugar, shown to be dextrose by its cupric reducing power, being also present.

In the swedes the sugars were found to be cane sugar, dextrose, and levulose in such proportions that the juice possessed practically no rotatory power. The polarimeter therefore could not be used for their determination, and recourse was had to the rapid volumetric method described in the preceding paper.

The authors regret that they were unable to determine the yield per acre except in the case of the mangels grown at the University Farm.

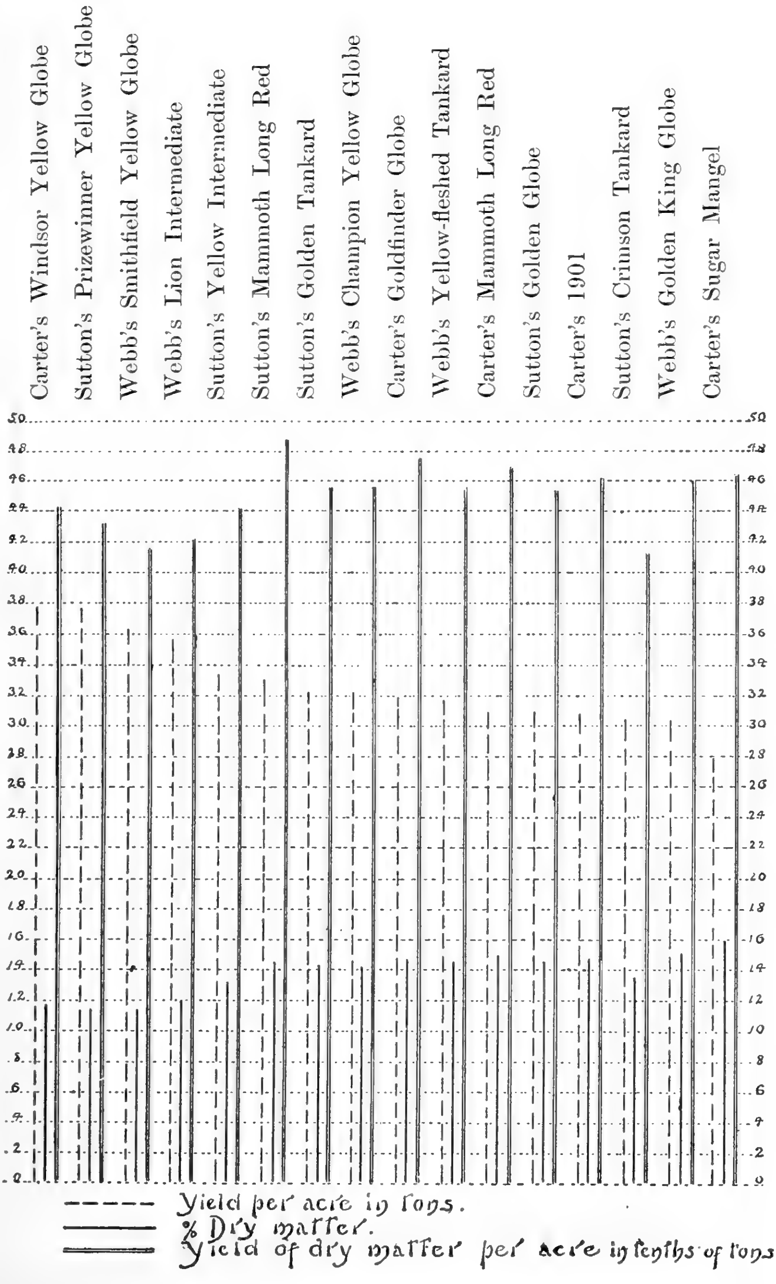
Mangels. The following tables give the yields per acre of mangels at the University Farm, and the percentages of dry matter and sugar in the mangels grown there and at the three stations in Norfolk.

TABLE III.

Mangels	University Farm			Whitlingham		Cawston		Saxlingham	
Varieties	Yield per acre, Tons	% Dry Matter	% Sugar	% Dry Matter	% Sugar	% Dry Matter	% Sugar	% Dry Matter	% Sugar
1. Sutton's Prize Winner Yellow Globe ...	37·6	11·5	6·7	10·3	6·2	9·7	5·7	10·4	8·2
2. Webb's New Smithfield Yellow Globe	36·2	11·5	7·2	12·1	7·7	10·5	6·8	12·2	8·1
3. Carter's Windsor Yellow Globe	37·8	11·7	6·9	11·1	7·6	9·9	6·2	11·5	7·7
4. Webb's Champion Yellow Globe.........	32·1	14·2	8·8	—	—	—	—	—	—
5. Sutton's Golden Globe	31·1	14·6	9·6	15·0	9·6	13·0	8·5	12·6	8·7
6. Webb's Golden King	30·3	15·2	9·7	—	—	—	—	—	—
7. Carter's Goldfinder	32·0	14·9	9·1	—	—	—	—	—	—
8. Sutton's Yellow Intermediate	33·5	13·2	8·1	—	—	—	—	—	—
9. Webb's Lion Intermediate	35·8	11·8	6·9	—	—	—	—	—	—
10. Sutton's Golden Tankard..................	32·1	14·3	8·8	13·6	8·7	12·5	8·4	14·6	9·5
11. Webb's Yellow Fleshed Tankard.........	31·8	13·7	8·6	—	—	—	—	—	—
12. Sutton's Crimson Tankard	30·7	13·5	8·4	—	—	—	—	—	—
13. Carter's 1901..................................	31·0	14·9	8·7	14·7	9·2	12·8	8·0	12·0	9·0
14. Carter's Sugar	27·9	16·0	10·9	15·7	9·9	14·7	9·4	15·8	11·0
15. Sutton's Mammoth Long Red	33·2	14·7	8·7	13·6	8·4	11·9	7·6	12·2	9·4
16. Carter's Mammoth Long Red	31·1	15·1	8·9	—	—	—	—	—	—

TABLE IV. MANGELS.

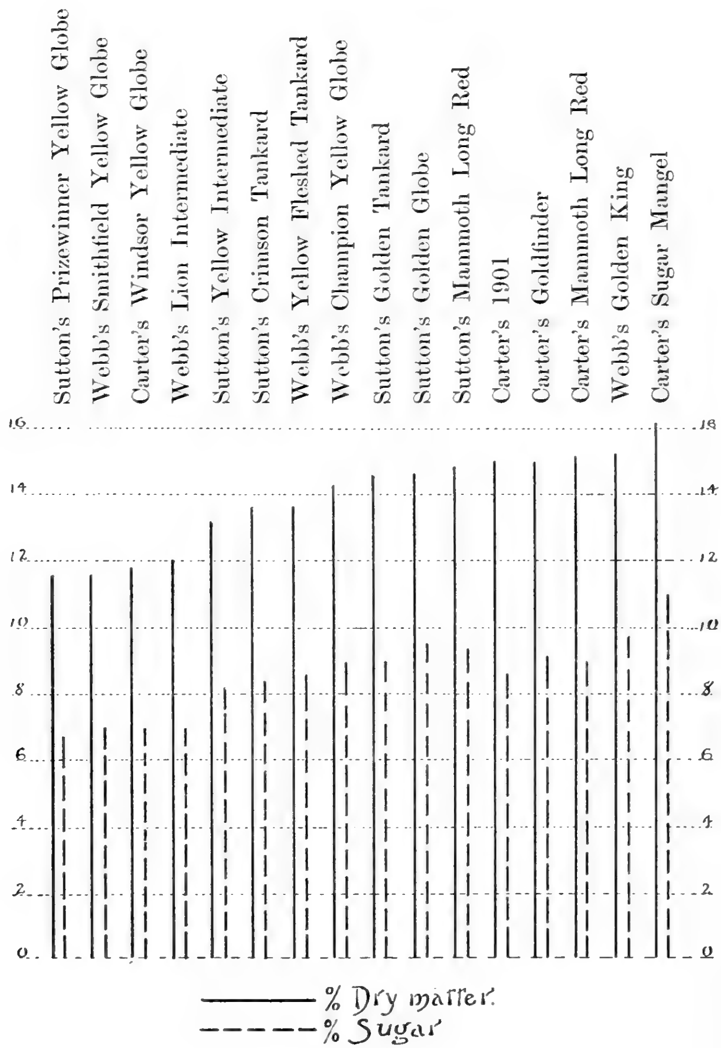
University Farm.



The tables expressing the results at the University Farm show the yield per acre, the percentage of dry matter, and the weight of dry matter per acre for each variety. It will be noticed that similar

TABLE V. MANGELS.

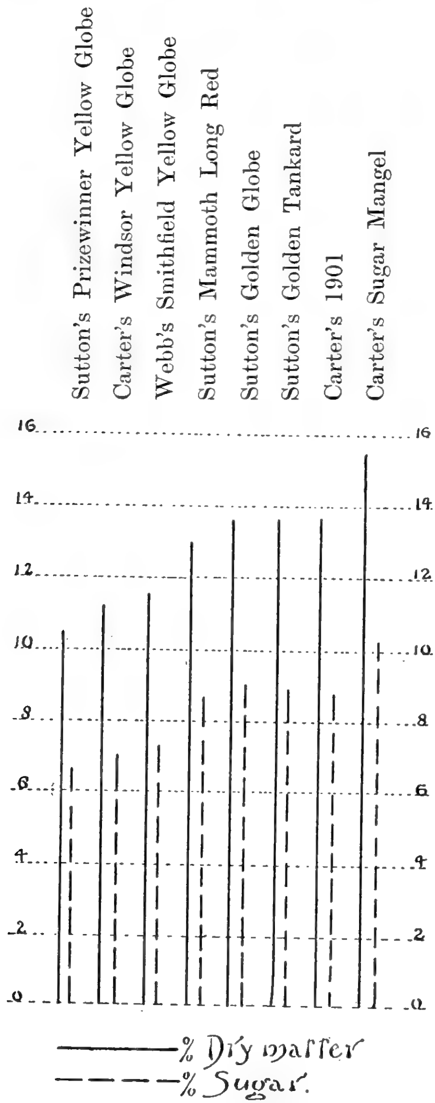
University Farm.



varieties agree in every respect. Thus Windsor, Prize Winner, and

TABLE VI. MANGELS.

Average of four Stations.



Smithfield yellow globes, which are really seedsmen's strains of the same variety, all agree closely in high yield per acre, and low percentage of dry matter. Again the two intermediates show the same concordance, as do also the two long reds, and the five golden or yellow fleshed globes. Such agreement between these obviously very closely related strains of the same varieties is very satisfactory as indicating the reliability of the general results. Crimson tankard appears to be quite a distinct variety, but neither its yield nor its composition is good.

Carter's sugar mangel, though a poor yielder, has such a large percentage of dry matter that it stands fourth in order of merit for yield of dry matter per acre. This variety is a hybrid between the long red mangel and the sugar-beet, and in its present condition possesses no particularly good points except its high content of dry matter and sugar.

It is by no means true to type, many pure white roots and red roots occurring as well as pink ones, it has a very pronounced tendency to run to seed, and a much-fanged root which buries itself deeply, entailing much trouble in lifting. It is nevertheless an interesting variety on account of its high content of food material, and is no doubt capable of improvement by judicious selection. Carter's 1901 is also a very interesting variety. It is stated to be the result of some years' selection, which has apparently been carried out by growing seed only from roots of high specific gravity. Its yield and composition are about on a par with those of the golden or yellow fleshed globes, which it resembles in other respects, but it does not grow so true to type as they do. Turning to the diagrams expressing the results at the other stations it will be seen that they confirm in almost every case those at the University Farm, and this confirmation lends further confidence to the results.

It is now possible to classify the varieties which have been submitted to experiment as follows:

White fleshed yellow globes. Yield good, but quality low.

Golden or yellow fleshed globes. Yield moderate, but quality much better.

Intermediates. Yield very fair, quality fair.

Long red. Quality good, but troublesome to lift and only suited to deep soils, on which it would have given a much better yield.

It was obvious early in the season that the yellow fleshed globes were among the best varieties, but roots for individual analysis for selection of mothers had to be saved before the complete results for all the varieties were ready. Consequently the varieties chosen for chemical selection, namely Sutton's golden

globe, Webb's golden king, and Carter's 1901 are not absolutely the best.

In the autumn 200 roots of each of these varieties were picked out, attention being given to shape, size, and general excellence of external characteristics. This was kindly done by Mr R. H. Biffen, M.A., of Emmanuel College, Botanist to the Department of Agriculture. For this and much other assistance the authors tender their best thanks. These picked roots were carefully stored over the Christmas vacation, and were then again picked over, weighed, numbered, and cored. Each core was dried to constant weight, and its percentage of dry matter calculated. Extraordinary variation was noticed in each case, as is shown in the following diagrams.

These diagrams, figs. 2 and 3, show several points of interest. First that the percentage of dry matter in individual roots of the same variety grown side by side and under conditions of as great uniformity as is obtainable in ordinary farm practice, varies very greatly—in each of the three cases from about 11 per cent. to about 20 per cent. Secondly, that the variation is of the usual type—that is to say a few individuals with very low percentages, a few with very high percentages, and a large number intermediate between these, rising by short steps continuously from the lower to the upper limit. Thirdly that (fig. 2) while there is a distinct tendency for the percentage of dry matter to be higher in small roots than in large ones, the inverse proportionality between size of root and content of dry matter is by no means so definite as is usually taken for granted. These diagrams furnish the necessary information for the commencement of chemical selection. Seed will this spring be grown from roots of each variety which contain the highest percentage of dry matter, precautions being taken to prevent cross-fertilization from plants of inferior quality.

This seed will not be ready to sow until the spring of 1904, and its progeny will not be ready for investigation until that autumn. In the meantime the work already done will be repeated, and extended. As yet the individual variation in percentage of nitrogen, sugar, and cellulose has only just been commenced.

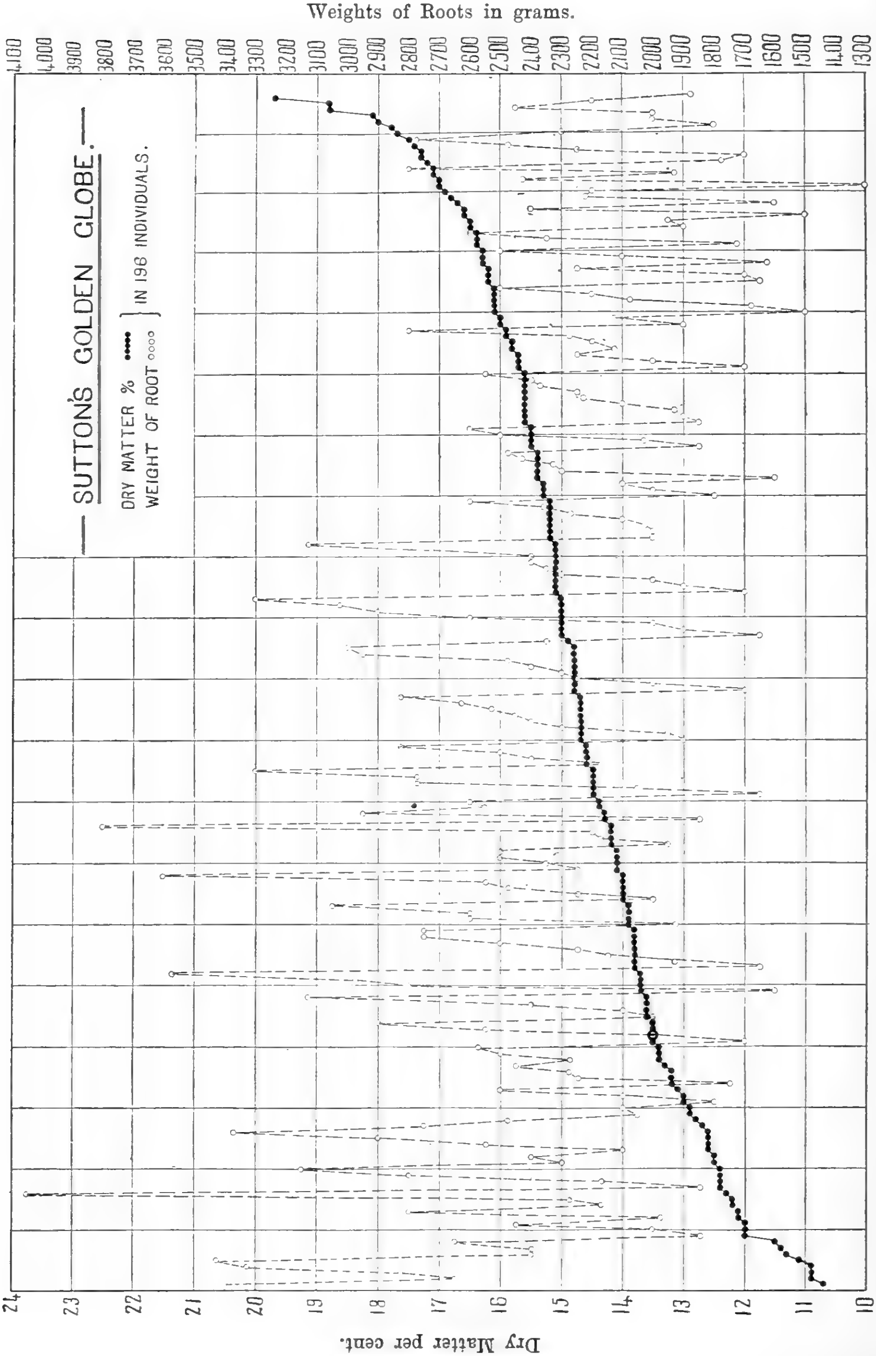

Weights of Roots in grams.
4100
4000
3900
3800
3700
3600
3500
3400
3300
3200
3100
3000
2900
2800
2700
2600
2500
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
SUTTON'S GOLDEN GLOBE.
DRY MATTER %
WEIGHT OF ROOT
IN 196 INDIVIDUALS.
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
Dry Matter per cent.

Fig. 2.

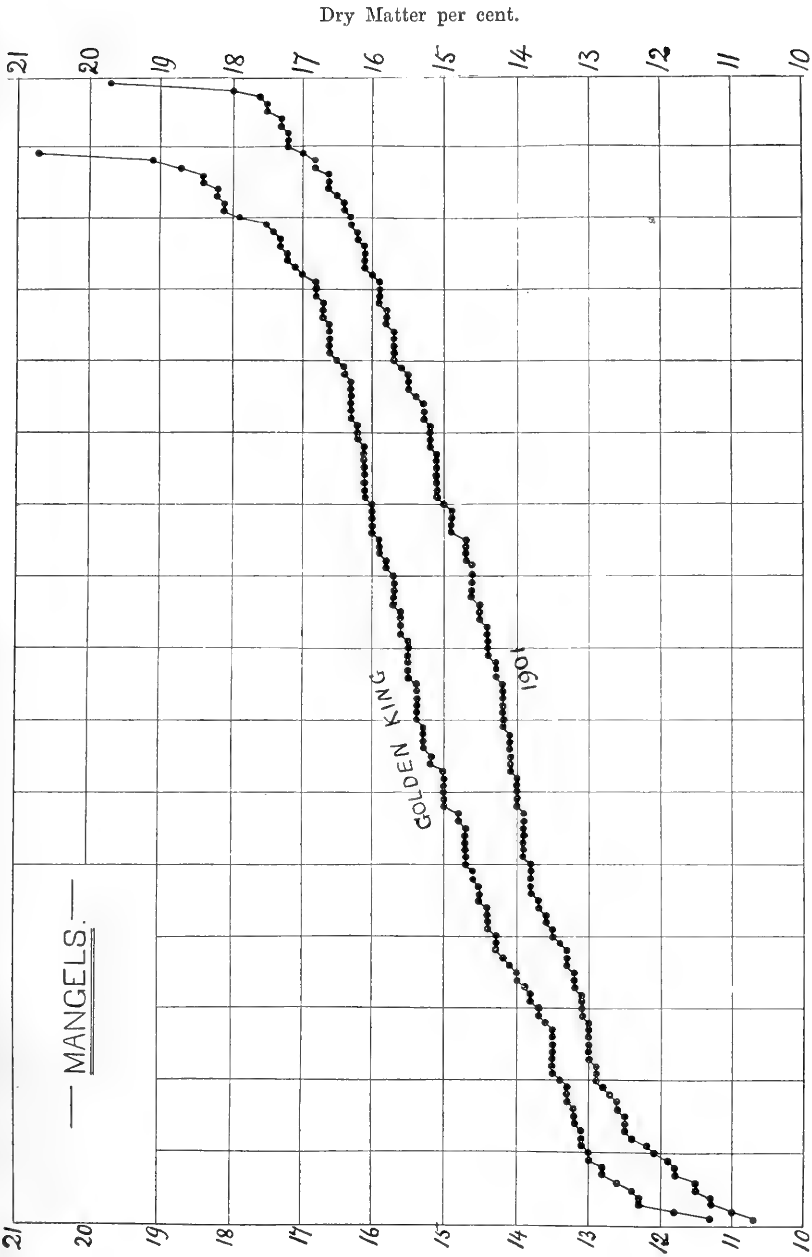

Dry Matter per cent.
21 20 19 18 17 16 15 14 13 12 11 10
MANGELS.
GOLDEN KING
1901

FIG. 3.

Swedes. The results of analyses of thirteen varieties of swedes grown at the three stations in Norfolk already mentioned are shown in the following table.

TABLE VII.

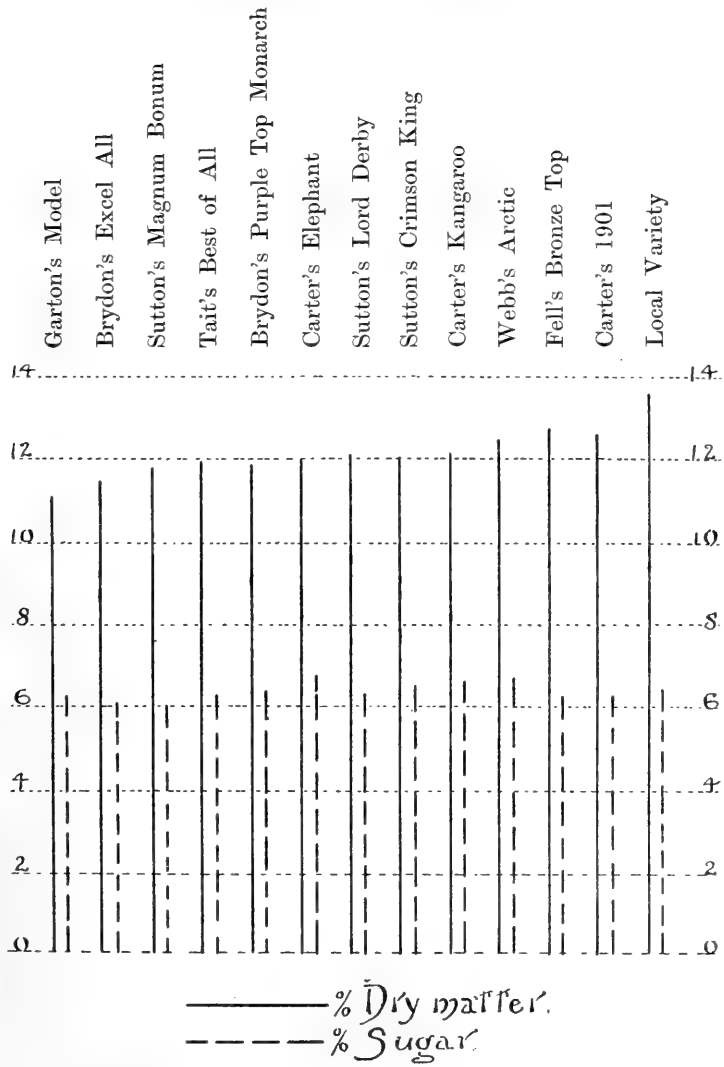
Swedes	Whitlingham		Cawston		Saxlingham	
Varieties	% Dry Matter	% Sugar	% Dry Matter	% Sugar	% Dry Matter	% Sugar
1. Sutton's Crimson King Purple top...	11·6	6·4	11·5	6·4	13·1	6·6
2. Sutton's Lord Derby Green top..	11·5	5·9	12·2	6·3	12·6	6·8
3. Sutton's Magnum Bonum Purple top	11·8	5·7	11·5	6·2	12·2	6·4
4. Carter's Kangaroo Bronze top	11·9	6·7	12·0	6·4	12·7	6·8
5. Carter's Elephant Purple top	11·3	6·7	12·3	6·7	12·2	6·6
6. Webb's Arctic Bronze top	12·9	7·1	12·4	6·6	12·1	6·4
7. Tait's Best of All Purple top	11·8	6·3	11·5	6·3	12·0	6·5
8. Fell's Bronze top	12·3	5·8	12·9	6·4	13·0	7·0
9. Brydon's Purple top Monarch	10·9	5·7	12·3	6·6	12·1	6·8
10. Brydon's Excel All Green Bronze top	11·5	6·3	11·5	6·4	11·4	5·8
11. Carter's 1901 ...	12·6	6·3	13·1	6·3	12·5	6·3
12. Garton's Model Green Bronze top .	11·5	6·5	10·6	7·2	10·9	6·1
13. Local Variety Green top	13·1	6·2	13·8	7·0	14·0	6·4

The results entirely confirm those already obtained by Collins (*Durham College of Science Report,* 1901). The same varieties appear to be good and bad in quality both in Norfolk and in Northumberland. The best swede however of those grown in Norfolk is a local variety, for the seed of which the authors are indebted to the kindness of Mr Herbert Smith of Stody Hall, Melton Constable, on whose farm it has been grown continuously from the same stock of seed for over 70 years. It is a very large

swede, grows a very big crop, and contains an exceptionally high percentage of dry matter and an average percentage of sugar. Its composition is being further investigated, and if found satisfactory, this swede should prove a very valuable one for the district.

TABLE VIII. SWEDES.

Average of three Stations.



Mr Smith kindly allowed 120 of his own selected mother roots to be cored for determination of dry matter. The results are shown in the accompanying diagram (fig. 4), which shows that

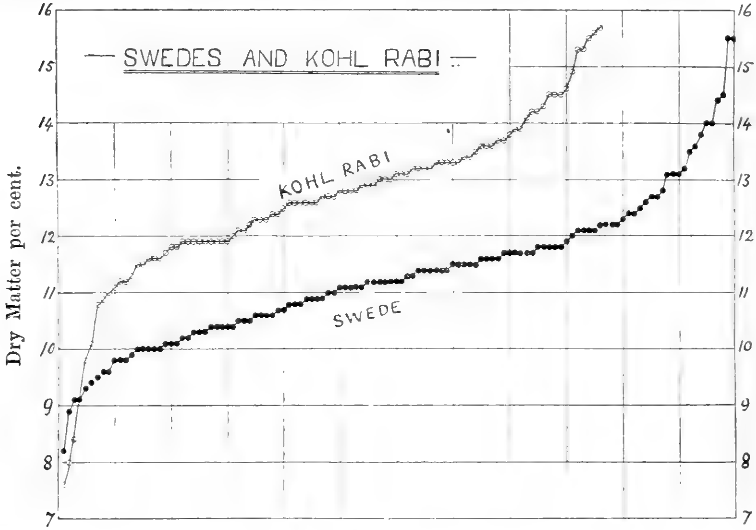


Fig. 4.

the variation of individual swedes is similar in every way to the variation among individual mangels. Seed will be grown separately from several selected roots as in the case of the mangels. In the meantime a further investigation is proceeding into the composition of the dry matter of the different varieties. The diagram also shows the variation in percentage of dry matter in 100 kohl rabis kindly presented by Mr Smith Rowley of Histon. They too vary just as the mangels, and the best roots have been picked out for growing seed.

The authors wish to express their thanks to Professor Middleton for advice in the conduct of the work, and for placing at their disposal the resources of the University Farm, to Mr Henry Giles, the superintendent of the field experiments of the Norfolk Chamber of Agriculture, and Mr H. Henshaw, manager of the University Farm, for the care they took in attending to the growing of the roots, to Mr J. Goodchild, B.A., of Clare College, who has sampled all the roots, and to Messrs S. F. D. Harwood, B.A., of St John's College, and W. Cartwright, B.A., of Clare College, who have given assistance when the samples came in too rapidly to be dealt with by the authors themselves.

Modes of Formation of Osones. By R. S. MORRELL, M.A., Caius College, and J. M. CROFTS, M.A., Emmanuel College.

[*Received* 23 March 1903.]

Hexahydric alcohols of the general formula

$$CH_2OH.(CHOH)_4CH_2OH,$$

admit of oxidation to give aldehyde alcohols, one of the terminal CH_2OH groups being oxidised to the aldehyde grouping. Several methods of oxidation have been used to bring about this change. The action of bromine and caustic soda on dulcitol gives rise to an aldehyde alcohol whose osazone is probably identical with galactosazone [E. Fischer, *Ber.* XX. 3390]. Hydrogen peroxide and ferrous sulphate acting on mannitol will transform the hexahydric alcohol into mannose [Fenton and Jackson, *J.C.S.* LXXV. 9]. If the hexose group of aldoses is represented by the general formula

$$CHO(CHOH)_4CH_2OH \quad \text{or} \quad \underbrace{CHOH.(CHOH)_2CH}_{O}\,CHOHCH_2OH,$$

the products of further oxidation of these aldoses may be monobasic or dibasic acids. Glucose may yield gluconic acid

$$COOH(CHOH)_4CH_2OH,$$

which on further oxidation can be transformed into *d* saccharic acid, $COOH(CHOH)_4COOH$. The oxidation may however proceed in another direction, the (CHOH) grouping adjacent to the aldehyde grouping in the aldoses or the primary alcoholic group adjacent to the ketone group in the ketoses may be oxidised.

$$\begin{array}{llcll}
CHO & CHO & & CH_2OH & CHO \\
(CHOH)_4 \longrightarrow & CO & & CO & CO \\
(CHOH)_3 & (CHOH)_3 & \text{and} & (CHOH)_3 \longrightarrow & (CHOH)_3 \\
CH_2OH & CH_2OH. & & CH_2OH. & CH_2OH \\
d\ \text{glucose.} & d\ \text{glucosone.} & & d\ \text{fructose.} & d\ \text{glucosone.}
\end{array}$$

[Morrell and Crofts, *J.C.S.* LXXV. 716.]

Evidence in favour of the oxidation proceeding in this direction is not new. The action of phenylhydrazine on carbohydrates

to give osazones points to the intermediate formation of such aldehyde ketone alcohols. The formation of an insoluble hydrazone of mannose and the transformation of mannose hydrazone by the action of excess of phenylhydrazine into mannosazone, identical with glucosazone, have been explained in the following way: the aldehyde grouping first reacts with phenylhydrazine, and then the excess of phenylhydrazine is enabled, by the withdrawal of two hydrogen atoms from the (CHOH) adjacent to the aldehyde group, to react with the ketone grouping thus formed to give an osazone [E. Fischer, *Ber.* XX. 821].

These ketone aldehyde alcohols (osones) were first prepared by E. Fischer, who removed from the corresponding osazones the two phenylhydrazine residues by careful treatment with concentrated hydrochloric acid [*Ber.* XXII. 87].

$$\begin{array}{lcl}
CH{-}N{-}NHC_6H_5\,. & & CHO \\
\vert & & \vert \\
C{-}N{-}NHC_6H_5\,. & \longrightarrow & CO \\
\vert & & \vert \\
(CHOH)_3 & & (CHOH)_3 \\
CH_2OH\,. & & CH_2OH\,. \\
\text{phenylglucosazone.} & & \text{glucosone.}
\end{array}$$

The yield of osone was good, if small quantities of the osazones were used. The isolation of pure glucosone was troublesome, since the osone had to be precipitated from solution by means of lead hydroxide, with which it formed a loose chemical compound. This method has not yielded the glucosone in a crystalline form, but only as a syrup. The characteristic properties of the osones were their strong reducing action on Fehling's solution, the readiness with which they reacted with phenylhydrazine at the ordinary temperature and the ease with which they were transformed into furfurol on being heated with water at 130° C. By this method the following sugars were transformed into corresponding osones: *d* glucose, galactose, and rhamnose [E. Fischer, *Ber.* XXII. 96]. Glucosone, obtained from phenylglucosazone, when reduced by zinc dust, was transformed into *d* fructose. This shewed that the aldehyde grouping of the osone is reducible to the primary alcoholic group CH_2OH [E. Fischer, *Ber.* XX. 94].

In 1902, E. Fischer [*Ber.* XXXV. 3141] obtained osones from the more soluble osazones of the disaccharides by means of Herzfeld's method, whereby the phenylhydrazine residue of a hydrazone was removed by benzaldehyde. Benzaldehyde and an osazone reacted to form benzaldehyde hydrazone and the corresponding osone.

$$2\,C_6H_5CHO + \underset{\text{maltosazone.}}{C_{24}H_{32}N_4O_9} = \underset{\text{maltosone.}}{C_{12}H_{20}O_{11}} + 2\,[C_6H_5CH{-}N{-}NHC_6H_5]$$

The maltosone was obtained as an uncrystallisable syrup.

The yield of maltosone, if small quantities of maltosazone were used, was very good, being 70 °/$_\circ$ of the theoretical amount [E. Fischer, *loc. cit.*]. We have not been so successful in obtaining so good a yield of maltosone when 20 grams of maltosazone were worked up in one operation.

The osones of several synthetical disaccharides, galactosidogalactose, glucosidogalactose and galactosidoglucose (melibiose ?) were obtained by this method [E. Fischer, *loc. cit.*]. The transformation of the osazones of the hexoses into osones is not so satisfactory, owing to the insolubility of these osazones in water. The addition of alcohol, although it brings the osazone into solution, does not accelerate the reaction of the osazone with benzaldehyde, since benzaldehyde hydrazone is also soluble in alcohol. Phenylglucosazone has not been transformed into glucosone by this method, although we have tried heating it with benzaldehyde and alcohol and water under pressure, and also with benzaldehyde and alcohol in a sealed tube to 125° C. Formalin does not transform phenylglucosazone into glucosone when heated with it in the presence of water. The osazone is undoubtedly attacked by the formalin, but the solid product is of an uninviting appearance, and the filtrate, after it has been shaken with ether and concentrated in a vacuum does not contain glucosone. By means of orthonitrobenzaldehyde glucosazone can be partially transformed into glucosone. The change does not occur when the osazone is boiled with water and with sufficient alcohol to keep the orthonitrobenzaldehyde in solution; but if orthonitrobenzaldehyde, dissolved in 50 °/$_\circ$ alcohol, is heated with glucosazone in a sealed tube to 125° C. interaction between the osazone and the aldehyde takes place, and about 20 °/$_\circ$ of the osazone is changed into glucosone. If rhamnosazone is heated with orthonitrobenzaldehyde and alcohol and water in a sealed tube at 125° C. for several hours, a small quantity of rhamnosone, about 6 °/$_\circ$, is obtained, but if the temperature is raised to 140°, no rhamnosone can be detected. Since glucosone is easily transformed into furfurol on being heated with water to 140° C., it is probable that the rhamnosone is transformed into a methyl furfurol derivative at this temperature. By applying Herzfeld's method to the preparation of rhamnosone from rhamnosazone, modified by the addition of a small quantity of alcohol in order to bring the osazone into solution, we have obtained a yield of nearly 20 °/$_\circ$ of rhamnosone.

Rhamnosone is very soluble in ethyl and methylalcohol. From an alcoholic solution acetone or ether precipitate rhamnosone in a syrupy form, but as yet we have been unable to obtain it crystalline. Rhamnosone, prepared by this method, possesses the characteristic properties of the osones; it reacts readily with substituted hydra-

zones, and reduces strongly a Fehling's solution. The dioxim has not been obtained crystalline. An aqueous solution of the dioxim, on being allowed to stand with phenylhydrazine at the ordinary temperature, gives phenylrhamnosazone (mp. 178°).

The third method for transforming carbohydrates into osones is by careful oxidation of hexoses or pentoses, in dilute aqueous solution, by hydrogen peroxide in the presence of a small quantity of ferrous sulphate [Morrell and Crofts, *J.C.S.* LXXV. 786; LXXVII. 1219, LXXXI. 666; Cross, Bevan and Smith, *J.C.S.* LXXIII. 463].

Hydrogen peroxide and ferrous sulphate, acting as oxidising agents, have been referred to earlier in this paper in connection with the oxidation of mannitol to mannose, and these combined oxidising agents have been used by Fenton to transform tartaric acid into dihydroxymaleic acid [Fenton, *J.C.S.* LXV. 899].

In the oxidation of a carbohydrate by hydrogen peroxide in the presence of ferrous sulphate, the peroxide must be added in small quantities at a time. The ferrous iron is first oxidised to the ferric state and slowly passes back to the ferrous condition. The action of the ferrous sulphate may be compared with that of manganous sulphate in cases of oxidation by oxydases.

According to Bertrand [*Comptes Rendus*, CXXIV. 1356] manganese is present in combination as a manganous salt in oxydases, and is transformed into manganese dioxide by the oxygen rendered active by the enzyme; the manganese dioxide acts as an oxidiser, and is reduced to manganous oxide, which combines with the acid albuminous material present to form a manganous salt.

Bach and Chodat [*Ber.* XXXV. 2466] have stated that they have discovered the formation of peroxides in living cells of plants, caused by the action of a class of enzymes known as oxydases or oxygenases. In addition to these enzymes Bach and Chodat have shewn the existence of peroxydases, which possess the power of making active the hydrogen peroxide formed by the oxygenases. It was difficult to separate peroxydases from oxygenases, though a partial separation could be effected by dialysis, the peroxydases passing through the dialyser into the water [Bach and Chodat, *Ber.* XXXVI. 606]. The peroxydases contained small quantities of manganese and aluminium, but no iron. They possessed no reducing properties towards a Fehling's solution, and were not oxidisers [Bach and Chodat, *Ber.* XXXVI. 600]. It would appear as if the oxydases were reducers, absorbing oxygen to form peroxides, which in the presence of an 'acceptor' give up their active oxygen. The acceptor passes this oxygen on to any easily oxidisable substance. Investigations on the action of autoxidisers such as benzaldehyde in the presence of metallic salts (peroxydases) are in

progress, and we hope at an early date to be able to make a communication on the results, which so far promise to agree with what was predicted. In the formation of the osones from hexoses or pentoses by the action of hydrogen peroxide in the presence of ferrous sulphate, it is advisable to allow part of the ferric iron formed during the oxidation to return to the ferrous state, before more hydrogen peroxide is added. When sufficient hydrogen peroxide has been added, one gram molecule of hydrogen peroxide for one gram molecule of the carbohydrate, the solution is treated with lead acetate and basic lead acetate, filtered, and the filtrate treated with excess of baryta water, whereby the precipitated lead hydroxide withdraws the osone from the solution as a loose lead hydroxide compound. After washing with water, the lead hydroxide compound is decomposed exactly by 30 °/₀ sulphuric acid, and the filtrate concentrated in a vacuum at 50° C. to a syrup. A certain amount of the osone in the lead hydroxide compound undergoes oxidation during the washing with water, yielding an acid, which gives an insoluble lead salt with normal lead acetate. The syrup containing the osone is not free from the parent sugar. In the case of fermentable sugars, fermentation with yeast will remove the carbohydrate, leaving the nonfermentable osone. The aqueous solution of the glucosone, obtained by this method of oxidation from glucose, reacts readily at the ordinary temperature with phenylhydrazine acetate and also with methyl phenylhydrazine acetate [Morrell and Crofts, *J.C.S.* LXXV. 786]. The methyl phenylosazone is identical with that obtained by Neuberg from *d* fructose [*Z. Ver. Rubenzucker Ind.* 1902, 237]. Paranitrophenylhydrazine dissolved in glacial acetic acid reacts immediately with glucosone at the ordinary temperature. The dark red osazone possesses the same melting-point as that obtained from the action of nitrophenylhydrazine on *d* glucose at 100° C. All these osazones were analysed and their melting-points compared with those obtained directly from *d* glucose or *d* fructose.

Glucosone obtained from *d* glucose or *d* fructose is feebly optically active. It is very soluble in water, in hot alcohol, and is insoluble in acetone or ether. On treatment of a methyl alcohol solution of glucosone with acetone the glucosone is precipitated as an amorphous powder, which on analysis gives numbers corresponding to the formulae $C_6H_{10}O_6$ or $C_6H_{12}O_6$. The amorphous solid possesses the characteristic properties of osones. [Morrell and Crofts, *J.C.S.* LXXXI. 666.]

Glucose is not the only carbohydrate which, on oxidation with hydrogen peroxide in the presence of ferrous sulphate, yields an osone. If *d* fructose is oxidised with these oxidising agents, glucosone is formed, which possesses all the properties of the same substance obtained from glucose. Mannose also yields the same

glucosone, which reacts with phenylhydrazine *at the ordinary temperature* to give glucosazone.

Galactose, on oxidation, has not as yet yielded the corresponding osone, and the investigation of the products is still being carried on. Of the pentose group of carbohydrates, arabinose and methyl pentose (rhamnose) have been investigated. Arabinose yields very readily an osone, which reacts with methyl phenylhydrazine at the ordinary temperature, giving an osazone identical with that obtained by Neuberg from the oxidation product of arabitol [*Z. Ver. Rubenzucker Ind.* 1902, **237**].

Rhamnose, on oxidation with hydrogen peroxide in the presence of ferrous sulphate, yields an osone. The osone has the same properties as that prepared from rhamnosazone by the second method described in the paper.

Saccharose, when oxidised by neutralised hydrogen peroxide in the presence of ferrous salt, is, at first, apparently hydrolysed to glucose and fructose, which on oxidation yield glucosone [Morrell and Crofts, *J.C.S.* LXXVII. 1219]. The disaccharides seem to react with the oxidising agent in dilute solution more slowly than the hexoses or pentoses. Maltose and lactose are, in comparison, only slightly acted on to give substances of an osone nature. In all these cases of oxidation ferrous sulphate seems to play the part of a peroxydase. It is possible that the oxydase in the root of the horse radish contains a substance with the properties of an aldehyde [Bach and Chodat, *Ber.* XXXVI. 603] which acts as an autoxydiser like benzaldehyde [P. Engler and Wild, *Ber.* XXX. 1669] to give a peroxide, which, in the presence of the peroxydase, furnishes active oxygen.

The oxidation of carbohydrates in plants to give substances of an osone nature is not highly improbable. If saccharose is supposed to be the first sugar formed by assimilation in plants [Horace Brown and Morris, *J.C.S.* 1893, 604], this sugar might be slowly acted on by the oxydases and peroxydases, being first inverted and then partially oxidised to an osone. We have noticed that fructose is more easily acted on by these oxidising agents, and the part played by the ferrous sulphate as peroxydiser is easy to follow in the oxidation of fructose by hydrogen peroxide. Oxidising agents such as bromine and nitric acid do not seem to act on fructose more rapidly than on glucose, and the stronger action of hydrogen peroxide on fructose is very peculiar. When once an osone is formed, it is able to react with any substance of a basic nature or which contains an amine group; moreover the osones are easily oxidised in the presence of alkalies [Fischer, *Ber.* XXII. 87], and in the presence of acid oxidising agents they seem to lose two carbon atoms to yield a dihydroxybutyric acid [Morrell and Crofts, *J.C.S.* LXXXI. 666].

The first oxidation product of glucosone would be oxygluconic acid, $COOH\ CO(CHOH)_3CH_2OH$, whose amino derivative would have the formula

$$COOH . C \begin{matrix} \diagup NH_2 \\ \diagdown OH \end{matrix} (CHOH)_3CH_2OH.$$

The amino acids obtainable from albumen by the action of hydrolysing agents have the amino group attached to the same carbon atom with which the grouping COOH is united [A. Kossel, 'Ueber den gegenwärtigen Stand der Eiweisschemie,' *Ber.* xxxiv. 3214]. Arginin, Ornithin, $CH_2NH_2CH_2CH_3CH\ NH_2COOH$, and Lysin, $CH_2NH_2CH_3CH_2CH_3CHNH_2COOH$, and chitaminic acid (glucosaminic acid), $CH_2OH(CHOH)_3CH\,NH_2COOH$ [E. Fischer and Leuchs, *Ber.* xxxv. 3787] all have the amino group in the α position. From arabinose by the action of hydrocyanic acid Fischer and Leuchs have obtained *l* glucosaminic acid, which is the optical antipode of chitaminic acid; they have also obtained from galactose the corresponding aminogalactoheptonic acid. Derivatives of α amino acids, containing a chain of five or six carbon atoms, are obtainable from aldehyde alcohols [Fischer and Leuchs, *Ber.* xxxv. 3787], and such acids might be obtained from the first oxidation products of the osones. We have not found any reference in the literature to the occurrence of osones in nature, and since the properties of these ketoaldehyde alcohols are so peculiar, a full investigation of their properties seems to be of the greatest importance.

Note on the Stereochemistry of Benzene. By H. O. JONES, M.A., Clare College, and J. KEWLEY, B.A., King's College.

[*Read* 2 March 1903.]

The question whether derivatives of benzene can shew optical activity, due to the asymmetry of the carbon atoms in the ring, or not, is one of great importance in deciding the constitution of benzene and its derivatives. According to the constitutional formulae proposed by Ladenburg, Claus and Vaubel, in all of which the six carbon atoms are not in a plane and each is singly linked to three other carbon atoms, all meta (1.3) disubstituted derivatives, and all trisubstituted derivatives have no plane of symmetry: such compounds should therefore be capable of existing in two enantiomorphously related, optically active forms, and, if they do not, these formulae are excluded. According to the other formulae proposed by Kekulé, Collie and others, in which all the carbon atoms are supposed to lie in the same plane or the molecule is assumed to be in a continual state of transition, such optical activity would not be expected.

The fact that naturally occurring derivatives which have the necessary structure, such for instance as vanillin

$$C_6H_3\,(CHO)\,(OCH_3)\,(OH)\,(1.3.4),$$

have never been found shewing optical rotatory power, even when prepared from compounds that are themselves optically active, is fairly strong evidence against the possibility of its occurrence.

The subject has also been attacked experimentally by several observers.

Le Bel (*Bull. Soc. Chem.* 1882, XXXVIII. 98) tried to resolve orthotoluidine by means of living organisms; Lewkowitsch (*Jour. Chem. Soc.* 1888, LIII. 781) attempted to resolve various substituted benzoic acids by the fractional crystallization of their salts with certain alkaloids; V. Meyer with Lühn and Sudborough (see Meyer and Jacobsen, *Lehrbuch*, II. 64) also tried some penta substituted benzene acids; but in all these cases no activity could be observed.

It has, of late, been shewn repeatedly that the methods introduced by Pope and his colleagues, which depend on the use of a strong acid and non-hydroxylic solvents, thus eliminating the effects of hydrolytic dissociation, have been successful in resolving compounds into their active components when the older methods had entirely failed to do so. Therefore it was

considered of some interest to see whether the application of these methods to some benzene derivatives would effect any resolution.

The dextro-camphorsulphonate of meta nitraniline was prepared by mixing molecular quantities of Reychler's acid and the base. It was found to be a crystalline substance, but was not very suitable for the purpose in hand.

The dextro-camphorsulphonate of 1. 3. 4 methyl-nitro-amino benzene was prepared by mixing molecular quantities of acid and base in ethyl acetate solution. The salt crystallizes readily in needles of a bright yellow colour which melt to a red liquid at 126—8° C. The salt is very sparingly soluble in water and its solution in dilute alcohol is of a deep red colour, so that it is unsuitable for polarimetric examination.

Some 1. 3. 4 methyl-chlor-amino benzene was prepared by the method of Chattaway and Orton (*Jour. Chem. Soc.* 1900, LXXVII. 792) from some of the red oil (3 chlor-para-acetotoluidide) kindly supplied by Dr Chattaway. The salts of this base were found to be suitable in every respect for the purpose in hand.

1. 3. 4 *methyl-chlor-amino benzene dextro-camphorsulphonate* was prepared by mixing molecular quantities of the base (10·1 grams) and Reychler's acid (17·9 grams) dissolved in dry ethyl acetate; on standing in a desiccator the mixture deposited a mass of fine needle crystals which were dried on a porous plate. The salt is colourless, is very soluble in chloroform, but very sparingly soluble in ethyl acetate and acetone. It was repeatedly recrystallized from a mixture of chloroform and ethyl acetate (1 : 2). Crystallizes as a felted mass of very fine needles which melt at 148·5—149·5° C.

A specimen on analysis gave the following results:

I. ·2120 grams gave ·1222 grams H_2O and ·4245 grams CO_2.

II. ·1990 " " ·1180 " H_2O " ·3990 " CO_2.

	Found. I.	Found. II.	Calculated for $(C_6H_3.CH_3.Cl.NH_2)(C_{10}H_{15}OSO_3H)$.
C.	54·57	54·7	54·62
H.	6·40	6·57	6·42

Determinations of the rotatory power of the salt in aqueous solutions were made with a triple field polarimeter by Schmidt and Haensch, using sodium light:

·995 grams in 25 c.c. gave $\alpha_D = 1·07°$ in a 2 dcm. tube, hence $[\alpha]_D = 13·44°$ and $[M]_D = 50·2°$.

·4130 grams in 25 c.c. gave $\alpha_D = ·45°$ in a 2 dcm. tube, hence $[\alpha]_D = 13·62°$ and $[M]_D = 50·9°$.

$[M]_D$ for a salt of the acid with any inactive base is found to be 51·7°. The base here is therefore inactive.

The base when recovered by the addition of potash was found to be quite inactive in ethereal solution.

1. 3. 4 *methyl-chlor-amino benzene dextro-brom-camphorsulphonate* was prepared by boiling the hydrochloride of the base with the requisite quantity of silver salt of Kipping and Pope's dextro-brom-camphorsulphonic acid and some moist ethyl acetate, filtering off the silver chloride and evaporating the solution to dryness on a water-bath. White solid, crystallizing in fine needles, very soluble in chloroform and sparingly soluble in ethyl acetate. It was recrystallized repeatedly from a mixture of ethyl acetate and chloroform, and it then melted at 208—9° C. with decomposition.

Analysis of a specimen dried in a vacuum desiccator gave the following result:

·1626 grams gave ·0816 grams H_2O and ·2536 grams CO_2.

	Found.	Calculated for $(C_6H_3.CH_3.Cl.NH_2)(C_{10}H_{14}BrOSO_3H)$.
C.	42·54	42·92
H.	5·5	5·3

The rotatory power was determined as in the case of the dextro-camphorsulphonate.

·292 grams in 25 c.c. gave $\alpha_D = 1{\cdot}38°$ in a 2 dcm. tube, hence $[\alpha]_D = 59{\cdot}1°$ and $[M]_D = 267{\cdot}4°$.

·668 grams in 25 c.c. gave $\alpha_D = 3{\cdot}22°$ in a 2 dcm. tube, hence $[\alpha]_D = 60{\cdot}2°$ and $[M]_D = 272{\cdot}9°$.

The value of $[M]_D$ for salts of the acid with inactive bases is always found to be about 270°. The base here is therefore inactive.

The oil recovered from the salt was found to be quite inactive when examined in ether solution.

Unless both the salts here examined exhibit the phenomenon of partial racemism, the above experiments shew that benzene derivatives cannot shew optical activity due to the asymmetry of a carbon atom in the ring, and that the formulae which represent the six carbon atoms of benzene as not in one plane are inadmissible.

A Method of detecting Nickel and Cobalt in the presence of each other. By F. W. DOOTSON, M.A., Trinity Hall.

[*Read* 2 March 1903.]

The process described by Rosenheim and Huldschinsky (*Ber.* 1901, 2050) for the quantitative separation of these metals, though somewhat long and laborious, may be so modified as to render it a rapid and easy qualitative test.

Nickel and Cobalt are separated from other metals and obtained as sulphides by the ordinary processes of analysis. These sulphides are then dissolved in the least possible quantity of hydrochloric acid, with the addition of a few drops of nitric acid, and the solution diluted if necessary. A large excess of acid materially diminishes the sensitiveness of the reaction, and should be removed, either by evaporation or by partial neutralisation with alkali.

To the acid solution sufficient ether is added to form a layer about half an inch in depth. Commercial ether is usually sufficiently free from alcohol for the purpose.

A dilute solution of ammonium or potassium thiocyanate (1 in 10) is then poured in, a little at a time, the test tube being vigorously shaken after each addition, till the ethereal layer acquires a distinct colour. The addition of large excess of thiocyanate, in presence of cobalt, is liable to mask the colour due to the nickel when only traces of the latter metal are present.

With nickel alone, the ethereal layer varies in tint from a pale pink to blood red according to the concentration of the solution. If nickel is absent, and cobalt present, the ether is tinted blue, and the colour is considerably deepened by the subsequent addition of methylated spirit.

When both metals are present the pink colour due to the nickel is the first to appear, and cobalt is detected by the cautious addition of methylated spirit after the solution has stood sufficiently long to allow the ether to separate completely. An intense blue colour appears where the ethereal and aqueous layers meet.

The mechanism of the process appears to be that double thiocyanates of nickel or cobalt with potassium or ammonium

are formed, and that the nickel compound is freely soluble in ether free from excess of acid. That of cobalt is much less so, unless alcohol is present, when it passes readily into the ether. It is possible that the addition of alcohol causes a different molecular arrangement in the case of the cobalt compound. (Vide Treadwell, *Zeit. Anorg. Chem.* 1901, 108; and Rosenheim and Huldschinsky, *loc. cit.*)

Experiments show that where decinormal solutions of the chlorides are used, one part of nickel may be detected in presence of 100 parts of cobalt, and one part of cobalt in presence of 50 parts of nickel, and that if the method is used comparatively even greater delicacy may be obtained. If either metal, but not both, be present the process is sufficiently sensitive to give the reaction distinctly with less than one-tenth of a milligram. In the hands of students the method has been found to be rapid and reliable.

On the Joule-Thomson Effect. By P. V. BEVAN, M.A., Fellow of Trinity College.

[*Received* 20 February 1903.]

The method which at first sight appears best adapted to determine empirically the characteristic equation for gases depends on the observations of the Joule-Thomson effect in gases forced through a porous plug. We shall see however that with the experimental results which we possess at present we are led to an equation which does not agree well with direct experiment on the pressure and volume of gases.

The well known expression for the change of temperature for a gas streaming steadily through a porous plug is

$$\Delta\theta = \left(\theta \left.\frac{\partial v}{\partial \theta}\right)_p - v\right) \frac{\Delta p}{C_p},$$

where $\Delta\theta$, Δp are the small changes of temperature and pressure, from one side of the plug to the other, C_p the specific heat at constant pressure, and p, v and θ are as usual the pressure, volume and temperature of the gas measured on the absolute scale. In actual experiments which have been performed the change of pressure from one side of the plug to the other has always been large so that the observed temperature change is

$$\Theta = \int \frac{\theta \left.\frac{\partial v}{\partial \theta}\right)_p - v}{C_p} . \Delta p,$$

where the limits of the integral are the values of the pressure on the two sides of the plug.

From Joule and Thomson's experiments the temperature change observed was found to be approximately proportional to the difference of pressure on the two sides of the plug, so that assuming this proportionality to hold, we should have $\frac{\Delta\theta}{\Delta p}$ independent of p; in other words that

$$\frac{\theta \left.\frac{\partial v}{\partial \theta}\right)_p - v}{C_p}$$

is a function of the temperature only.

We have therefore

$$\theta \left.\frac{\partial v}{\partial \theta}\right)_p - v = C_p f(\theta) \quad \text{.....................(1)},$$

where $f(\theta)$ represents the cooling effect.

Differentiating equation (1) with respect to θ we obtain

$$\theta \left.\frac{\partial^2 v}{\partial \theta^2}\right)_p = \frac{\partial}{\partial \theta}(C_p f(\theta)).$$

Now we have, if u is the total energy of the gas, ϕ the entropy and dq the heat added in any process which is only an infinitesimal change

$$dq = \theta d\phi = du + pdv,$$

and therefore
$$C_p = \theta \left.\frac{\partial \phi}{\partial \theta}\right)_p,$$

and by Maxwell's fourth thermodynamical relation

$$\left.\frac{\partial \phi}{\partial p}\right)_\theta = -\left.\frac{\partial v}{\partial \theta}\right)_p,$$

so that
$$\frac{\partial^2 \phi}{\partial p \partial \theta} = -\left.\frac{\partial^2 v}{\partial \theta^2}\right)_p = \frac{1}{\theta}\left.\frac{\partial C_p}{\partial p}\right)_\theta,$$

or
$$-\theta \left.\frac{\partial^2 v}{\partial \theta^2}\right)_p = \left.\frac{\partial C_p}{\partial p}\right)_\theta.$$

We have therefore

$$\left.\frac{\partial C_p}{\partial p}\right)_\theta + \frac{\partial}{\partial \theta}\{C_p f(\theta)\} = 0;$$

multiplying this equation by $f(\theta)$ and writing $\int \frac{d\theta}{f(\theta)} = z$, we have

$$\frac{\partial}{\partial p}\{C_p f(\theta)\} + \frac{\partial}{\partial z}\{C_p f(\theta)\} = 0,$$

and therefore
$$C_p f(\theta) = F(p - z).$$

We thus obtain from equation (1)

$$\theta^2 \frac{\partial}{\partial \theta}\left(\frac{v}{\theta}\right)_p = F(p - z).$$

Integrating this equation we obtain

$$\frac{v}{\theta} = \phi(p) + \int \frac{F(p - z)}{\theta^2} d\theta \quad \text{..................(2)},$$

where $\phi(p)$ is a function of p only.

The term $\int \frac{F(p-z)}{\theta^2} d\theta$ is the term depending on the cooling effect and we notice as has been frequently pointed out that even if the cooling effect vanishes, the substance does not necessarily obey Boyle's law.

If now we know the form of the cooling effect and can evaluate the integral on the right-hand side of equation (2), we should be well advanced in determining by this method the characteristic equation. Thomson and Joule first assumed that the cooling effect could be represented by the expression $\frac{a}{\theta^2}$ where a is a constant. With this assumption we have

$$f(\theta) = \frac{a}{\theta^2},$$

whence

$$z = \frac{\theta^3}{3a},$$

and therefore

$$aC_p = \theta^2 F(3ap - \theta^3).$$

Now if p be small we may assume that C_p tends to a limiting constant value C_0, we thus have

$$aC_0 = \theta^2 F(-\theta^3),$$

which determines the form of the function F, and we have

$$aC_p = aC_0\theta^2 (\theta^3 - 3ap)^{\frac{2}{3}},$$

and therefore

$$\frac{v}{\theta} = aC_0 \int \frac{d\theta}{\theta^4 \left(1 - \frac{3ap}{\theta^3}\right)^{\frac{2}{3}}} + \phi(p),$$

or

$$v = \theta\phi(p) + \frac{aC_0\theta}{3p}\left(1 - \frac{3ap}{\theta^3}\right)^{\frac{1}{3}*}.$$

We know however that the Thomson-Joule formula for the cooling effect is inapplicable owing to the existence of the inversion temperature—the temperature at which the sign of the effect considered changes. It is therefore not worth while to investigate the equation derived on this assumption any further.

* For this special case see Planck's *Thermodynamics*. An equation for this case is also deduced in Thomson and Joule's paper "On the Thermal Effects of Fluids in Motion," but in this equation C_p is regarded as constant all through. Joule's *Scientific Papers*, Vol. II., p. 359.

Consider again the original equation for the cooling effect

$$\theta \left.\frac{\partial v}{\partial \theta}\right)_p - v = C_p f(\theta),$$

the actual cooling effect $\dfrac{\Delta\theta}{\Delta p}$ being $f(\theta)$.

To integrate this equation we are not justified in regarding C_p as independent of θ and p and using any empirical formula for $f(\theta)$ which fits the observations more or less accurately. The change of C_p with temperature is considerable when p is large and θ small. For example at 10 atmospheric pressures the value of C_p for air at -50° C. is ·254 and at -140° C. it is $-$·408*.

The dependence of C_p on the pressure is however of not so much account as that on the temperature. Regnault assumed that C_p was independent of the pressure. This is only an approximation, but it leads to results not very far from expressing the experimental data. If we make this assumption we have

$$\left.\frac{\partial C_p}{\partial p}\right)_\theta = 0,$$

and therefore by the relation we have already used

$$\left.\frac{\partial^2 v}{\partial \theta^2}\right)_p = 0,$$

and integrating this

$$\left.\frac{\partial v}{\partial \theta}\right)_p$$

is a function of p only, and

$$v = \theta\phi(p) + \psi(p).$$

This equation has a fairly wide range of applicability, but is of course not generally true, as is at once shewn by the experimental numbers for the expansion coefficients for gases at constant pressure. However, for a wide range the variation of $\left.\dfrac{\partial v}{\partial \theta}\right)_p$, owing to changes in θ, is small; and so the quantity $\left.\dfrac{\partial C_p}{\partial p}\right)_\theta$ is at any rate very small, and for an approximation we may assume that in the small term, representing the cooling effect, C_p is independent of p.

Equation (2) with this assumption becomes

$$\frac{v}{\theta} = \phi(p) + \phi_1(\theta) \qquad \text{.......................(3)},$$

* *Rapports, Congrès de Physique* (Paris, 1900) I. p. 686.

since z is a function of θ and therefore θ alone occurs in the quantity under the integral sign.

We may write equation (3) in the form

$$p = \psi\left(\frac{v}{\theta} - \phi_1(\theta)\right),$$

where $\phi_1(\theta)$ is the term depending on the Joule-Thomson effect and is small in the range to which our equations are applicable, that is over a range where the rate of change of the coefficient of expansion at constant pressure with the temperature is small compared with the coefficient of expansion itself.

If now we assume that Boyle's Law is obeyed as a limit when p is very small and θ is not very small, this equation must reduce to $p = \frac{R\theta}{v}$, when $\phi_1(\theta)$ is negligible. We have therefore for an infinite range of v and θ

$$\frac{R\theta}{v} = \psi\left(\frac{v}{\theta}\right),$$

and therefore the form of ψ is determined and our equation reduces to

$$p = \frac{R}{\frac{v}{\theta} - \phi_1(\theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4),$$

or if we neglect squares of $\phi_1(\theta)$,

$$p = \frac{R\theta}{v}\left(1 + \frac{\theta}{v}\phi_1(\theta)\right),$$

or

$$pv = R\theta\left(1 + \frac{p\phi_1(\theta)}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

So far we have derived the equation for a gas from the cooling effect with the aid of some justifiable approximations. We shall now make use of results of experiments made to directly determine relations between p, v, and θ. Ramsay and Young* found for certain liquids and gases the relation

$$p = b\theta - a$$

for constant volume; a and b being constants. Amagat† has discovered an equivalent result $\left.\frac{\partial p}{\partial \theta}\right)_v$ = constant. These two relations we may express in the form

$$p = \theta f_1(v) - f_2(v) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$$

* *Phil. Mag.*, 23, 1887.

† *Rapports, Congrès de Physique* (Paris, 1900).

This equation for a great many substances appears to be very accurately followed, even at temperatures in the region of the critical point. This equation represents then the best empirical formula which has as yet been obtained for the behaviour of gases and liquids. The quantities a and b in Ramsay and Young's formula are however found not to be any simple function of the volume, so that this equation alone does not carry us very far.

If however we consider the equation

$$p = \frac{R\theta}{v}\left(1 + \frac{\theta}{v}\phi_1(\theta)\right),$$

and the equation

$$p = \theta f_1(v) - f_2(v),$$

we obtain

$$\phi_1(\theta) \text{ in the form } \frac{c}{\theta} + \frac{d}{\theta^2},$$

so that equation (5) becomes

$$pv = R\theta\left(1 + \frac{p}{R}\left(\frac{c}{\theta} + \frac{d}{\theta^2}\right)\right) \quad \text{.................(7).}$$

In thus deriving a form for the function $\phi_1(\theta)$ we obtain forms for the functions $f_1(v)$ and $f_2(v)$. But these functions determined in this way do not give very good values for the numbers obtained by experiment by Ramsay and Young. The functions are far too simple to give anything but a fairly good approximation to these numbers.

The equation (7) is equivalent to van der Waals' equation if we neglect products of the small quantities, for we have from van der Waals' equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = R\theta,$$

$$pv = R\theta + bp - \frac{a}{v}$$

$$= R\theta\left(1 + \frac{p}{R}\left(\frac{b}{\theta} - \frac{a}{R\theta^2}\right)\right).$$

The form we obtain for the cooling effect from equation (7) is

$$\frac{1}{C_p}\left(\frac{A}{T} - B\right).$$

If we regard C_p as constant this is the same form as used by Rose Innes in an empirical formula which agrees well with the results of Thomson and Joule's experiments. In the case of carbon-

dioxide, however, the constants thus used do not agree with the constants as deduced from van der Waals' equation *.

Equation (7) does not therefore lead to very encouraging results as far as agreement with experimental results is concerned. We can see that the same inconsistency applies to equation (4) from which equation (7) is derived.

Equation (4) is equivalent to

$$pv = R\theta + p\theta\phi_1(\theta),$$

or

$$p(v - \theta\phi_1(\theta)) = R\theta \quad \text{.....................} (8),$$

and for a given value of θ the curves representing pv and p are straight lines. But we know from Amagat's results that the pv and p curves are straight lines approximately only in the case of hydrogen. In all other cases these curves are convex to the pressure axis.

Our equation is therefore inconsistent with the results of experiment, and we must examine closely the assumptions we have made to find where the inconsistency has been introduced.

First of all let us consider the assumption that C_p is, as far as the small term representing the Joule-Thomson effect is concerned, independent of the variation of pressure.

For gases which are with difficulty liquefiable the value of $\left.\frac{\partial C_p}{\partial p}\right)_\theta$ is undoubtedly very small at pressures which are not very great, and at temperatures which are not very low. We may assume in virtue of the relation

$$\left.\frac{\partial C_p}{\partial p}\right)_\theta = \theta \left.\frac{\partial^2 v}{\partial \theta^2}\right)_p,$$

that the ratio of $\left.\frac{\partial C_p}{\partial p}\right)_\theta$ to C_p is of the order of the departure of the gas from Boyle's Law. In the small term therefore representing the cooling effect, the result of assuming C_p independent of the pressure will be to introduce errors of the second order in the quantities involved. This cannot account for the differences between Amagat's pv and p curves and the curves represented by equation (8), which are of the first order in the same small quantities.

The other assumption on which we have proceeded is that the Joule-Thomson effect is proportional to the pressure fall from one side of the plug to the other.

The experimental results on which this assumption is founded are not in good agreement, and it appears that this assumption is not justifiable.

* See Callendar, *Phil. Mag.*, Jan. 1903, p. 59.

The following numbers are given by Joule and Thomson* for the cooling effect divided by the difference of pressure in question for air.

·0170	for a difference of pressure	7
·0179	,, ,, ,, ,,	8
·0165	,, ,, ,, ,,	22
·0196	,, ,, ,, ,,	19
·0177	,, ,, ,, ,,	39
·0172	,, ,, ,, ,,	65
·0174	,, ,, ,, ,,	69

The pressures are measured in pounds per square inch.

The individual experiments on which these numbers are founded differ very considerably among themselves, the variations being in some cases very large. For example the cooling effects from which the first number in the above table was deduced varied from ·094° C. to ·125° C. For the fourth number in the above table there were four observations, ·358°, ·348°, ·380° and ·373°. The best series of observations was the series for the last number where the variations amounted to only about 6 per cent. of the mean value. There is thus room for a considerable variation of the cooling effect in the difference of pressures in the case of air.

For carbon dioxide the numbers are in better agreement, being for the cooling effect divided by the pressure difference:

·0770	for a pressure difference of	6
·0778	,, ,, ,, ,,	18
·0789	,, ,, ,, ,,	37
·0833	,, ,, ,, ,,	60

For the last number the temperature was about 70° C. lower than for the others, and so the observation is not comparable with those taken at nearly the same temperature.

The observations for carbon dioxide are in much closer agreement than in the case of air, but the number of comparable observations being only three with a not very large range of pressure difference, room is left for a considerable dependence of the cooling effect on the pressure.

With the experimental results at present available we cannot, therefore, obtain any reliable approximation to the characteristic equation for fluids. Until more accurate experiments have been made by which the relation of the cooling effect to the pressure of the gas can be determined, this method cannot lead to any equation which will represent the results of direct experiment on p, v, and θ.

* Joule, *Scientific Papers*, Vol. II., p. 258.

On a Sensitive Gold-Leaf Electrometer. By C. T. R. WILSON, M.A., F.R.S., Sidney Sussex College.

[*Read* 2 March 1903.]

The electrometer has the form shown in the accompanying figure. (Fig. 1.)

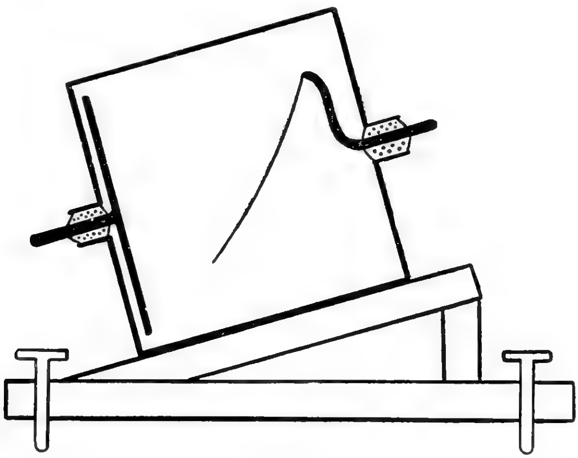


FIG. 1.

The case consists of a brass box with flat sides, of dimensions 4 cms. × 4 cms. × 3 cms.

The gold-leaf is a narrow one, less than 2 mm. wide and about 3 cms. long, with straight, clean-cut edges. It is attached to the end of a brass wire which passes through a sulphur plug in the centre of one of the narrower walls of the vessel. Opposite it, and supported on a brass rod passing through a similar sulphur plug, is a rectangular brass plate nearly equal in size to the inner surface of the wall, from which it is distant about 1 mm.

Small glass windows in the square sides of the case allow the position of the gold-leaf to be read with a microscope provided with a micrometer scale. The magnifying power was such that 54 micrometer divisions corresponded to a movement of 1 mm.

To use the instrument the insulated plate is maintained at a constant potential. The electrometer is placed in an inclined position, as shown in the figure, the angle of inclination and the potential of the plate being adjusted by trial to give the desired sensitiveness. The gold-leaf being initially connected to the case, the microscope is adjusted so that the image of the gold-leaf is near the centre of the scale. A rise or fall of the potential of the gold-leaf through a small fraction of a volt can then readily be measured.

In the experiments made to test the apparatus the plate was maintained at a constant potential by means of a battery of storage cells, of which one terminal was connected to the case and

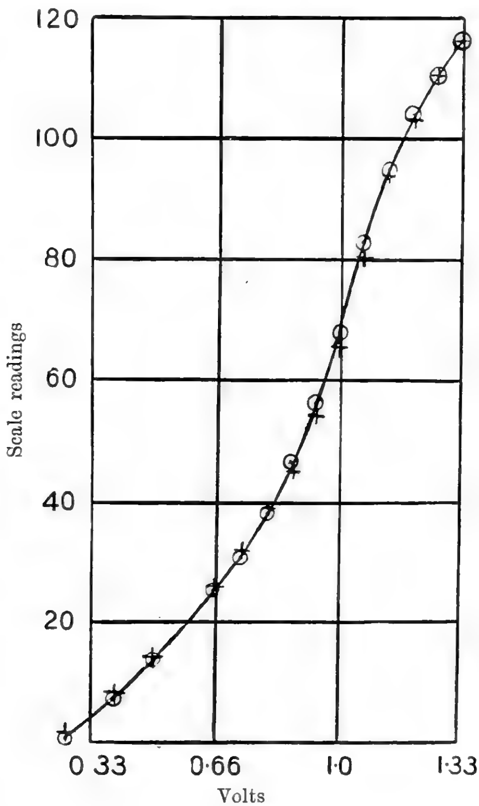


Fig. 2.

the other to the plate. A wire leading from the terminal supporting the gold-leaf to the sliding contact of a potentiometer, of

which one point was connected to the case, allowed the potential of the gold-leaf to be varied as desired. The current through the potentiometer was supplied by a storage cell.

When the plate was maintained at a potential of 207 volts and the apparatus was tilted to give the maximum sensitiveness (the plate being then inclined at an angle of about 30° to the vertical) a change of potential of one volt caused the gold-leaf to move almost completely across the 120 divisions of the micrometer scale, and the sensitiveness over the central portion of this range exceeded 200 divisions per volt. The curve (fig. 2) shows the relation between the deflections, read on the horizontal scale of the eye-piece micrometer, and the potential of the gold-leaf, the case being earthed and the plate kept at 207 volts. The crosses and circles indicate observations taken in ascending and descending order respectively.

Fig. 3 shows the results obtained when the gold-leaf was kept in metallic connexion with the plate and the potential of the plate was varied over a wide range; a low-power microscope was used

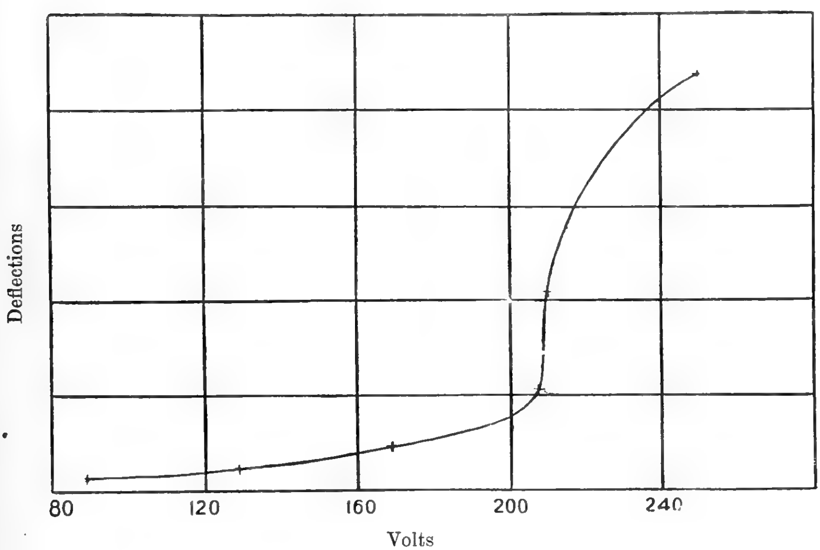


Fig. 3.

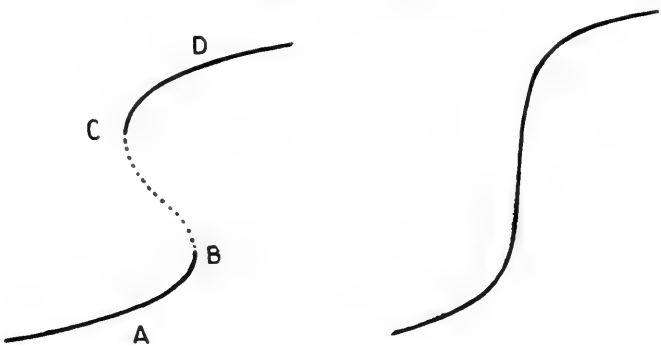
for these observations. It will be observed that the change in deflection in passing from 207 to 209 volts is as great as the whole deflection for 207 volts.

For any particular angle of tilt of the apparatus there is a

difference of potential between the plate and gold-leaf, in the immediate neighbourhood of which the sensitiveness is a maximum. The less the tilt the greater is this potential and the greater is the maximum sensitiveness, until a certain critical inclination is reached below which there is instability over a certain range of deflections. The range over which there is instability increases as the instrument is less and less tilted, and when there is no tilt, the plate being then vertical, the leaf is unstable over the greater part of its range.

The greatest sensitiveness is obtained when the inclination is as little as possible beyond the critical value.

When the inclination of the apparatus is below the critical value, there is a certain range of potential differences between the plate and the gold-leaf within which there are, for a given potential difference, two positions of stable equilibrium for the leaf. The behaviour of the instrument under these conditions will be understood more readily with the aid of the diagrams which follow (fig. 4). For these, it must be understood, only a qualitative



a *b*

Fig. 4.

correctness can be claimed; they show the general nature of the curves connecting deflection with difference of potential. The portion of the curve with which we are specially concerned is S-shaped. In the central dotted portion of the S, where the deflection increases as the voltage diminishes, we get instability, and the curve here cannot be followed experimentally. The rest of the curve shown by full lines can be experimentally realized. If we start with a small difference of potential and gradually increase it, the deflection increases more and more rapidly as the point B is approached. At this point the gold-leaf is found suddenly to jump to the position D, and if the potential difference

be further increased the deflection now increases less and less rapidly with the potential difference. If we retrace the curve by diminishing the potential difference, the sensitiveness continually increases till it becomes very great near C, and there follows a sudden drop to A, the lower branch of the curve being then retraced from this point as the potential is further diminished.

If the tilt of the apparatus be increased beyond the critical amount, the curve has the form shown in fig. 4 *b*. Here the deflection for a given potential is single-valued, and there is no part of the curve indicating instability. Exactly at the critical angle there will be a portion of the curve vertical, indicating infinite sensitiveness, and the nearer this is approached the more sensitive will the instrument be, when the plate is kept at the corresponding potential.

The practical limit to the sensitiveness is probably fixed by the degree of constancy which can be attained in the potential of the plate.

The instrument may be carried about without risk of injury to the gold-leaf; it is only necessary to charge up the plate to such a potential that the gold-leaf (connected to the case) is stretched straight out towards the plate, which it is just too short to touch. The apparatus may then safely be inverted or carried in any position; the gold-leaf remains steadily pointing towards the plate.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The Origin of the Thorough-bred Horse. By Professor Ridgeway (communicated by A. E. Shipley).

[*Read* 24 November 1902.]

The author said that he had shown (*Academy*, Jan., 1891, p. 91) that not only, as had been long observed, did the Homeric Greek drive the horse before they rode him, but that the same was true of all ancient peoples—Egyptians, Canaanites, Assyrians, Aryans of Rig-Veda, Umbrians, Celts,—and that the explanation of this was given by Herodotus (v. 9), who in speaking of the Sigynnae, the only tribe north of the Danube whose name he knew, said that they had small horses, with large flat noses, and very long hair, which though not able to carry a man, were excellent under chariots: "wherefore they used chariots." Dio Cassius likewise says that the Britons used chariots in war, because their horses were "small, though active." The description of the horses of the Sigynnae tallies exactly with the abundant remains of the primitive horse of Europe, eaten in great quantities and delineated on antlers by the men of the Stone Age. He was a small animal about 10 hands high with a big head. Even after domestication he remained very small, as witness bits of bronze and horn found in Swiss Lake-dwellings, the shoes found at Silchester, and in camps on the Roman Wall, etc. Authorities are agreed that from this primitive horse has been developed the cart-horses of the Continent and these islands, whilst our blood-horses have come from an Eastern stock of slight build and smart appearance. Our problem is to ascertain the original habitat of this superior horse. He has not come from Upper Asia, as the Mongolian pony is taken as the type of the coarse,

thickset horse, from which sprang the cart-horse. The Mongolian pony probably represents the Scythian horses, which continued to be of a small size down to Strabo's time, and they were derived either from the tarpan or Prezevalsky's horse. The Mongolian pony, though surefooted and enduring, is slow of pace. Neither China, Siam, nor Burmah has any indigenous horse answering to the blood-horse. India could never breed horses, says Marco Polo, in whose time India was supplied either with Mongolian ponies from Yunnan, or with Arabs from South Persia, Aden and other Arabian ports. These Arabs fetched enormous prices, equivalent to £200. It has hitherto been universally held that Arabia is the original home of the blood-horse. This is a baseless assumption. In the Old Testament the Arabs are never mentioned as riding anything but camels and asses. Though the author of Job knew of the war-horse, yet Job did not own a single horse, his equine possessions consisting of 500 she asses. Herodotus (VII. 87) enumerates the nations (including the Libyans) that supplied cavalry to Xerxes' host, but the Arabs only furnish a camel corps. Agatharchides (cited by Strabo) describes the Arabs as camel keepers.

Finally, Strabo (*flor.* A.D. 1) expressly states that neither the peoples of Arabia Felix nor those of Arabia Petraea bred horses. Naturally then Scaurus after defeating the Arab king Aretas put on his coins Aretas leading his camel. It is clear then that down to the Christian era the Arabs bred no horses. It is therefore clear that though the Persian kings in the 5th cent. B.C. bred the largest and best horses in Asia, these were not of an Arab strain. These horses were kept largely in Armenia, and are described by Strabo as similar to the Parthian horses, and as differing from the horses bred in Greece and the other kinds of horses known in the Roman empire. There can be little doubt that they were the same horses as Marco Polo found in great numbers in Armenia (1270 A.D.) known as Turquans, the Turcoman ponies well known in Persia to-day. The Persian horses cannot then have been the ancestors of the thoroughbred, though it is quite possible that their superiority was due to their having a cross of thoroughbred blood, for already by 900 B.C. Solomon imported horses from Egypt (1 Kings x.), and "so for all the kings of Syria and for all the kings of the Hittites" Egypt could not breed horses, neither could she have got them from the Arabs, who bred none even 1000 years later. But she could and did get them from the Libyans, who from the dawn of history are masters of the most famous horses. Cyrene sent the best horses to the games of Greece (Pindar, *Pyth.* IV. etc.). It is noteworthy that it was in the same century as the founding of Cyrene that the four-horse chariot and the race-horse were added to the Olympic events. The Phoenician settlers

at Carthage found the Libyans using these beautiful horses, and when they struck coins placed a horse or a horsehead on them as the badge of Libya, and used a similar type on their coins struck in Sicily, whither doubtless they carried the Libyan breed. This accounts for the extraordinary fame of the horses of Etna and Syracuse, and the famous steeds of Tarentum. It is now clear that the Arabs never owned a good horse until they had become masters of North Africa and the Barbary horses, from whom are sprung our own racing stock through Lord Godolphin's Barb. North Africa therefore and not Arabia or any other part of Asia is the original home of the thoroughbred.

Now though the pedigree of the cart-horse type can be traced to the coarse, thickset little horses of Europe and Asia, the wild ancestor of the Barb is yet to seek, for Africa has no wild horse, such as tarpan or Prezevalsky's, though she has an ass and four zebras, including the quagga, now extinct. Can the Barb be sprung wholly or in part from a zebra? Arab foals at birth constantly have zebra markings, sometimes retained when full grown, as by Prof. Ewart's filly Fatima. Strabo too notices that the horses of the Libyan Garamantes have longer hoofs than any other horses. Prof. Ewart's hybrids from Burchell's zebra and various mares show the markings not of a Burchell's zebra, but of a Somaliland zebra, from which it has been inferred that the remote ancestor of both Equus caballus and Burchell's zebra was striped like the Somaliland and mountain zebra. But is it necessary to go back so far? May not the Somaliland zebra stripes in the hybrid be due to the circumstance that the dam in each case had a certain amount of Barb blood in her, which was derived from either the Somaliland zebra or a closely allied species? He (Prof. Ridgeway) had crossed a Muscovy drake with a common white duck, derived from the common wild duck, with the result that all the offspring are coloured and their colouring resembles that of the mallard. No one would say that the hybrids show a reversion to a remote common ancestor of both mallard and Muscovy, for it is obvious that the colouring is simply that of the white duck's immediate ancestors. Authorities like Capt. Hayes have pointed out the great similarity in form between Burchell's and the Somaliland zebra to a well-bred horse, *i.e.* a horse that has Barb blood in him. He therefore suggested that the Barbary horse, from which he had shown all the fine horses of the world have sprung, was derived either from the zebra of North-east Africa, or, as is more likely, from some very closely allied species now extinct, which like Prezevalsky's horse may have had castors on its hind legs like Equus caballus.

The Theory of the Rate of Recombination of Ions in Gases. By O. W. Richardson, B.A., Coutts Trotter Student, Trinity College.

[*Received* 17 June 1902.]

The object of the present paper is to calculate the rate of recombination of ions in gases. Recombination is supposed to take place when the ions collide, the number of collisions being increased by the mutual attraction of the ions. In a paper on the "Viscosity of Gases and Molecular Force," Mr Sutherland* has given a method for calculating the increase produced by molecular attraction, in the number of collisions of two gaseous molecules of equal mass, when the molecules are moving with a given relative velocity. Mr Sutherland's mode of calculation when applied to two molecules of unequal mass yields an absurd result. This is due to the fact that he takes as his uniformly moving origin one of the moving molecules which is really accelerated. The proper assumption to make is that, whatever be the nature of the action between the molecules, it cannot affect the motion of their centre of gravity. I have therefore calculated on this assumption the number of collisions which take place between two molecules of different masses whose relative velocity is r. From this I have obtained the total number of collisions which take place, by integrating over all values of r which occur according to the Maxwellian distribution. Finally the formulae so obtained have been applied to calculate the coefficient of recombination of X-ray ions in gases.

In what follows it will be assumed that a molecule may be represented with sufficient accuracy as a hard sphere. A collision would then ensue when two such spheres come in contact, and the average number of collisions made by a molecule per second would be $\sqrt{2}\pi S^2\Omega N$ where S is the diameter of a molecule, Ω is the mean speed, and N the number of molecules in a cubic centimetre. In this case the number of collisions is the same as the number of molecules whose centres lie within the volume described by a sphere of radius $\sqrt[4]{2}S$ moving with the molecule; if the molecules attract one another, then the number of collisions will be greater than this, the effect of the attractive forces being

* *Phil. Mag.* xxxvi. 507.

to increase the apparent diameter of the molecule in a certain ratio.

Objection may be raised to the assumption of a definite size for molecules on the ground that it is too specialized. The only alternative seems to be the view of centres of force developed by Maxwell*. One objection to the consideration of molecules as mere centres of force is urged by Mr Sutherland†. Another is the values of the ratio of the specific heats for mono- and diatomic molecules which seem to indicate a definite geometrical distribution of mass points. A further objection lies in the great mathematical difficulties furnished by this view which makes its application to any particular case almost impossible. In addition to this the excellent agreement of Mr Sutherland's results with the experimental determination of the variation of viscosity of gases with temperature furnishes a strong argument in favour of molecules of definite shape.

Even if molecules really consist of an aggregation of centres of force, in some cases the view which regards them as hard spheres possesses a definite meaning. Looking at a molecule as a centre of repulsive force varying extremely rapidly with the distance, it has a certain mean free path and therefore a certain apparent sphere of action whose radius is, say, S. If now a field of force which varies much less rapidly with the distance is superposed on the first, the number of molecules which are deflected through a given angle will be changed. If the second force is attractive a greater number, if repulsive a smaller number, of molecules will be pulled into the stronger part of the field, so that the number of deflections through a given angle would in the first case be greater, in the second, smaller than before. The actual alteration will be much the same as if the molecules were hard spheres of mean radius S attracting or repelling one another with the second system of forces. As an imaginary illustration we might take the case of a gas containing positive and negative ions, which are supposed to have the following constitution. A positive ion is to be regarded as a molecule from which a corpuscle has been taken away and a negative ion a molecule to which a corpuscle has been added. The number of collisions between ions of opposite signs could then be calculated, assuming that the molecules were rigid spheres of radius equal to that given by experiments on viscosity, attracting one another according to the inverse square law: the result thus obtained would then be true, at any rate approximately, even if the molecules were really pure centres of force.

Turning to the actual calculation, we shall assume that the

* *Scientific Papers*, II. 35.

† *Phil. Mag.* XXXVI. 509.

motion of the centre of gravity of the two particles is uniform and uninfluenced by their mutual action.

Let us consider two particles of mass m_1 and m_2 respectively, and let us measure all the coordinates from their centre of gravity. The distance between the two particles at any time t is $\rho = \rho_1 + \rho_2$ where $m_1\rho_1 = m_2\rho_2$ and ρ_1, ρ_2 are the respective distances of the particles from the origin. Then if $\dot{\tau}$ be the angular velocity of each particle about a straight line through the origin moving with the uniform velocity of the centre of gravity of the system, the equations of motion of the two particles are

$$m_1\ddot{\rho}_1'' - m_1\rho_1\dot{\tau}^2 = -F(\rho_1 + \rho_2) = m_2\ddot{\rho}_2'' - m_2\rho_2\dot{\tau}^2,$$

where $F(\rho_1 + \rho_2)$ is the attraction between the two molecules when their centres are $\rho = \rho_1 + \rho_2$ cms. apart; together with

$$\frac{d}{dt}(\rho_1{}^2\dot{\tau}) = \frac{d}{dt}(\rho_2{}^2\dot{\tau}) = 0.$$

Integration of the last two equations gives

$$\dot{\tau} = \left(\frac{1 + \dfrac{m_1}{m_2}}{\rho}\right)^2 h_1 = \frac{a^2}{\rho^2} h_1$$

$\left(\text{if } 1 + \dfrac{m_1}{m_2} = a\right)$ where h_1 is twice the area swept out by the vector ρ_1 per second. Moreover if V be the relative velocity of the two molecules when they are so far apart as to be uninfluenced by their mutual attractions, and b be the least perpendicular distance between the straight paths they originally pursued, then

$$h_1 = \frac{bV}{a^2}.$$

Substituting these values for $\dot{\tau}$ the first equations of motion reduce to one, viz.

$$m_1\ddot{\rho}'' - m_1 b^2 V^2 \rho^{-3} = -aF(\rho).$$

The integral of this is

$$\tfrac{1}{2}\dot{\rho}^2 = C - \tfrac{1}{2}b^2V^2\rho^{-2} + \frac{a}{m_1}\int_\rho^\infty F(\rho)\,d\rho.$$

Further, when $\quad \rho = \infty, \quad C = \tfrac{1}{2}V^2,$

so that $\quad \dot{\rho}^2 = V^2 - b^2V^2\rho^{-2} + 2\dfrac{a}{m_1}\displaystyle\int_\rho^\infty F(\rho)\,d\rho.$

The smallest distance within which the molecules approach is given by $\quad \dot{\rho} = 0,$

i.e., by $\quad V^2 - b^2\rho^{-2}V^2 + \dfrac{2a}{m_1}\displaystyle\int_\rho^\infty F(\rho)\,d\rho = 0.$

For a collision this must be less than s the radius of the sphere of action of the two molecules (supposed forceless). Hence the criterion for a collision is

$$b^2 \ngtr s^2 \left\{1 + 2\frac{m_1 + m_2}{m_1 m_2 V^2}\int_s^\infty F(\rho)\, d\rho\right\}.$$

Hence the effect of the intermolecular forces is to apparently increase the area of cross section of the sphere of action in the ratio $1 + 2\frac{m_1 + m_2}{m_1 m_2 V^2} W$ where W is the work done by the attraction of the particles when they are brought together from an infinite distance.

If we put $m_1 = m_2 = m$ we can compare this with the result obtained by Mr Sutherland for two particles of equal mass. The correction term in the above formula is exactly twice as great as that found by Mr Sutherland; the discrepancy is due to the different assumptions made, as was pointed out at the beginning of the calculation.

The formula first found shows that a collision takes place when the straight paths of two molecules whose relative velocity is r approach within a distance b which is less than

$$s^2 \left\{1 + 2\frac{m_1 + m_2}{m_1 m_2 r^2}\int_s^\infty F(\rho)\, d\rho\right\}.$$

From this we can find the total number of collisions which take place in the gas by the use of a formula given by Maxwell*. Maxwell shows that, if we have a space containing two gases, of which there are N and N' molecules per c.c. and whose mean velocity squares are α^2 and β^2, respectively; then the number which have a relative velocity between r and $r + dr$ and whose straight paths approach within a distance b, is given by

$$NN' \frac{4\sqrt{\pi}}{(\alpha^2 + \beta^2)^{\frac{3}{2}}} b^2 r^3 e^{-\frac{r^2}{\alpha^2+\beta^2}} dr.$$

Hence taking molecular attraction into account, the total number of collisions per second is given by

$$\int_0^\infty 4\sqrt{\pi} NN' \frac{s^2}{(\alpha^2 + \beta^2)^{\frac{3}{2}}} \left\{1 + 2\frac{m_1 + m_2}{m_1 m_2 r^2}\int_s^\infty F(\rho)\, d\rho\right\} r^3 e^{-\frac{r^2}{\alpha^2+\beta^2}} dr$$

$$= 2\sqrt{\pi} NN' s^2 (\alpha^2 + \beta^2)^{\frac{1}{2}} \left\{1 + \frac{2}{\alpha^2 + \beta^2}\,\frac{m_1 + m_2}{m_1 m_2}\int_s^\infty F(\rho)\, d\rho\right\}.$$

If we replace the mean velocity squares by the mean velocities,

* *Scientific Papers*, I. 385.

viz., $\Omega_1 = \frac{2}{\sqrt{\pi}}\alpha$, $\Omega_2 = \frac{2}{\sqrt{\pi}}\beta$: we find that the total number of collisions made by a particle of each gas per second is given by

$$\Gamma_1 = \sqrt{2}\pi N_1 s_1^2 \Omega_1 \left\{1 + \frac{8}{\pi m_1 \Omega_1^2}\int_{s_1}^{\infty} F_1(\rho)\, d\rho\right\}$$
$$+ \pi N_2 \sigma^2 \sqrt{\Omega_1^2 + \Omega_2^2}\left\{1 + \frac{8(m_1+m_2)}{m_1 m_2 (\Omega_1^2 + \Omega_2^2)}\int_{\sigma}^{\infty} F_0(\rho)\, d\rho\right\},$$

$$\Gamma_2 = \sqrt{2}\pi N_2 s_2^2 \Omega_2 \left\{1 + \frac{8}{\pi m_2 \Omega_2^2}\int_{s_2}^{\infty} F_2(\rho)\, d\rho\right\}$$
$$+ \pi N_1 \sigma^2 \sqrt{\Omega_1^2 + \Omega_2^2}\left\{1 + \frac{8(m_1+m_2)}{m_1 m_2 (\Omega_1^2 + \Omega_2^2)}\int_{\sigma}^{\infty} F_0(\rho)\, d\rho\right\};$$

where $F_1(\rho)$, s_1 : $F_2(\rho)$, s_2 and $F_0(\rho)$, σ are the respective attractions and radii of spheres of action of the three possible kinds of pairs of molecules. If the gas is all at the same temperature

$$\Omega_2^2 = \frac{m_1}{m_2}\Omega_1^2,$$

so that

$$\Gamma_1 = \sqrt{2}\pi N_1 s_1^2 \Omega_1 \left\{1 + \frac{8}{\pi m_1 \Omega_1^2}\int_{s_1}^{\infty} F_1(\rho)\, d\rho\right\}$$
$$+ \pi N_2 \sigma^2 \sqrt{1 + \frac{m_1}{m_2}}\,\Omega_1 \left\{1 + \frac{8}{\pi m_1 \Omega_1^2}\int_{\sigma}^{\infty} F_0(\rho)\, d\rho\right\}, \text{ etc.}$$

In applying these formulae to compute the rate of recombination of ions in gases we shall assume that practically every collision results in recombination. This requires that the kinetic energy which the two ions possess at the moment of collision should be very rapidly dissipated. The necessary dissipation probably takes place in two ways.

(1) By direct radiation. The phenomena of a discharge tube indicate that recombination is accompanied by the emission of a considerable amount of radiant energy.

(2) By the direct transference of the energy to the other modes of vibration of the molecule.

From the preceding formulae it follows that if there are N_1 negative ions, of mass m_1 and mean velocity Ω_1, and N_2 positive ions, of mass m_2, per cubic centimetre: then the number which recombine per second is

$$-\frac{dN_1}{dt} = -\frac{dN_2}{dt} = \pi N_1 N_2 \sigma^2 \sqrt{1 + \frac{m_1}{m_2}}\,\Omega_1 \left\{1 + \frac{8}{\pi m_1 \Omega_1^2}\int_{\sigma}^{\infty} F(\rho)\, d\rho\right\},$$

where σ is the distance between the centre of a positive and that of a negative ion, when in contact, and $\int_\sigma^\infty F(\rho)\,d\rho$ is the work done when two ions recombine. On comparing this with the equation given by Professors Thomson and Rutherford*, which for the case of unequal numbers of the two ions takes the form

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = -\alpha N_1 N_2,$$

we see that the coefficient of recombination α is equal to

$$\pi\sigma^2 \sqrt{1 + \frac{m_1}{m_2}}\,\Omega_1 \left\{1 + \frac{8}{\pi m_1 \Omega_1^2} \int_\sigma^\infty F(\rho)\,d\rho\right\}.$$

It is evident from this expression that α is independent of the pressure of the gas, as has been found to be the case by Mr McClung†.

In order to make further deductions from the formula we require to know the value of $\int_\sigma^\infty F(\rho)\,d\rho$ the work done on an ion during recombination. We shall assume that this is equal to the energy necessary to ionize a molecule. It is evident that even if the two things are not identical they must at any rate be of the same order of magnitude. Now Professor Townsend‡ has shown that the energy required to ionize a molecule in air is equal to that acquired by a corpuscle in falling through a potential difference of five volts. Thus the value of $\int_\sigma^\infty F(\rho)\,d\rho$ is $\frac{5e}{300}$ where e is the charge on an ion in electrostatic units. If we substitute the following numerical values, viz.,

(1) for the number of molecules in a cubic centimetre of a gas at 0° C. and 760 mm. $n = 2 \times 10^{19}$,

(2) for the charge on an ion $e = 6{\cdot}5 \times 10^{-10}$ electrostatic units,

(3) for the mean velocity square for a hydrogen molecule

$$\alpha^2 = 3{\cdot}24 \times 10^{10}\,(\text{cms. per sec.})^2,$$

we find

$$\frac{8}{\pi m_1 \Omega_1^2} \int_\sigma^\infty F_0(\rho)\,d\rho = 148;$$

so that the additional term due to the attraction is large compared with unity.

* *Phil. Mag.*, Nov. 1896.
† *Phil. Mag.*, March, 1902.
‡ *Phil. Mag.* 6. I. 198.

As an illustration I have used the above formula to calculate the coefficient of recombination on the assumption that the negative ions are corpuscles and the positive ions gas molecules each possessing a charge equal to that of a corpuscle. Assuming Townsend's result that the mean free path of a corpuscle is four times that of a gas molecule, the only other physical quantities required for the calculation are

(1) the collision frequency of a molecule of the gas,

(2) the electrochemical equivalent of hydrogen, and

(3) the mean velocities of agitation of a corpuscle and a gas molecule.

The value of the coefficient of recombination found in this way is $1{\cdot}14 \times 10^{-6}$. Very little stress can be laid on the agreement of this number with the value ($2{\cdot}2 \times 10^{-6}$) which has been found experimentally; for a large number of results have been obtained which indicate that X-ray ions in air have a far more complex structure than that assumed in this calculation.

Since the value of $\dfrac{8}{\pi m \Omega^2}\displaystyle\int_\sigma^\infty F(\rho)\,d\rho$ is great compared with unity we may write α very approximately in the form

$$\alpha = \frac{8\sqrt{1+\dfrac{m_1}{m_2}}\,\sigma^2}{m_1 \Omega}\int_\sigma^\infty F(\rho)\,d\rho.$$

From this we see that for the same gas α varies inversely as the square root of the absolute temperature; the coefficient of recombination should therefore diminish slowly as the temperature increases. If the effect of the attraction between the ions were negligible compared with their velocities of agitation α would have been *directly* proportional to the square root of the absolute temperature.

If we apply this method to the calculation of the mean free path (λ) of an ion in air—a quantity required for the computation of the velocity (v) of an ion under unit field, according to the formula $v = \dfrac{1}{2}\dfrac{e}{m}\dfrac{\lambda}{\Omega}$ we are placed in difficulties owing to our ignorance of the attraction between an ion and a molecule. Moreover, on this view, for very high velocities the number of collisions made by an ion with the molecules of a gas, in going through one centimetre of it, becomes independent of the velocity of the ion. This result is, at any rate, open to criticism, since it is probable that an ion makes fewer and fewer collisions the greater its velocity.

If we assume that an ion and a molecule act on one another purely as centres of force varying as a power of the distance

between them, then the formulae given by Maxwell* enable us to obtain the law of force between an ion and a molecule from the variation, with the velocity of the ion, of the number of collisions made by an ion in travelling through a centimetre of gas. These formulae show that the number of deflections of path through an angle θ is unchanged, provided

$$b\left\{\frac{V^2 M_1 M_2}{K(M_1+M_2)}\right\}^{\frac{1}{n-1}}$$

remains constant: where b is the smallest perpendicular distance between the original straight paths of the particles whose masses are M_1 and M_2, V is their relative velocity, and K is the magnitude, when the particles are unit distance apart, of the force, which varies inversely as the nth power of the distance between them. If we keep the law of force and the mass of the particles constant, we find that the number of deflections through a given angle is unaltered provided $bV^{\frac{2}{n-1}}$ remains constant. We see therefore that the number of collisions per centimetre is proportional, *ceteris paribus*, to $V^{-\frac{4}{n-1}}$; so that if we determine the way in which the number of collisions varies with the velocity we can at once obtain the value of n, which determines the law of force. The following numbers indicate the way in which the number of collisions made in passing through a centimetre varies with the velocity for different laws of force, on the hypothesis that the particles can be regarded purely as centres of repulsive force, whose magnitude varies as the $-n$th power of the distance between them.

Values of n	$-\infty$	-3	-1	0	$+\cdot5$	$+\cdot66$	$+1$	$+1\cdot5$	$+2$	$+3$	$+5$	$+9$	$+\infty$
No. of collisions per cm. vary as	independent of V	V	V^2	V^4	V^8	V^{12}	$V^{\pm\infty}$	V^{-8}	V^{-4}	V^{-2}	V^{-1}	$V^{-\frac{1}{2}}$	independent of V

In conclusion, I wish to acknowledge my indebtedness to Professor J. J. Thomson for valuable suggestions.

NOTE [*added* April 20, 1903].

In the application of the formulae deduced in the preceding paper to the calculation of the coefficient of recombination of ions

* *Scientific Papers*, II. 36.

the simplest possible constitution of the ions has been assumed. Many facts however point to the conclusion that the negative ions in air are more than mere corpuscles and that the positive ions are more than a molecule from which a corpuscle has been removed. Among such facts which have long been known are: (1) the numerical values of the velocities and coefficients of diffusion of the ions which indicate that they are of greater than molecular dimensions, (2) the fact that the velocities of the positive and negative ions are of the same order of magnitude, and (3) the effect of moisture on the velocities of the ions.

Recently several other results have been obtained which point to the same conclusion. Langevin* has found that at low pressures the velocity of the ions increases more rapidly than the inverse ratio of the pressure which is required by the formula

$$v = \frac{1}{2}\frac{e}{m}\frac{\lambda}{\Omega}$$

before mentioned. The effect is specially marked in the case of the negative ions, the positive ions showing very little increase at the pressures investigated.

As a result of some work at present being carried out in the Cavendish Laboratory, Mr McClung finds that the coefficient of recombination of X-ray ions in air increases very rapidly when the temperature is raised, instead of varying inversely as the square root of the absolute temperature, as the simple theory requires. Thus Mr McClung finds that at 270° C. the coefficient of recombination is about eight times as big as at 15° C.

Both these results can only be explained by supposing that X-ray ions in air at atmospheric pressure are complicated structures which become smaller as the pressure is lowered or the temperature raised.

It is at once evident that we cannot apply the foregoing theory to the case of X-ray ions in air without making considerable reservations. For example, it will not give the correct variation with conditions which change the nature of the ions, such as temperature and pressure (at low pressures). In addition, our present ignorance of the nature of the ions prevents the absolute value of the coefficient being calculated. The theory also assumes that the diameter of the ions is small compared with the mean free path. This condition however appears to be fulfilled by X-ray ions at atmospheric pressure†.

* *Thèses de l'Université de Paris*, A 431. Gautier Villars, 1902.
† See Townsend, *Phil. Trans.* A. vol. 193, p. 156.

On Mendelian Heredity of three characters allelomorphic to each other. By W. BATESON, St John's College.

[*Read* 4 May 1903.]

The object of this note was to call attention to various possibilities attainable by a modification of the Mendelian method. In the ordinary method the constitution of the gametes in the first cross (F_1) is tested by breeding such individuals *inter se* or with a pure recessive. The ensuing generation (F_2) will consist of a mixture of dominant and recessive individuals; but if the proportions depart from the expected 3 : 1 or 1 : 1, it is not possible to tell whether such departure is due to change in relative numbers of dominant and recessive gametes, to imperfect segregation of characters, or to change in dominance. This question can in part be answered by a method which consists in crossing F_1 produced from a parent having one dominant character, with another heterozygous individual having a different dominant character (the same recessive being used in both cases). In the poultry experiments described, rose comb (R) and pea comb (P) were used as the two dominants, single comb (S) being the recessive. When RS was crossed with PS, F_2 showed the four expected forms RS, PS, SS, and RP, each well characterized. The RP combs have a highly peculiar structure, resembling the "walnut" comb of some Malay fowls, the comb almost always being *feathered* to some extent, and generally crossed by a curious band of small feathers.

In F_3, RS gave rise to only R and S birds. No doubt the PS would give only P and S, and the S birds, S only. But if R and P had been completely segregated in F_1 the RP birds should be incapable of producing any singles in F_3. In the experiments $RP \times RP$ did give some singles, and therefore, assuming that S is not a hypallelomorph of R or P (which is being tested), a proof is provided that the segregation of R (or perhaps P) from S may be imperfect. That it may also be perfect for other R and P gametes has of course been previously established in several cases by breeding from dominants in F_2, some proving homozygous dominants, while others were heterozygotes. The ratios in which the several sorts of gametes are produced by the "walnut" birds have

not been clearly made out. Complications arise from the fact that the segregation of R from P may be imperfect, for some "walnuts" occur in F_3 from walnut × single. At present it seems likely that R, P, RP, and S are produced in equal numbers, but various experiments are needed to settle the point. A note of these qualitative results seemed of value as indicating possible lines of work, which if pursued on several subjects may considerably extend our knowledge of gametic differentiation.

Possibly the F and D forms raised by de Vries from *Antirrhinum* (*Mutationsth.* II. 197 and 353) are somewhat analogous examples of imperfect segregation. An indication might be got by breeding from the reds formed by extracted F × extracted D. If whites then occur we should have proof that F was one hypallelomorph + white, and D another hypallelomorph + white, and the problem would be greatly simplified.

On the Difference of Potential between the Terminals of a Vacuum Tube. By W. A. D. RUDGE, M.A., St John's College.

[*Read* 4 May 1903.]

During the course of some experiments which were undertaken with the object of investigating the influence (if any) of a magnetic field on the deposition of the metal volatilized from the cathode in a vacuum tube, it was observed that the metal was concentrated at certain places in the tube, especially along a line at right angles to the direction of the field where the end of the positive column was deflected upon the glass.

It was suggested by Prof. J. J. Thomson that it would be of interest to observe the effect produced by placing a piece of wire-gauze in the path of the cathode discharge.

When this was done the gauze acted as a secondary cathode, and a deposit of metal was obtained from it as well as from the original cathode.

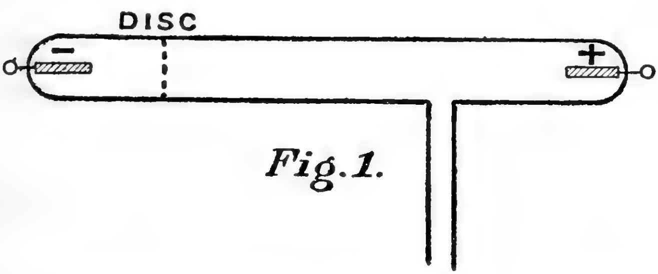


Fig.1.

The tubes used were shaped as in fig. 1. The electrodes were of iron or copper and the gauze was composed either of the same metals as the electrodes or of different metal. The metal volatilized from the primary cathode passed readily through the meshes of the gauze, whether the latter was composed of the same metal or of a different metal, and a deposit of both metals was found on the sides of the tube beyond the gauze.

The most interesting thing observed was that the presence of the gauze caused the potential difference between the electrodes to

be largely increased when the discharge was passing through the tube, the potential being inferred from the length of spark between the knobs of a spark micrometer joined in parallel with the tube.

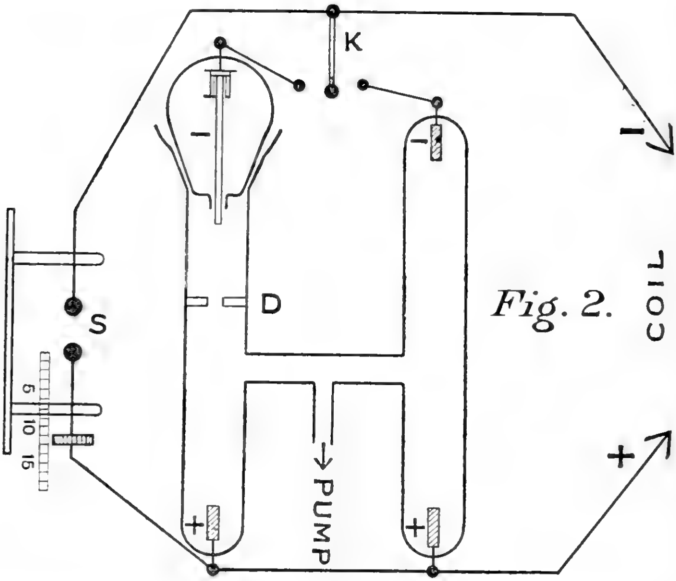


Fig. 2.

The potential varied with the nature of the metal composing the gauze and also with the mesh or area of aperture.

A series of experiments was then undertaken in order to investigate the relation between the nature of the metal and the increase in potential of the tube. This increase was very great, the potential being from two to six times that of a similar tube without gauze and at the same degree of exhaustion.

Instead of wire-gauze it was found more convenient for purposes of comparison to use discs of metal perforated with the same number of holes and drilled with the same drill, so that the aperture areas were the same in each case. The discs were fixed in the tubes by means of two wire rings of the same metal as the discs.

In order that the potentials of two tubes with discs of the same or different metals might be compared at the same pressure an apparatus was constructed as in fig. 2.

The vertical tubes were about 20 cm. in length and 1·5 cm. in diameter. On the tubes used at first the electrodes were attached to brass caps which were fastened to the tubes by sealing-wax.

The discs were placed at the same distance, about 2 cm., from the electrodes in each tube. By the switch K, the current could be sent through either tube at will, and from the reading of the spark micrometer S, the potentials of the tubes could be inferred. The apparatus was attached to the pump at T and exhausted until a good phosphorescence was obtained when the discharge passed. The coil was adjusted to give a spark of from 4 to 6 cm. in air at atmospheric pressure.

In the first experiment the electrodes were of iron and the discs in both tubes were either all of aluminium or of copper. Then the reading of the spark micrometer was the same for each tube. Next, aluminium discs were inserted in one tube and copper discs in the other. The spark lengths were as follows:

	Al tube	Cu tube
Length of spark	1·5 cm.	·5 cm.
,,	1·9	·6
,,	1·2	·3
,,	2·4	·9
,,	2·0	·7
,,	1·8	·6

or approximately Al : Cu = 3 : 1.

The sealing-wax joints gave considerable trouble owing to the tube becoming so hot that the wax melted, and therefore the tube shown in the figure was devised.

The stopper was hollow and had a small aluminium cap attached to the platinum wire. Into this cap different electrodes could be fitted. Some little indentations were made in the side of the tube so that the various discs could be placed at the same distance from the cathode. The discs were really shallow caps turned to fit the tube snugly, the hole being of course of the same size, 0·4 cm. each in diameter.

The tube without a disc was used as a standard with which to compare the potential of the other tube. The use of a second disc close to the anode was found to exert but little effect, and therefore was not used further.

The mode of conducting an experiment consisted in exhausting the tubes until a good phosphorescence was obtained, the dark space extending up to or beyond the disc. Then the standard tube being in connection with the coil the micrometer was adjusted until the sparks passed between the knobs and the discharge through the tube. The micrometer reading was then taken. The current was then switched from the standard to the one with the disc, and a micrometer reading taken as before. The ratio of the spark lengths was taken to be the ratio of the potential differences.

The following is a typical example:

Spark given by induction coil 5 cm.

Iron Electrodes. Aluminium Disc.

Spark length by Micrometer.

Standard tube	Al disc tube	Ratio
0·5 cm.	1·6 cm.	3·2
·4	1·6	4·0
·3	1·1	3·6
·6	2·0	3·3
·6	2·2	3·6

In a similar manner the tube with disc of copper, iron, and silver was compared with the standard and, taking the mean of many experiments, the following figures were deduced:

	Standard	Al	Cu	Fe	Ag
Potential differences from spark length	1·	3·5	1·35	1·35	1·25

The pressure of the air in the tube was about ·05 mm. in each case.

A magnesium disc was next used and this gave a perceptibly higher potential than was the case with aluminium, but owing to the breaking of the tube it could not be directly compared with the series given above. It was however compared with aluminium in another tube. Lead and zinc were also compared with iron, and the following potential ratios obtained:

$$\mathrm{Al} : \mathrm{Mg} = 1 : 1{\cdot}2, \quad \mathrm{Fe} : \mathrm{Pb} = 1 : 0{\cdot}75,$$
$$\mathrm{Fe} : \mathrm{Zn} = 1 : 1.$$

Arranging the metals experimented on in the order in which they (when used as perforated discs near to the cathode) increase the potential of a vacuum tube, the apertures of the discs being of the same size, we got the order:

Pb	Ag	(Cu Fe Zn)	Al	Mg
1	1·25	1·35	3·5	3·8

that is, if the introduction of a lead disc increased the potential difference of a certain tube by a definite amount, then an aluminium disc would increase the potential by about 3·5 times the amount.

It was found that a glass disc produced very much the same effect as Fe.

As the potential of the tube evidently depended upon the position occupied by the disc some experiments were made to find out at what part of the tube the disc exerted the maximum effect. With this object a tube was constructed about 30 cm. in length, and having an iron disc which could be moved into any required position by means of a magnet and the spark potential found by the micrometer for the various positions. Starting with the disc in contact with the anode and moving it towards the cathode, the potential rose slightly at first, then fell a little, and remained constant until the disc neared the cathode. The potential then increased, reached a maximum and fell off a little when in contact with the cathode.

The rise in potential began when the disc entered the cathode dark space, and the position of the maximum depended upon the degree of exhaustion.

The following is a typical example:

Distance of disc from cathode	Spark length
3 cm. and beyond	0·3 cm.
2 cm.	0·5 cm.
1·5 cm.	0·9 cm.
1·0 cm.	1·2 cm.
0·5 cm.	1·4 cm.
·25 cm.	1·4 cm.
0 cm.	1·2 cm.

The length of the dark space was about 2·5 cm. but it was difficult to locate the exact termination owing to the small diameter of the tube and the bright phosphorescence of the glass. See fig. 3 and curve (1).

As it seemed probable that the increase in potential was due to the cathode rays being drawn together by the disc, a tube was made in which the cathode was an aluminium wire 1 mm. in diameter, and covered with glass except at the extremity so that the discharge proceeded from practically a point, fig. 3, *A*, *B*, *C*.

With this tube, using an iron disc, the following results were obtained (curve 2):

Distance of disc from cathode	Spark length
0 cm.	0·4 cm.
0·7 cm.	0·8 cm.
1·0 cm.	0·6 cm.
1·2 cm.	0·35 cm.
2·0 cm. and beyond	0·25 cm.
1·5 cm.	0·33 cm.
1·0 cm.	0·55 cm.
0·8 cm.	0·7 cm.

The length of the dark space could not be determined with this tube, but it was probably about 2 to 3 cm.

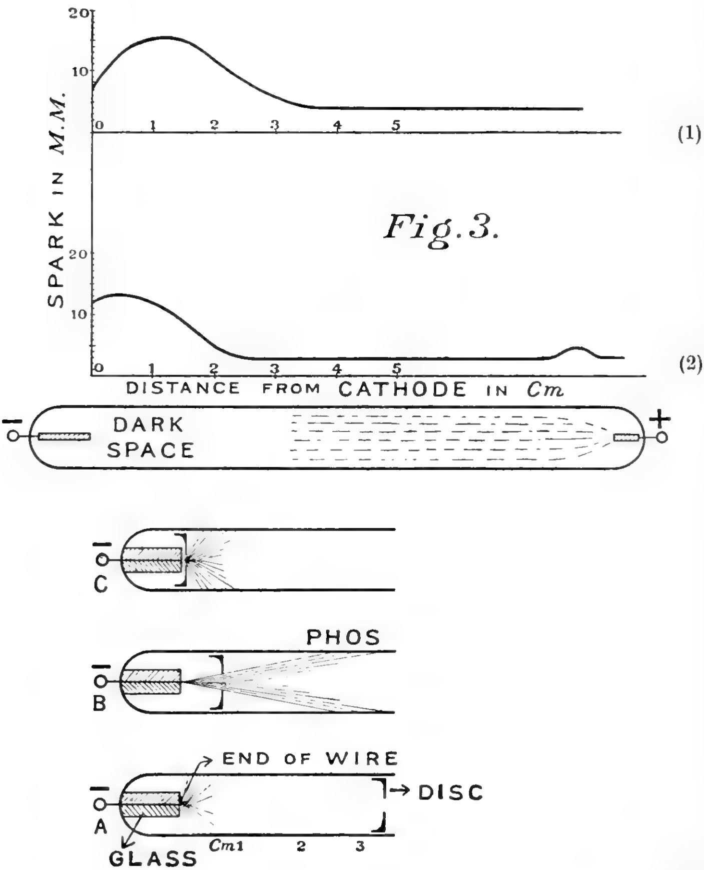


Fig. 3.

When the disc was actually behind the point cathode (the hole in the disc was large enough to pass over the glass covering the cathode) the potential was rather more than when at the central parts of the tube, and a sudden rise occurred just as the disc came over the cathode. The phosphorescence (A, fig. 3) which previously existed just opposite the cathode was thrown down the tube, the aperture pulling the rays together to form a cone having its apex at the cathode (B, fig. 3), and the base of the cone, *i.e.* the position where the phosphorescence of the tube

begins, being determined by the distance the aperture of the disc was from the cathode, fig. 3.

As pointed out at the beginning the potential difference was connected with the area of the aperture, and a comparison was made between the rise in potential caused by a disc with no aperture, and with discs having different apertures.

An apparatus, somewhat similar to fig. 2, was used, but both tubes had ground stoppers and aluminium electrodes. This was first exhausted without any discs being present and the potentials of each tube observed. The ratio was found to be 1 : 1.

Then a solid copper disc was inserted in one tube and a perforated copper disc in the other.

When the diameter of the hole was about ·25 cm. the ratio of the potential of the tube with the solid disc to that of the tube with the perforated one was about 5 : 1. With smaller apertures the ratio was smaller, and with larger apertures greater. Approximately, between 0·1 and 0·5 cm. the potential was inversely as the size of the hole but exact figures could not be obtained owing to the difficulty of securing the same degree of exhaustion, after changing the discs in the tube.

As it seemed likely that the disc acted as a secondary cathode*, the effect of having a second disc in the tube was next tried. The electrodes were of aluminium and the discs of iron.

The tube was exhausted and one disc moved up close to the cathode, and the potential compared with that of a standard tube exhausted at the same time. The second disc was then moved up close to the first, and the potential was then found to be increased to nearly twice as much as with the one disc.

The rise in potential caused by the approach of the second disc to the first was of the same character as that seen when using one disc only, viz., a maximum occurred just a little way from the first disc, *i.e.* the first disc behaved as though it were a cathode itself†.

The greater part of the observations were made when the tube had been filled with air before exhaustion, but a number of observations were also taken after filling the tube with hydrogen, or carbonic acid, or nitrogen. The results were practically the same as with air.

* Goldstein has shown (*Phil. Mag.* 10, Pl. 173) that when a discharge passes through a constriction in a vacuum tube, the constriction acts as a secondary cathode, and also that "the cathode itself may be replaced by a system of small holes or closely packed pores congruent with the cathode."

† Goldstein, working with movable cathodes and secondary cathodes with variable apertures (*Phil. Mag.* 14, Pl. 366) showed that the number of stratifications depended upon the aperture area, the smaller the aperture the fewer the number of stratifications.

The following conclusions may be drawn from the experiments:—

(1) The presence of a perforated disc in a vacuum tube increases the resistance of the tube in the electrical discharge.

(2) The increase in resistance is inversely proportional to the area of the aperture of the disc.

(3) The increase in resistance depends upon the position occupied by the disc, the maximum effect being produced at a short distance from the cathode.

(4) The increase in resistance depends upon the nature of the metal forming the disc, the lower the atomic weight the greater the increase.

My very best thanks are due to Professor J. J. Thomson for the kindly advice he has given me during the course of the experiments.

The determination of curves satisfying given conditions. By H. BATEMAN, Trinity College.

[*Read* 4 May 1903.]

Since the co-ordinates of a point on a curve only involve one parameter, it follows that a line associated with a point on the curve will not take up every direction in space, but only a singly infinite number of directions.

Accordingly, if from a fixed point we draw lines in the particular directions, they will all lie on a certain cone which will cut a unit sphere with the fixed point as centre in a curve.

This curve has been called the indicatrix of the former curve; when the associated line is the tangent, principal normal, or binormal at the point, the curve will be distinguished as the tangent, principal normal, or binormal indicatrix.

Corresponding to a given indicatrix we have a group of curves which possess certain properties in common: for example, if the tangent indicatrix is a small circle, all the curves will be helices.

If we start with a given curve and derive a series of curves from it by some definite process, the tangent indicatrix of the original curve will in general be an indicatrix (for some associated direction) of the derived curves. For example, if we consider the evolutes of a curve, the principal normal of an evolute is parallel to the tangent to the curve; accordingly the tangent indicatrix of the curve is the principal normal indicatrix of the evolutes.

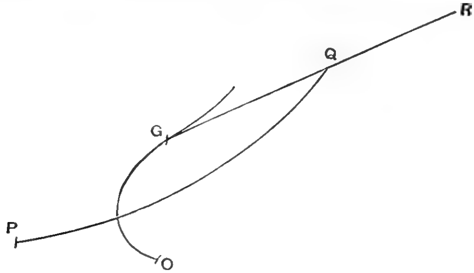
One method by which a series of curves can be derived from a given curve is as follows.

Let O be the origin, P a fixed point on a curve, Q a variable point; G the centre of gravity of a weight w at O and the arc PQ, where the density at distance s from P (measured along the curve) is $\frac{du}{ds}$: u being any function of s. If the co-ordinates of P and Q are (α, β, γ) and (x, y, z), the co-ordinates of G will be (ξ, η, ζ) where

$$\left.\begin{aligned} \xi\{w+u\} &= \int_P^Q x\frac{du}{ds}\,ds \\ \eta\{w+u\} &= \int_P^Q y\frac{du}{ds}\,ds \\ \zeta\{w+u\} &= \int_P^Q z\frac{du}{ds}\,ds \end{aligned}\right\} \quad \text{..................(1)},$$

the constant in u having been chosen so that u vanishes at P.

When we add the small element QQ' the new centre of gravity will in the limit lie in GQ: therefore GQ is the tangent at G to the locus of G.



Thus the equations (1) are those of a curve which passes through an arbitrary point O, and whose tangent always meets the given curve.

They can also be written in the form

$$\left.\begin{aligned}\xi w + u(\xi - x) &= \int_P^Q u\frac{dx}{ds}ds\\ \eta w + u(\eta - y) &= \int_P^Q u\frac{dy}{ds}ds\\ \zeta w + u(\zeta - z) &= \int_P^Q u\frac{dz}{ds}ds\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(2);$$

accordingly, if we produce GQ to R so that $GR = \frac{u}{w}GQ$, the co-ordinates of R will be $\frac{1}{w}\int u\frac{dx}{ds}ds$, etc.: and therefore R describes a curve having the same tangent indicatrix as the original curve.

Consider now a group of curves having the same tangent indicatrix. Starting with one curve, the equations of any other curve of the group are of the form

$$\left.\begin{aligned}x' &= \int u\frac{dx}{ds}ds\\ y' &= \int u\frac{dy}{ds}ds\\ z' &= \int u\frac{dz}{ds}ds\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

It is easy to verify that for the new curve (with the usual notation)

$$\rho' = \rho u, \quad \sigma' = \sigma u, \quad ds' = u\,ds, \quad dx' = u\,dx.$$

Now the quantity u is at our disposal, accordingly if we choose u so that

$$F\{\rho u, \sigma u, u\,dx, u\,dy, u\,dz\} = 0,$$

the new curve will possess the property

$$F\{\rho', \sigma', dx', dy', dz'\} = 0,$$

thus for a curve of constant torsion

$$u = \frac{1}{\sigma}.$$

Of the curves in which there is a special relation between the curvatures, Bertrand's curves form an interesting class: they possess the characteristic property that their principal normals are also the principal normals of another curve. The relation in this case is of the form

$$\frac{a}{\rho} + \frac{b}{\sigma} = c,$$

this includes helices $\left(\frac{\sigma}{\rho} = \text{constant}\right)$, curves of constant curvature, and curves of constant torsion.

Curves of constant torsion have been much studied during recent years: in 1884 Darboux proposed the problem of finding the algebraic curves of constant torsion, several attempts have been made and some particular curves have been obtained*.

Returning to equations (3) it is evident that we shall obtain the same results whichever curve of the group we start with; now as there will always be a number of curves of the group which can be drawn on a sphere, suppose we choose one of these as our initial curve.

If χ be the angle between the radius of the sphere and the principal normal of the curve, we have the formulae

$$\rho = a \cos \chi, \quad \sigma = \frac{d\chi}{ds}.$$

The following are the forms for the equations of the derived curve, corresponding to different properties:

$\sigma' = c,$ $\qquad x' = c \int \frac{dx}{ds}\, d\chi,$

$\rho' = c,$ $\qquad x' = c \int \sec \chi \,.\, dx.$

Curve on a sphere having same indicatrix

$$x' = c \int dx \,\{a + b \tan \chi\},$$

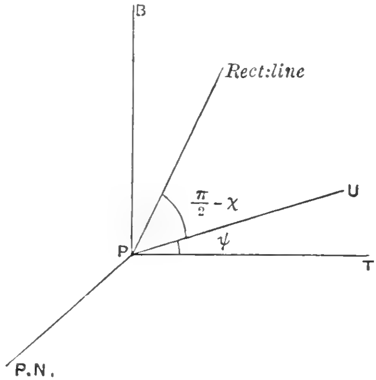
* Cf. Darboux, *Théorie générale des surfaces*, Vol. IV. p. 429.

$$\frac{d\rho}{d\tau} = \text{constant}, \qquad x' = c \int dx \,.\, \chi \sec \chi,$$

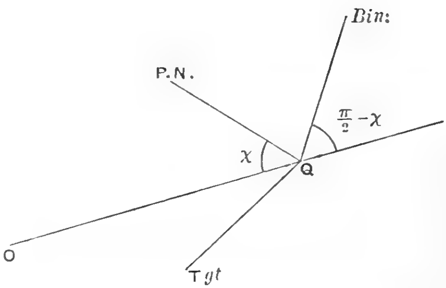
$$\sigma = \lambda s, \qquad x' = \int dx e^{\lambda\chi} \frac{d\chi}{ds},$$

$$\frac{d\rho}{d\tau} \propto \rho, \qquad x' = \int dx e^{\lambda\chi} \sec \chi.$$

When we require a special relation between the curvatures it is convenient to make use of the indicatrix. Now it is evident that a group of curves which have the same tangent indicatrix have also the same indicatrix for any line which is fixed relative to the axes of the curve.



Draw a line OQ of unit length parallel to a line PU in the rectifying plane making a constant angle ψ with the tangent. The tangent at Q to this indicatrix is parallel to the principal normal of the curve, while the binormal of the indicatrix is parallel to the rectifying line of the curve.



Let χ be the angle between the principal normal of the indicatrix and the radius of the sphere.

Then $\frac{\pi}{2} - \chi =$ angle between the binormal and radius, $= \alpha - \psi$, where α is the angle between the rectifying line and tangent of the curve; so that $\tan \alpha = \frac{\sigma}{\rho}$, therefore

$$\tan \chi = \cot(\alpha - \psi)$$

$$= \frac{1 + \frac{\sigma}{\rho} \tan \psi}{\frac{\sigma}{\rho} - \tan \psi}.$$

Let the direction cosines of the axes of the curve be given by the scheme

Tangent	$l_1,$	$m_1,$	$n_1,$
Prin. Norm.	$l_2,$	$m_2,$	$n_2,$
Binormal	$l_3,$	$m_3,$	$n_3.$

Then for the indicatrix

$$x_0 = l_1 \cos\psi + l_3 \sin\psi,$$

therefore $$dx_0 = l_2 \{d\epsilon \cos\psi - d\tau \sin\psi\}$$

by the Serret-Frenet formulae;

thus $$ds_0 = d\epsilon \cos\psi - d\tau \sin\psi.$$

Now $$x_0 ds_0 = [l_1 \cos\psi + l_3 \sin\psi][d\epsilon \cos\psi - d\tau \sin\psi],$$

and

$$\begin{aligned} z_0 dy_0 - y_0 dz_0 &= [(n_1 m_2 - n_2 m_1)\cos\psi + (n_3 m_2 - n_2 m_3)\sin\psi]\, ds_0 \\ &= [-l_3 \cos\psi + l_1 \sin\psi][d\epsilon \cos\psi - d\tau \sin\psi]; \end{aligned}$$

therefore

$$x_0 ds_0 \cos\psi + (z_0 dy_0 - y_0 dz_0)\sin\psi = l_1 [d\epsilon \cos\psi - d\tau \sin\psi].$$

Consider $$u = \left\{\frac{\cos\psi}{\rho} - \frac{\sin\psi}{\sigma}\right\} F \left\{\frac{\dfrac{\cos\psi}{\sigma} + \dfrac{\sin\psi}{\rho}}{\dfrac{\cos\psi}{\rho} - \dfrac{\sin\psi}{\sigma}}\right\},$$

this implies that for the new curve

$$\left\{\frac{\cos\psi}{\rho'} - \frac{\sin\psi}{\sigma'}\right\} F \left\{\frac{\dfrac{\cos\psi}{\sigma'} + \dfrac{\sin\psi}{\rho'}}{\dfrac{\cos\psi}{\rho'} - \dfrac{\sin\psi}{\sigma'}}\right\} = 1.$$

The equations of this curve, viz.

$$x' = \int l_2 u\, ds,$$

at once reduce to the form

$$x' = \int l_1 \{\cos\psi\, d\epsilon - \sin\psi\, d\tau\}\, F(\tan\chi),$$

or $$x' = \int [x_0 ds_0 \cos\psi + (z_0 dy_0 - y_0 dz_0)\sin\psi]\, F(\tan\chi).$$

The object of introducing the angle ψ is to make the function F as simple as possible.

When $\psi = 0$, $$x' = \int x_0 ds_0 F(\tan\chi)$$

gives a curve for which

$$\frac{1}{\rho} F \left\{\frac{\rho}{\sigma}\right\} = 1,$$

and when $\psi = \frac{\pi}{2}$, $\quad x' = \int F(\tan\chi)(z_0 dy_0 - y_0 dz_0)$

gives a curve for which

$$-\frac{1}{\sigma} F \left(-\frac{\sigma}{\rho}\right) = 1.$$

Consider two such curves obtained from a given curve on the sphere, the tangent at any point on one is parallel to the binormal at the corresponding point of the other, whilst the principal normals are parallel.

Suppose now that we divide the line joining corresponding points on the two curves in a constant ratio $\cos\psi : \sin\psi$, then the co-ordinates of this point will be

$$x = \frac{1}{\cos\psi + \sin\psi} \int F(\tan\chi)\,[x_0 ds_0 \cos\psi + (z_0 dy_0 - y_0 dz_0)\sin\psi],$$

accordingly it will describe a curve for which

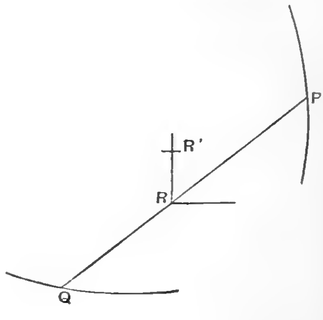
$$\left\{\frac{\cos\psi}{\rho} - \frac{\sin\psi}{\sigma}\right\} F \left\{\frac{\dfrac{\cos\psi}{\sigma} + \dfrac{\sin\psi}{\rho}}{\dfrac{\cos\psi}{\rho} - \dfrac{\sin\psi}{\sigma}}\right\} = \text{constant};$$

also the tangent at any point of this curve will make angles ψ and $\frac{\pi}{2} - \psi$ with the tangents at the corresponding points of the other two curves, and the principal normals of the three curves will be parallel.

If we form a surface by dividing the line joining any two points on the two curves in the ratio $\cos\psi : \sin\psi$, the previous curve will be a geodesic on this surface.

Let P and Q be the points on the curves, R the corresponding point on the surface.

Now if we keep Q fixed the tangent plane at R will be parallel to the tangent at P, for a similar reason it is also parallel to the tangent at Q; therefore when we keep Q fixed the tangent planes at two consecutive points R, R' will intersect in a line parallel to the tangent at Q; accordingly the directions of the tangents at P and Q are conjugate directions of the surface at R.



Now when P and Q are corresponding points on the two curves, the tangents at P and Q will be perpendicular, and therefore the normal to the surface at R, being perpendicular to the tangent and binormal at P, will be parallel to the principal normals of the three curves.

Thus the third curve is a geodesic on the surface; moreover, since the tangents at P and Q are perpendicular, and also parallel to conjugate lines at R, they are the directions of the lines of curvature at R; but the tangent to this geodesic makes a constant angle with these two tangents: accordingly we have a geodesic on the surface which cuts the lines of curvature at a constant angle.

Further, if R_1 and R_2 are the principal radii of curvatures of the surface at any point, we have along this geodesic

$$\frac{1}{\rho} = \frac{\cos^2\psi}{R_1} + \frac{\sin^2\psi}{R_2},$$

$$\frac{1}{\sigma} = \sin\psi\cos\psi\left\{\frac{1}{R_1} - \frac{1}{R_2}\right\}:$$

thus we can find the relation between R_1 and R_2 which is satisfied along this geodesic.

Returning to the determination of special curves, the following are convenient forms in certain cases:

$\rho = a,\qquad x' = a\int x_0 ds_0,$

$\sigma = a,\qquad x' = a\int(z_0 dy_0 - y_0 dz_0),$

or $\qquad x' = a\int x_0 \tan\chi\, ds_0,$

$\dfrac{\cos\psi}{\rho} - \dfrac{\sin\psi}{\sigma} = \dfrac{1}{a},\qquad x' = a\int x_0 ds_0 \cos\psi + (z_0 dy_0 - y_0 dz_0)\sin\psi,$

$\dfrac{1}{\sigma^2} - \dfrac{1}{\rho^2} = \dfrac{1}{a^2},\qquad x' = a\int\sqrt{\tan\chi}\,\{x_0 ds_0 + z_0 dy_0 - y_0 dz_0\},$

$\dfrac{1}{\sigma^2} + \dfrac{1}{\rho^2} = \dfrac{1}{a^2},\qquad x' = a\int x_0 d\epsilon_0.$

As an example of the method, suppose we take as our indicatrix a helix on a sphere: its equations may be written

$$x_0 = \cos^2\frac{\alpha}{2}\sin\left(2\theta\sin^2\frac{\alpha}{2}\right) - \sin^2\frac{\alpha}{2}\sin\left(2\theta\cos^2\frac{\alpha}{2}\right),$$

$$y_0 = \cos^2\frac{\alpha}{2}\cos\left(2\theta\sin^2\frac{\alpha}{2}\right) - \sin^2\frac{\alpha}{2}\cos\left(2\theta\cos^2\frac{\alpha}{2}\right),$$

$$z_0 = -\sin\alpha\,.\cos(\theta\cos\alpha).$$

The direction cosines of the tangent are

$$\sin\alpha\sin\theta,\quad \sin\alpha\cos\theta,\quad \cos\alpha;$$

therefore $$d\epsilon_0 = \sin\alpha\, d\theta.$$

Thus a curve for which $\dfrac{1}{\sigma^2}+\dfrac{1}{\rho^2}=\dfrac{1}{a^2}$ is given by

$$x = a\int \sin\alpha\, d\theta\left[\cos^2\frac{\alpha}{2}\sin\left(2\theta\sin^2\frac{\alpha}{2}\right)-\sin^2\frac{\alpha}{2}\sin\left(2\theta\cos^2\frac{\alpha}{2}\right)\right],$$

$$y = a\int \sin\alpha\, d\theta\left[\cos^2\frac{\alpha}{2}\cos\left(2\theta\sin^2\frac{\alpha}{2}\right)-\sin^2\frac{\alpha}{2}\cos\left(2\theta\cos^2\frac{\alpha}{2}\right)\right],$$

$$z = -a\int \sin^2\alpha\, d\theta\cos(\theta\cos\alpha),$$

which on integration give

$$x = \frac{a}{2}\sin\alpha\left[\tan^2\frac{\alpha}{2}\cos\left(2\theta\cos^2\frac{\alpha}{2}\right)-\cot^2\frac{\alpha}{2}\cos\left(2\theta\sin^2\frac{\alpha}{2}\right)\right],$$

$$y = \frac{a}{2}\sin\alpha\left[\cot^2\frac{\alpha}{2}\sin\left(2\theta\sin^2\frac{\alpha}{2}\right)-\tan^2\frac{\alpha}{2}\sin\left(2\theta\cos^2\frac{\alpha}{2}\right)\right],$$

$$z = -a\sin\alpha\tan\alpha\sin(\theta\cos\alpha);$$

when $\tan^2\dfrac{\alpha}{2}$ is commensurable this curve is algebraic.

Again, for a curve of constant curvature we have

$$x = \int x_0\, ds_0.$$

Now $$ds_0 = \sin\alpha\sin(\theta\cos\alpha)\, d\theta;$$

therefore

$$x = \int \sin(\theta\cos\alpha)\, d\theta\left[\cos^2\frac{\alpha}{2}\sin\left(2\theta\sin^2\frac{\alpha}{2}\right)-\sin^2\frac{\alpha}{2}\sin\left(2\theta\cos^2\frac{\alpha}{2}\right)\right],$$

$$y = \int \sin(\theta\cos\alpha)\, d\theta\left[\cos^2\frac{\alpha}{2}\cos\left(2\theta\sin^2\frac{\alpha}{2}\right)-\sin^2\frac{\alpha}{2}\cos\left(2\theta\cos^2\frac{\alpha}{2}\right)\right],$$

$$z = \int -\sin\alpha\cos(\theta\cos\alpha)\sin(\theta\cos\alpha)\, d\theta;$$

therefore

$$2x = \frac{\cos^2\frac{\alpha}{2}}{1-2\cos\alpha}\sin\theta(1-2\cos\alpha)+\frac{\sin^2\frac{\alpha}{2}}{1+2\cos\alpha}\sin\theta(1+2\cos\alpha)-\sin\theta,$$

$$2y = \frac{\cos^2 \frac{\alpha}{2}}{1 - 2\cos\alpha} \cos\theta\,(1 - 2\cos\alpha) + \frac{\sin^2 \frac{\alpha}{2}}{1 + 2\cos\alpha} \cos\theta\,(1 + 2\cos\alpha) - \cos\theta,$$

$$2z = \tan\alpha \cos^2 (\theta \cos\alpha);$$

as before, if $\tan^2 \frac{\alpha}{2}$ is commensurable, this curve is algebraic.

The following are some special curves;

$$\left.\begin{aligned} x &= \sin\phi, \\ y &= -\cos\phi, \\ z &= \log\tanh\frac{\phi}{2}, \end{aligned}\right\} \qquad 4\sigma^2 = \frac{\rho^4}{\rho^2 - 1};$$

$$\left.\begin{aligned} x &= \sin\phi \sinh\phi + \cos\phi \cosh\phi, \\ y &= \sin\phi \cosh\phi - \cos\phi \sinh\phi, \\ z &= 2\phi, \end{aligned}\right\} \qquad \rho^2 = 4\sigma,$$

$$\left.\begin{aligned} x &= \tfrac{1}{5} \sinh 2\phi \sin\phi + \tfrac{2}{5} \cosh 2\phi \cos\phi, \\ y &= \tfrac{2}{5} \cosh 2\phi \sin\phi - \tfrac{1}{5} \sinh 2\phi \cos\phi, \\ z &= 2 \sinh\phi, \end{aligned}\right\} \qquad \rho^3 = 8\sigma^2.$$

On the Existence of a Radio-active Gas in the Cambridge Tap-water. By Professor THOMSON.

[*Read* 4 May 1903.]

When Cambridge tap-water is boiled the air given off is mixed with a radio-active gas. The existence of this gas is easily demonstrated by electrical means, for if the air expelled by prolonged boiling from about 10 litres of water is introduced into a closed vessel whose volume is about 600 c.c. the amount of ionization in the vessel (as measured by the saturation current) is increased five or six times. When the water has once been well boiled the gas expelled on any subsequent reboiling is not appreciably radio-active. The gas can also be extracted from water at the temperature of the room by vigorously bubbling air through it; the air as it bubbles through the water gets mixed with the radio-active gas and carries it along with it. When water which has been treated in this way is boiled no radio-active gas is given out; nor is the gas given off when air is bubbled through water which has been well boiled.

The gas extracted in this way from the water retains its radio-active properties after bubbling through strong sulphuric acid, or caustic potash after passing over red-hot copper, or through a narrow platinum tube kept at a white heat: it does not seem appreciably affected when sparks are passed through it.

The gas can diffuse through a porous plate, and by comparing its rate of diffusion with that of CO_2 through the same plate its density can be determined by Graham's law: preliminary measurements of this kind indicate that two different gases are present, of which one has a density about twice, the other between six or seven times that of CO_2. The gas obtained by boiling the water always diffused faster than that procured by bubbling air through the water; it seems possible that in the latter case the gas may get loaded with water-vapour to a greater extent than in the former.

A negatively electrified surface exposed to the gas becomes radio-active, the induced radio-activity dying away to half its value in about 45 minutes. Mr Adams has shown that a positively electrified surface also becomes radio-active when exposed to the gas, though to a smaller extent than if it had been negatively

electrified; an unelectrified surface does not become radio-active. In this respect the gas differs from the emanation from radium, which according to Rutherford produces much more induced radio-activity in an unelectrified surface than in a positively electrified one.

The rate of diffusion through a porous plate of the gas obtained by bubbling air through distilled water containing a trace of radium is not the same as that of the gas got by bubbling through tap-water.

If the gas is confined in a closed space its radio-activity slowly diminishes. Mr Adams found that the gas contained in a vessel of about 300 c.c. capacity lost when not exposed to an electric field about 5 % of its activity in 24 hours, under a strong electric field the rate of loss was doubled. Water drawn from the tap and left exposed in a bucket for a fortnight gave off very little of the gas when subsequently boiled. I have not found any of the gas in any of the numerous samples of rain and surface water which I have tested.

Professor Dewar (to whom I am greatly indebted for assistance and advice) was kind enough to subject the gas obtained by boiling the water to treatment by liquid air. Two samples were treated, one containing about 80 litres of gas, obtained from the coppers of the Star Brewery, Cambridge, by the kindness of Mr Armstrong (to whom I wish to express my thanks), was passed slowly through a bath of liquid air and samples of the emergent gas collected: this on testing was found to have no radio-activity, though it was strongly radio-active before passing through the liquid air: it is evident therefore that at the temperature of liquid air the radio-active gas is frozen out. The other sample of 20 litres prepared in the laboratory was actually liquefied: the liquid was then allowed to boil away, the gas coming off at the commencement of boiling was collected, and also that coming off when the liquid had all but boiled away. On testing the samples for radio-activity the former was found to be slightly radio-active but not nearly so much so as before liquefaction, while the second was extraordinarily radio-active, its activity being quite thirty times that of the original gas: thus showing, as we should expect from its great density, that the radio-active gas is much more easily liquefied than air.

The liquid obtained in the preceding experiment had a very strong smell of coal-gas. I must again express my thanks to Professor Dewar and Mr Lennox for their kindness in making these experiments.

A discharge tube was filled with strongly radio-active gas obtained as above, and the spectrum was most kindly investigated by Mr Newall, who photographed it and measured the lines; no

new lines were however discovered, the lines present being mainly those due to hydrocarbons.

I add a list of the various specimens of water I have examined; 'yes' means that the water contains the gas, 'no' that it does not.

Cambridge tap-water (yes). Rain water (no). Water from ditch round Botanical Garden (no). Water from Trinity College well on the Madingley Road (yes). Water from artesian well in Mr Whetham's garden, Chaucer Road (yes). Water from shallow well in same garden (no). Water from well at Star Brewery (yes). Artesian well in Trinity Hall Cricket Ground (yes). Artesian well at Girton (yes). Ely Town's water (yes). Birmingham Town's water (yes). Ipswich Town's water (yes).

In concluding this preliminary account I have much pleasure in thanking my assistant, Mr E. Everett, for his help in this investigation.

On the continuous spectrum. By T. H. HAVELOCK, B.A., St John's College. Communicated by Professor Thomson.

[*Read* 4 May 1903.]

The continuous spectrum may be regarded as produced by radiation constituted of an irregular sequence of impulses; in other words, it arises from the analysis of purely heat radiation. The problem of the constitution of the radiation has generally been limited to the case of the hypothetical black body, this radiation being defined more strictly as the equilibrium state of the radiation within an enclosure maintained at constant temperature. Then from thermodynamic considerations it can be shown that

$$E \alpha \theta^4; \quad \lambda_m \theta = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1),$$

where E = density of total energy of the radiation,

θ = absolute temperature of the walls of the enclosure,

λ_m = wave length for which the energy is a maximum.

Also if the energy E can be written in the form

$$E = \int_0^\infty E_\lambda d\lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

it follows that E_λ is of the form

$$E_\lambda = \lambda^{-5} \phi(\lambda\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3),$$

and the problem has been to determine the correct form of the function ϕ.

Some years ago Lord Rayleigh (*Phil. Mag.* 1889, p. 460) gave a simple form of average impulse which was capable of representing the complete radiation of a substance at a given temperature; he obtained in this way the formula of H. F. Weber, viz.

$$E_\lambda = c_1 \lambda^{-2} e^{-\frac{c_2}{\lambda^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4),$$

which represented approximately the experimental results known at that time. It may be of interest to show how this method can be extended so as to include the temperature factor and give results similar in form to those indicated by recent experiments and by the indirect theoretical methods.

The method is based upon the expression of a pulse of radiation as a Fourier integral, this corresponding to its physical analysis into periodic constituents; thus we may put an impulse into the form

$$\phi(t) = \int_0^\infty R \cos ut\,du \quad\text{.....................(5)},$$

where $\qquad u$ is proportional to frequency,

and $\qquad R$ is a function of u.

And as a consequence the energy of the pulse can be expressed by

$$E = \int_0^\infty R^2 du \quad\text{..........................(6)}.$$

Consider, for instance, the type of pulse given by

$$\phi(t) = Ae^{-c^2t^2} \quad\text{..........................(7)}.$$

When c is very large this pulse is intense and narrow, while for smaller values it becomes less abrupt. To express it as a Fourier integral, we notice that for large values of c all the effective part is contained within a small region, so that without appreciable error we may take $\pm\infty$ to be the limits for the time t. Then we have

$$\begin{aligned}\pi\phi(t) &= \int_0^\infty \int_{-\infty}^\infty \phi(\omega) \cos u(\omega - t)\,du\,d\omega \\ &= 2\int_0^\infty \cos ut\,du \int_0^\infty Ae^{-c^2\omega^2} \cos u\omega\,d\omega \\ &= \frac{A}{c}\int_0^\infty e^{-\frac{u^2}{4c^2}} du \quad\text{.................................(8)},\end{aligned}$$

and the energy of the pulse is given by

$$E = \frac{A^2}{c^2\pi^2}\int_0^\infty e^{-\frac{u^2}{2c^2}} du \quad\text{....................(9)}.$$

Now the heat radiation of a substance is supposed to consist of an irregular sequence of pulses, thus giving rise to a continuous spectrum; and it seems probable that this also holds for a gas, the individual impulse being given out during the short time of an encounter between two molecules. Any such impulse, when it reaches the observer, may be considered to be a function of the relative velocity of the two molecules and of the temperature of the gas as a whole; thus taking the type of pulse given in (7) and introducing r and θ in such a way as to conform to the general character of a pulse due to an impact, we have

$$\phi(t) = Ar^m e^{-\mu^2 r^2 \theta t^2} \quad\text{.......................(10)},$$

where A, m, μ are constants;

$$\theta = \text{absolute temperature of the gas},$$

$$r^2 = (\xi_1 - \xi_2)^2 + (\eta_1 - \eta_2)^2 + (\zeta_1 - \zeta_2)^2,$$

(ξ_1, η_1, ζ_1) and (ξ_2, η_2, ζ_2) being the velocities of the two molecules.

If we consider Maxwell's law of distribution of velocities still to hold, the number of such encounters with relative velocity r in a short time is proportional to

$$N^2\beta^3 r e^{-\beta(\xi_1^2+\eta_1^2+\zeta_1^2+\xi_2^2+\eta_2^2+\zeta_2^2)}\, d\xi_1 d\eta_1 d\zeta_1 d\xi_2 d\eta_2 d\zeta_2 \quad \ldots(11),$$

where
$$\left.\begin{array}{l} N \propto \text{density of the gas,} \\ \beta \propto 1/\theta. \end{array}\right\}$$

Now any physical analysis of the radiation is an average taken over a short, but sensible, time; so we can find the energy of each pulse and then sum up for all the pulses.

We have seen that for a pulse of type (10) the energy is

$$E = \frac{A^2}{\theta\mu^2} r^{2(m-1)} \int_0^\infty e^{-\frac{u^2}{2\mu^2\theta r^2}}\, du \quad \ldots\ldots\ldots\ldots(12).$$

Then multiplying by (11) and summing for all the molecules of the gas, we find for the average intensity of the radiation the expression

$$E = CN^2\beta^3\theta^{-1} \int_0^\infty du \int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{+\infty} r^{2m-1} e^{-\frac{u^2}{2\mu^2\theta r^2}} e^{-\beta(\xi_1^2+\eta_1^2+\zeta_1^2+\xi_2^2+\eta_2^2+\zeta_2^2)} \, d\xi_1 d\eta_1 d\zeta_1 d\xi_2 d\eta_2 d\zeta_2 \ldots(13).$$

If in the last six integrals we change to new variables given by

$$\left.\begin{array}{lll} \xi_1 - \xi_2 = u; & \eta_1 - \eta_2 = v; & \zeta_1 - \zeta_2 = w \\ \xi_1 + \xi_2 = U; & \eta_1 + \eta_2 = V; & \zeta_1 + \zeta_2 = W \end{array}\right\} \quad \ldots\ldots\ldots(14),$$

we can perform the integrations with respect to U, V, W. Then changing to variables given by

$$u = \psi \sin\phi \sin s; \quad v = \psi \cos\phi \sin s; \quad w = \psi \cos s \ldots(15),$$

we can integrate with respect to ϕ and s.

And we are left with

$$E = CN^2\beta^{\frac{3}{2}} \int_0^\infty du \int_0^\infty \psi^{2m+1} e^{-\beta\psi^2 - \frac{\alpha^2\beta u^2}{\psi^2}}\, d\psi$$

$$= CN^2\beta^{\frac{3}{2}-m} \int_0^\infty du \int_0^\infty x^{2m+1} e^{-x^2 - \frac{\alpha^2\beta^2 u^2}{x^2}}\, dx \quad \ldots\ldots\ldots(16).$$

And if $2m+1$ is even, we obtain E finally in the form

$$E = CN^2\beta^{\frac{3}{2}-m}\int_0^\infty \left\{A + Bu\beta + \ldots + K(u\beta)^{\frac{2m+1}{2}}\right\} e^{-2au\beta}\,du \ldots (17).$$

Changing now from frequency u to wave length λ, and from β to θ, we obtain the energy in the form

$$E = \int_0^\infty E_\lambda\, d\lambda,$$

where $$E_\lambda = CN^2(\lambda\theta)^{-2}\lambda^{-\frac{2m+1}{2}}\left[A(\lambda\theta)^{\frac{2m+1}{2}} + \ldots + K\right] e^{-\frac{c_2}{\lambda\theta}}$$

$$= C_1N^2\theta^{\frac{2m+1}{2}} f(\lambda\theta)\, e^{-\frac{c_2}{\lambda\theta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18).$$

Also integrating (17) with respect to u, we find that the total energy of radiation varies as $\theta^{m-\frac{1}{2}}$; while if λ_m be the wave length for which E_λ has its maximum value E_m, we have

$$\lambda_m\theta = \text{constant}; \quad E_m \propto \theta^{m+\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots (19).$$

These relations are of the required form, and we notice that for $m = 9/2$ they reduce to those which hold for black radiation.

Indirect methods have the advantage of avoiding assumptions concerning the mechanism of radiation, their results only applying, of course, to black radiation; regarding the latter as an aggregate of irregular impulses, it may be possible to determine its constitution by a theory of the statistical equilibrium of aether motions. However, the direct method of summation of elementary pulses serves to emphasize what is considered to be the real nature of radiation giving a continuous spectrum.

Experiments on the Thomson Effect in Alloys of Bismuth and Tin. By S. C. LAWS, St John's College, '1851 Exhibition' Scholar.

[*Read* 4 May 1903.]

The variation with composition of some of the electrical properties of alloys of bismuth and tin have been investigated by Rollmann*, von Ettinghausen and Nernst†, Hutchins‡, Spadavecchia§, and more recently Schulze‖, and many interesting results obtained. The present paper contains the results of some experiments on the Thomson effect or specific heat of electricity in such alloys.

The value of the Thomson effect in bismuth has already been found by Batelli¶, but for the sake of comparison experiments were first made with pure bismuth and then with alloys containing increasing amounts of tin.

The bismuth used in the experiments was obtained as pure as possible by procuring the pure oxide and reducing this in porcelain crucibles with pure potassium cyanide.

In this way bismuth containing no impurity other than 0·02 per cent. of iron was obtained.

The tin used was supplied as pure precipitated metal.

The specimens with which the experiments were made were cast in the form of cylindrical rods about 35 cms. in length and 5 mm. in diameter.

In the case of the alloys the moulds in which the rods were cast consisted simply of glass tubes of the required length and diameter; these were heated to about 240° C. in a bath of oil, and the molten metal, which had been previously kept liquid and well stirred for some hours, poured in and allowed to slowly cool.

* *Pogg. Ann.* 83, p. 78, 1851.
† *Wied. Ann.* XXXIII. p. 474, 1888.
‡ *Amer. Journ. of Science*, XLVIII. p. 226, 1894.
§ *Nuov. Cim.* IX. p. 432, 1899.
‖ *Ann. der Phys.* IX. p. 555, 1902.
¶ *Accad. delle Sci. di Torino, Atti*, XXII. p. 548, 1887.

In this way it was possible to obtain uniform solid rods with a perfectly smooth surface.

The glass was then removed from the rods and these annealed by heating in an oil bath to a temperature as near the melting point as possible and very slowly cooling.

Owing to the greater expansion of bismuth on solidifying, glass tubes could not here be used as moulds; in this case a mould of slate was employed.

This was obtained in two pieces by clamping together two long slabs of slate rectangular in cross section and drilling a hole of the required diameter along the length of the block with its axis coinciding with the line down the centre of the plane of junction.

This mould was placed inside a cylinder, around which a coil of German-silver was wound, and heated to about 250° C. by means of a current in the coil, when the liquid metal was poured in.

By means of an iron clamping arrangement the two slate blocks could be kept in close contact until the molten metal had solidified, and then by separating the two halves of the mould the rod could be withdrawn and annealed.

The method adopted for measuring the "specific heat" is that employed by Haga in his experiments with mercury and platinum* and similar to that originally used by Le Roux†.

It consists essentially in comparing the change of temperature produced when the direction of a current, passing along a temperature gradient, is reversed with the rise in temperature produced at the same point by the passage of a current in the rod when this is at an uniform temperature throughout.

In this latter case the amount of heat evolved per unit length per unit time is known from Joule's law; so that, assuming that the amount of heat produced is proportional to the change of temperature, the amount of heat evolved or absorbed in the former case may be calculated.

The "specific heat," σ, is then obtained from the relation $H = C\sigma d\theta$, where H is the quantity of heat evolved or absorbed per unit time by a current C in passing between two sections whose difference of temperature is $d\theta$.

To increase the magnitude of the effect to be measured and also to eliminate as far as possible any errors due to the want of homogeneousness of the material two rods were used.

These were placed parallel at a distance of 12 cms. apart with one end of each passing through an indiarubber stopper into a bath in which water could be kept boiling and the other end in a bath which might contain melting ice or through which a current of cold water could be circulated.

* *Ann. de l'école polytechnique de Delft*, I. p. 145, 1885, III. p. 43, 1887

† *Ann. de Chim. et de Phys.* X. p. 258, 1867.

The ends in the hot bath were joined by a copper rod, so that a current could be sent through the two rods in series.

To the other ends of the rods leads were soldered and the circuit completed through a battery of accumulators B, adjustable resistance R, ammeter A and reversing key K_1 (fig. 1).

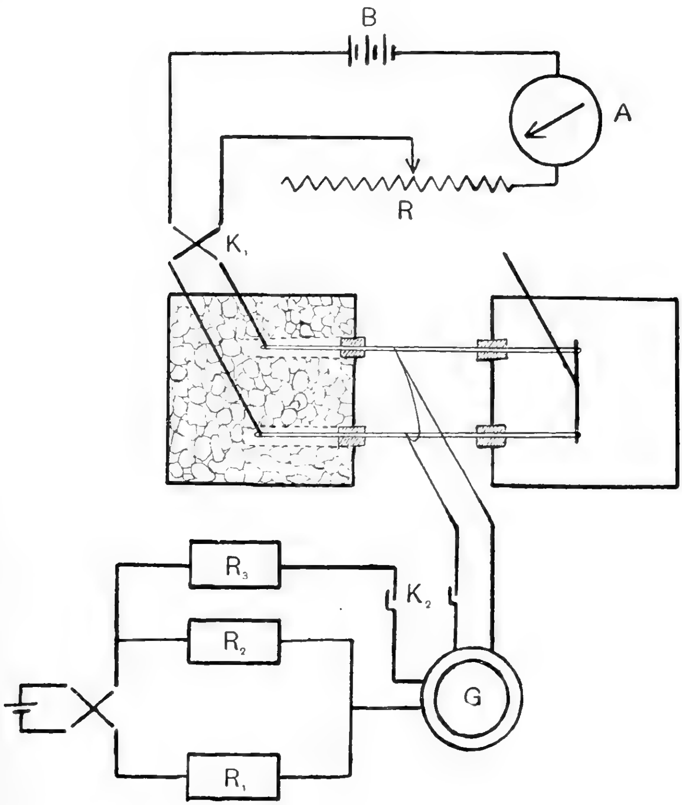


Fig. 1.

To measure the changes of temperature thermo-couples of iron and nickel were used, one junction being fastened at a point on one bar, the other at a point on the other bar which assumes as nearly as possible the same temperature as the former.

The junctions were separated from the bars by thin strips of mica and both rods and baths closely packed with cotton-wool.

The free ends of the couples were joined up in series with one set of coils of a sensitive differential Thomson galvanometer G of low resistance.

The resistance of this circuit was about 1·5 ohms and the galvanometer could easily be arranged to give a deflection of 1 division of a scale reflected in a telescope for a current of 10^{-7} amperes.

The thermo-electric power of the couple was found to be about 33×10^{-6} volts per 1° C., so that a deflection of one division corresponds to a change of temperature of $\frac{1}{220}$th degree centigrade.

The current thus sent through the galvanometer by the E.M.F. of the couple was compensated by adjusting a current in the other set of coils of the galvanometer.

This compensating current was obtained from a single secondary cell joined up with a set of known resistances. From a part R_2 (fig. 1) of the main circuit a current was taken off to pass through the galvanometer and high resistance R_3.

The required magnitude of this compensating current was obtained by first choosing a suitable value for R_2, then adjusting first R_1 and finally R_3; the current is then calculated from the E.M.F. of the cell and known resistances.

Each galvanometer circuit included a key K_2 consisting of a vertical copper rod dropping into a mercury cup; the two rods were insulated from each other and connected to the same movable upright worked by a string over a pulley, their lengths being adjusted so that they touched the surfaces of the mercury in the cups at the same instant.

By this means the compensating current could be adjusted so that the galvanometer gave no deflection when the upright was released and the circuits completed.

This value of the compensating current is then proportional to the difference of temperature between the points of the rods to which the junctions are attached.

The method of carrying out an experiment is then as follows:

The hot bath is filled with boiling water and this is kept boiling either by means of a current in a coil of wire placed inside the bath or by means of a small gas flame underneath; the cold bath is filled with melting ice, the ice being prevented from coming into contact with the rods by means of wire gauze cages surrounding their free ends.

When the temperature of the rods has become steady throughout a constant current—3 or 4 amperes—is passed through them.

The temperature again becoming constant, as indicated by the constant value of the current in the junction circuit, the difference of temperature of the corresponding points of the two rods—or, rather, the current in the compensating circuit necessary to counterbalance the current in the galvanometer due to this difference of temperature of the junctions—is observed.

The current through the rods is then reversed, when, by virtue

of the Thomson effect, the temperature of any point in the rods changes, one rod experiencing an evolution, the other an absorption of heat. After an interval ranging from 30 minutes to 2 hours, according to the magnitude of the temperature change, the temperatures again become steady and the value of the compensating current is again observed.

The current through the rods is then again reversed and so on.

The change of temperature measured in this way is then four times that due to the Thomson effect.

The second part of any experiment consists in measuring the rise in temperature produced by a current in the rods.

In this case the rods are at the same temperature throughout, and by means of a lead soldered to the strip of copper joining them the current is sent through each in turn.

The current is sent first through one rod and when the temperature has become steady the difference of temperature of corresponding points of the two rods measured as in the former experiment; this circuit is then broken and the current passed in the same direction through the other rod. Sufficient time having elapsed for the temperatures to become steady another reading of the compensating current is taken and the current changed back again to the first rod, but flowing in this case in the opposite direction.

The change of temperature thus observed is then twice that due to the evolution of heat in either rod separately.

The evolution or absorption of heat in the first experiment is obtained on the assumption that the heat produced is proportional to the change of temperature, and the value of the "specific heat" calculated.

For if H_1 is the heat produced or absorbed per unit length per unit time when a current flows along a temperature gradient, then

$$H_1 = C_1 \sigma \frac{d\theta}{dx} \text{ ergs},$$

where C_1 is the current in C.G.S. electro-magnetic units,

σ the specific heat of electricity,

and $\frac{d\theta}{dx}$ the temperature gradient,

or
$$H_1 = \frac{C_1 \sigma \frac{d\theta}{dx}}{10} \text{ ergs},$$

when C_1 is measured in amperes.

Also if H_2 is the heat evolved per unit length per unit time by a current C_2, according to Joule's law

$$H_2 = C_2^2 r \text{ ergs},$$

where r is the resistance per unit length, and both C_2 and r are measured in C.G.S. units.

Or $$H_2 = C_2^2 r \times 10^7 \text{ ergs}$$
when C and r are expressed in amperes and ohms respectively.

Hence if δ_1, δ_2 are the changes in the compensating currents observed in the two cases these are proportional to the corresponding changes of temperature, and

$$\frac{H_1}{H_2} = \frac{\delta_1/4}{\delta_2/2} = \frac{\delta_1}{2\delta_2},$$

or $$\frac{C_1\sigma\dfrac{d\theta}{dx}}{10} = H_1 = \frac{\delta_1}{2\delta_2} \,.\, C_2^2 r \times 10^7 .$$

That is, $\sigma = \frac{10^8}{2} \cdot \frac{C_2^2}{C_1} \cdot \frac{r}{d\theta/dx} \cdot \frac{\delta_1}{\delta_2}$ ergs per C.G.S. unit current per 1° C.

The validity of the above assumption that the heat produced is proportional to the change of temperature observed is evident from the following data for the rise in temperature due to different currents in the rods.

Current (C)	Compensating current (y)	y/C^2
1·5 amperes	9·97	4·43
1·6 ,,	11·1	4·34
3·0 ,,	39·6	4·40

It will be seen that within the errors of experiment the rise in temperature is proportional to the square of the current or, by Joule's law, to the heat developed.

In practice it was found that the heat produced at a given point by a current in the rod when this was at an uniform temperature throughout was not altogether independent of the direction of the current, or that something analogous to a Peltier effect occurred throughout the length of the rod.

Hence in these experiments the change in temperature produced by a reversal of the current is measured both when there is a temperature gradient along the rods and when these are at the same temperature throughout, and the latter applied as a correction to the former.

Another series of observations has then been made with the rods reversed.

To obtain the temperatures at various points along the rods a number of thermo-couples of copper and German-silver were

attached at intervals along each rod, one junction of each being fastened to the rod, from which it was separated by a strip of mica, the other being outside the covering of cotton-wool so that it could be immersed in a bath of water.

Each couple was then joined up in turn with the galvanometer and the water heated until there was no current in the circuit when this was completed.

The temperature of the bath then indicated the temperature of the rod at the point observed, and thus the temperature and temperature gradient at any point could be found.

Experiments with Bismuth.

The specific heat of electricity in this metal has been previously measured by Batelli*, who also found that the value is proportional to the absolute temperature.

Mean diameter of rods $= 5{\cdot}7$ mm.

Mean resistance of rods $= 0{\cdot}000050$ ohms per mm.

Some observations taken are shown in Tables I and II, in which H is the time at which the observation was made, γ_1 the value of the compensating current when the current flows in one direction through the rods, γ_2 the corresponding value when the current in the rods is reversed, and δ the change in the compensating current due to the reversal of the current in the rods.

TABLE I.

Thomson effect. $\theta = 30°$ C.

$C = 3$ amperes. $\dfrac{d\theta}{dx} = 0{\cdot}525$.

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	δ
3·34	– 4·562		
4·29		– 4·228	·381
5·24	– 4·656		
6·15		– 4·308	·388
7·8	– 4·738		
8·7		– 4·404	·359
8·59	– 4·785		

Mean value for δ : 0·376
Correction + ·003

0·379

* *loc. cit.*

TABLE II.

Joule effect.
$C = 2$ amperes.

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	d
12·24		+ 0·97	
1·26	− 1·79		2·84
2·22		+ 1·12	
3·27	− 1·72		2·76
4·23		+ 0·96	
5·18	− 1·75		2·77
6·15		+ 1·08	
7·11	− 1·77		2·83
8·5		+ 1·05	
9·0	− 1·68		2·76
·55		+ 1·12	

Mean value for d : 2·79.

Substituting in the expression for σ given above (p. 184) the value $\sigma_{30^\circ} = -863$ ergs per C.G.S. unit current per 1° C. is obtained.

At 23°·8 the values − 847, − 857 were obtained with currents of 3 and 4 amperes respectively.

The value found by Batelli was $-12{\cdot}7 \times 10^{-6}$ calories or − 532 ergs.

The discrepancy between the two results may well be attributed to some slight difference of composition, for Riecke gives the value $+24{\cdot}5 \times 10^{-6}$ calories for commercial bismuth*. Moreover, Le Roux† in his comparative results obtains the same values for bismuth and iron whilst Batelli finds for iron the value − 1130 ergs‡.

Alloy No. 1, containing 1·23 per cent. of tin.

Mean diameter of rods = 4·85 mm.

Mean resistance of rods = 0·00019 ohms per mm.

One series of observations is shown in Tables III and IV from which the value $\sigma_{48^\circ\cdot 8} = -11{,}700$ is calculated.

* *Experimentalische Physik*, II. p. 345.
† *loc. cit.*
‡ *loc. cit.* p. 539.

TABLE III.

Thomson effect. $\theta = 48°{\cdot}8.$

$C = 3$ amperes. $\dfrac{d\theta}{dx} = 0{\cdot}66.$

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	δ
3·45	+ 0·90		
4·26		− 10·12	10·8
5·8	+ 0·54		
·45		− 10·50	10·9
6·25	+ 0·23		
7·3		− 10·63	10·9
8·16	+ 0·30		
·52		− 10·37	10·8
9·30	+ 0·38		

Mean value for δ : 10·9
Correction − 0·4
10·5

TABLE IV.

Joule effect.
$C = 1{\cdot}6$ amperes.

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	d
3·26	+ 4·91		
4·25		− 6·66	11·0
5·20	+ 3·78		
6·15		− 6·87	11·2
7·10	+ 4·96		
8·6		− 6·49	10·9
9·3	+ 3·81		

Mean value for d : 11·0.

Other results obtained are:

$\sigma_{40^\circ \cdot 2} = -10{,}700$ ergs per C.G.S. unit current per 1° C.
$\sigma_{53^\circ \cdot 8} = -12{,}600$ „ „ „ „
$\sigma_{67^\circ \cdot 5} = -14{,}600$ „ „ „ „
$-14{,}700$ „ „ „ „

These values increase much more rapidly than the absolute temperature; in fact a straight line drawn through the points cuts the axis of temperature at -40° C. (fig. 2).

It will be observed that these values are about 12 times as large as those found for bismuth; the largest value previously found was obtained by Le Roux for an alloy of bismuth and antimony and is about 2·4 times that given for bismuth.

Alloy No. 2, containing 3 per cent. of tin.

Mean diameter of rods = 4·82 mm.
Mean resistance of rods = 0·000177 ohms per mm.

From the data given in Tables V and VI the value

$\sigma_{42^\circ \cdot 7} = -12{,}900$ is obtained.

TABLE V.

Thomson effect. $\theta = 42^\circ \cdot 7$.

$C = 3$ amperes. $\frac{d\theta}{dx} = 0{\cdot}585$.

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	δ
1·4	− 4·32		
3·0		+ 6·33	10·10
4·56	− 3·21		
7·2		+ 6·80	9·86
9·2	− 2·92		
12·10	− 3·94		
2·13		+ 6·49	10·11
4·13	− 3·29		
6·11		+ 6·52	9·82
8·8	− 3·31		
10·4		+ 6·71	9·92

Mean value for δ : 9·96
Correction − ·06
9·90

Table VI.

Joule effect.
$C = 2$ amperes.

H	$\gamma_1 \times 10^5$	$\gamma_2 \times 10^5$	d
1·11		+ 8·11	
3·0	− 7·07		15·4
4·52		+ 8·60	
6·50	− 7·08		15·6
8·42		+ 8·44	
10·33	− 6·99		15·5

Mean value for d : 15·5.

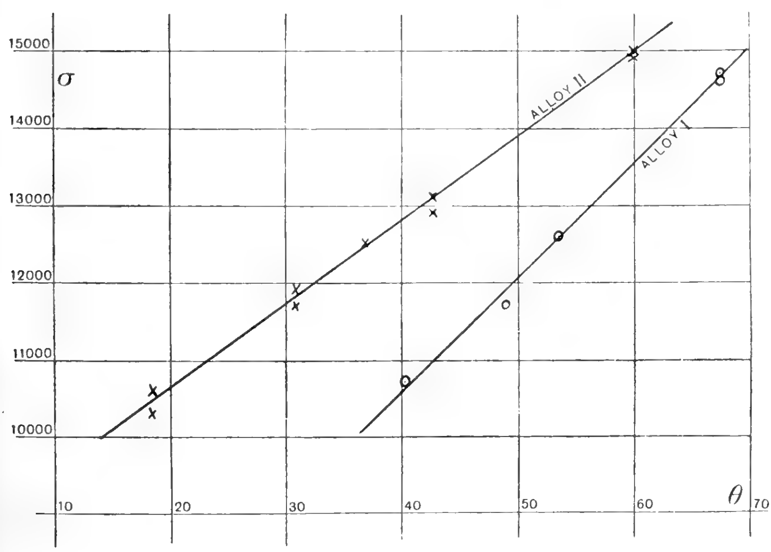


Fig. 2.

Other values found in this way are:

$\sigma_{18^\circ\cdot 6} =$	$-10,300$	ergs per C.G.S.	unit current	per	1° C.
	$-10,600$	,,	,,	,,	,,
$\sigma_{30^\circ\cdot 8} =$	$-11,700$	,,	,,	,,	,,
	$-11,900$	,,	,,	,,	,,
$\sigma_{36^\circ\cdot 9} =$	$-12,500$	,,	,,	,,	,,
$\sigma_{42^\circ\cdot 7} =$	$-13,100$	,,	,,	,,	,,
$\sigma_{60^\circ\cdot 0} =$	$-14,900$	,,	,,	,,	,,
	$-15,000$	,,	,,	,,	,,

These values are of about the same magnitude as those found for alloy No. 1.

They also increase more rapidly than the absolute temperature, a straight line drawn through the points cutting the axis at -80° C. (fig. 2).

Some experiments at present being made with an alloy containing 10 per cent. tin indicate a value for σ somewhat smaller than those obtained for the above alloys. The results of further experiments I hope to publish shortly.

In conclusion I desire to express my best thanks to Prof. J. J. Thomson for his advice and suggestions throughout the course of the work.

A Preliminary Account of an Investigation of the effect of Temperature on the Ionization produced in Gases by the action of Röntgen Rays. By R. K. McClung, Trinity College, '1851 Exhibition' Scholar.

[*Read* 4 May 1903.]

During the course of an investigation of the effect of temperature on the recombination of ions produced by Röntgen rays, the question of how the ionization was affected by the temperature of the gas arose, and it was thought advisable to make a separate investigation of this question. A number of measurements were made on it, using the apparatus which was used in the experiments on the recombination of the ions, but in this apparatus the air which was being experimented upon was free to expand into the outside air when heated. Since the amount of ionization depends upon the density of the gas, it was considered necessary to confirm these results obtained by making some experiments, using an air-tight vessel, so that the gas might be kept at a constant density, In this paper a preliminary account is given of some of these experiments which are still in progress.

Description of Testing Apparatus.

The chief difficulty which arose at the outset of the investigation was that of obtaining a metal vessel, of necessity containing joints, which must be made gas-tight, and which would stand both temperature and pressure. After trying several methods, the apparatus which is shown in detail in fig. 1 was adopted, and proved very satisfactory.

AB is a circular brass cylinder about 7 cms. in diameter and 24 cms. in length. *FF* is a thick brass flange brazed on the end of the cylinder. *P* is an aluminium plate 6 mm. in thickness, which is recessed at the centre to a depth of 5 mm. in order to allow the Röntgen rays to pass into the cylinder without much absorption. This plate is made to fit closely to the flange *FF*. Between the plate and the flange is placed a lead wire, as shown in

the diagram, neatly overlapping at the junction. The plate P is then screwed down as tightly as possible to the flange with heavy

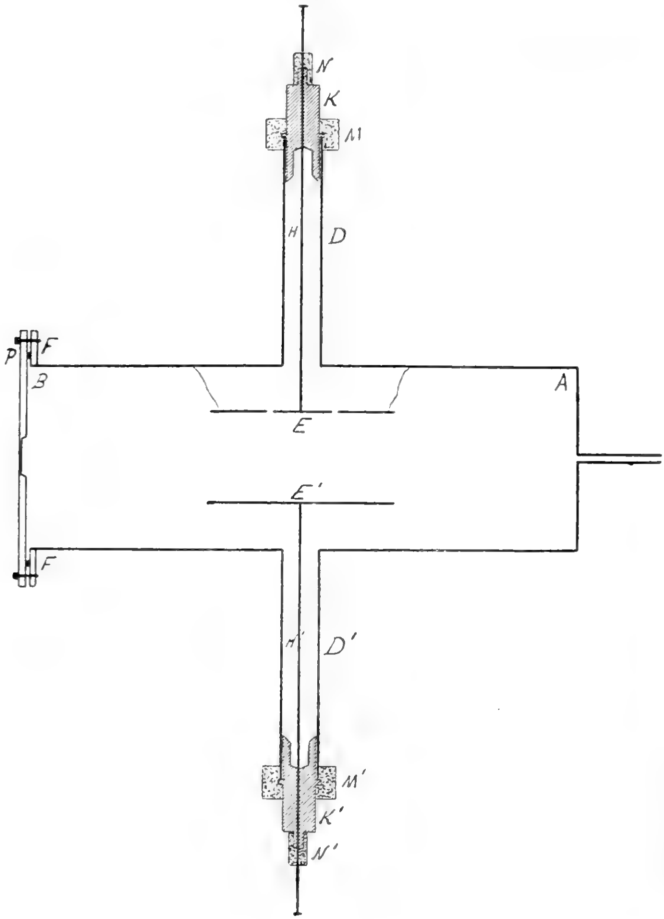


Fig. 1.

screws until the lead is flattened down to about half its thickness. By this means an air-tight junction was obtained which has proved very satisfactory, and which would stand the highest temperature and pressure employed in the experiment. This device was the solution of one of the greatest difficulties in designing the apparatus. The end of the cylinder must be of aluminium, as being the only metal which could be satisfactorily employed as a window to allow the rays to pass into the cylinder, and it was also necessary to have

the plate adjustable so that it might be removed in order to gain access to the interior of the cylinder when desired.

E and *E'* are two parallel brass plates acting as electrodes between which the gas is ionized and between which the rate of leak is measured. *E* is surrounded by a guard-ring connected to earth. These plates are supported by brass rods *H* and *H'*, which pass through heavy ebonite corks fitting tightly into the

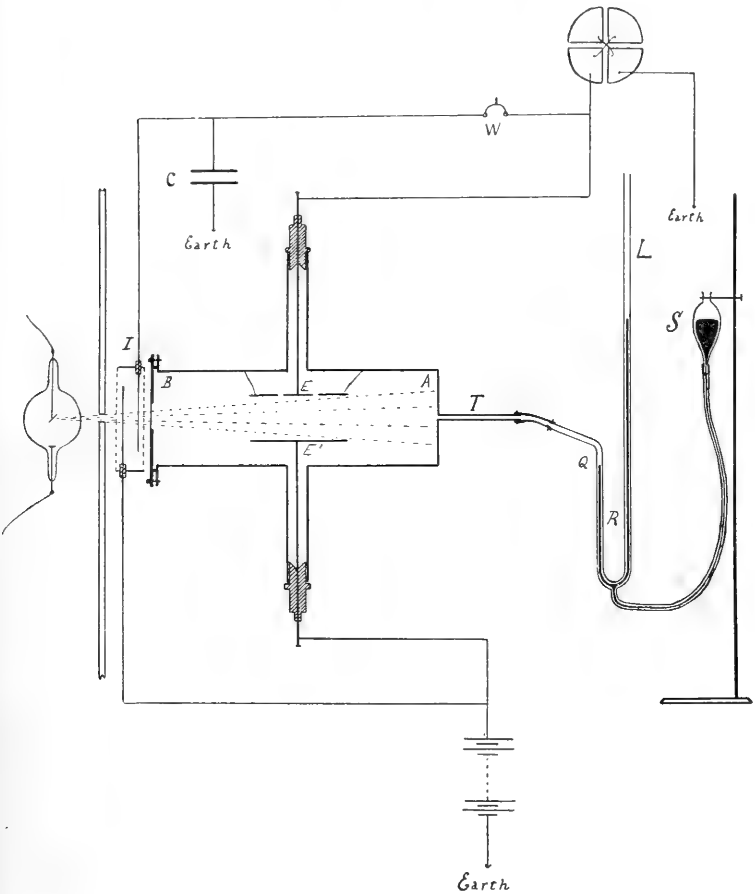


Fig. 2.

brass tubes *D* and *D'*. The joints between the ebonite and the brass are made gas-tight by means of heavy rubber corks bored to fit tightly over the ebonite and the brass and then bound round

tightly with wire, and also by heavy rubber compression tubing N and N' adjusted in a similar manner. These joints have also proved very satisfactory.

General arrangement of Apparatus.

The general arrangement of the whole apparatus is shown in fig. 2. The cylinder AB was connected through the tube T to a mercury pressure-gauge R, and the level of the mercury contained in the arm nearest the cylinder could always be brought back to a fixed point Q by means of a mercury reservoir S whose height could be adjusted. By this means the quantity of gas in the cylinder could be kept at a constant volume and density. When the cylinder was heated the amount of pressure exerted by the gas was measured by the height of the mercury column in the arm L of the pressure-gauge. As this mercury gauge was necessary in any case it was utilized also to determine the temperature of the enclosed gas when heated, by observing the amount of pressure exerted by the gas. This obviated the necessity of introducing an ordinary thermometer into the cylinder and thus the number of joints which had to be made gas-tight was lessened. This method of determining the temperature gave the mean temperature of the gas inside the cylinder.

The cylinder and enclosed gas were heated by means of an electric current passing through a coil of wire wound closely round the outside of the cylinder. The wire was insulated from the cylinder by means of sheet asbestos. Above the layer of wire there was a covering of asbestos and also of felt in order to keep the heat from radiating as much as possible. By this method the cylinder could be heated up to the maximum temperature, employed quite quickly, and the temperature could also be kept quite steady.

The electrode E, as shown in the figure, was connected to one pair of quadrants of an electrometer while the other pair was connected to earth. The electrometer used was one of the Dolezalek type, and gave about two thousand scale divisions for a difference of potential of one volt between the quadrants when the needle was charged to 120 volts. The other electrode, E', was connected to one pole of a battery of accumulators while the other pole was connected to earth.

The Röntgen ray bulb used in the experiments was one of the focus-tube type, with an automatic vacuum regulator attached. The bulb along with the induction coil was enclosed in a lead-covered box, and the rays emerged through a well-defined rectangular orifice, as shown in the diagram. Although it worked very satisfactorily, even with one of these tubes the intensity of

the rays given out is apt to vary slightly during a series of observations extending over several hours, and it was therefore deemed advisable to have some sort of test of the intensity of the rays, apart from the ionization in the cylinder, since the condition of the gas with regard to temperature was to change in the cylinder during the course of the experiment. For this purpose a small test apparatus was introduced in the path of the rays between the bulb and the cylinder, as shown in the diagram. This consisted simply of a short rectangular cylinder made of sheet lead, the ends of which were closed with ordinary white paper. Inside this were placed two insulated parallel plate electrodes, which consisted of exceedingly thin aluminium foil, stretched on metal frames for support. The rays easily passed through the thin aluminium foils and ionized the air between them and the measure of the ionization between the plates was a test of the intensity of the rays from time to time, for any change in intensity would of course be indicated by a corresponding change in the ionization produced. One of these plates was connected to the same pole of the battery as the electrode E' while the other plate was connected, in parallel with a condenser C through the key W, to the same pair of quadrants of the electrometer as the electrode E.

Method of observations.

In measuring the ionization the observations were taken in the following manner. Both pairs of quadrants of the electrometer and the electrode E were connected to earth and also the electrode I was earthed. The Röntgen ray bulb was started and allowed to run for twenty seconds so that the ionization in the cylinder AB might reach a steady state. At the end of the twenty seconds the one pair of quadrants of the electrometer along with the electrode E was insulated. The electrode I was also insulated and disconnected from the electrometer and the electrode E by opening the key W *at the same instant* that E was insulated. Both the key W and the one insulating E were worked at a distance by means of cords. After insulating these electrodes the bulb was allowed to run for five seconds, at the end of which it was shut off. Both the electrodes E and I were thus allowed to charge up for exactly the same time, and the gas in each case was ionized by the same cone of rays and consequently any change in intensity in one case would produce a corresponding change in the other. The air in the standard apparatus was thus always ionized at precisely the same time as the gas in the testing cylinder, and since the ionization in the standard was independent of what was going on in the testing cylinder it formed a check on the intensity of the rays

independent of any variation which might take place in the testing cylinder AB. The charge imparted to E, which is proportional to the amount of ionization produced in the gas between the plates E and E', was indicated by the deflection of the electrometer needle. This reading being taken, the electrometer quadrants were discharged and then the charge contained in I and the condenser C was thrown into the electrometer by insulating the quadrants and closing the key W, and the deflection corresponding to the charge in I was observed. Several of these readings at any given temperature were taken and then the mean taken of each set.

The first sets of observations were taken by measuring the ionization at the ordinary temperature of the room and then heating the cylinder and contained air up to a given steady temperature and again measuring the ionization at that temperature. Then it was again heated to a still higher temperature and the ionization was again measured, and so on to the highest temperature observed. To take measurements at several temperatures over a range of about 200° occupied considerable time, and it was found that by reversing the order and heating to the highest temperature first and then cooling down step by step, the observations could be obtained in much less time and that the experiment could be worked more satisfactorily. It is desirable to take the observations as quickly as possible, for the liability of variations of the X-ray bulb and of other conditions is lessened by shortening the length of time occupied by a series as much as possible. The latter order of taking the readings was therefore adopted in the later experiments.

Results of experiments.

The first test was made of course with air. Several series of readings were made with the cylinder filled with air, and it was found that for a range of temperatures of nearly 200° C. there was no change in the amount of ionization produced in the air when its temperature was altered. Observations were taken at several temperatures from about 14° C. to a little over 200°, and the ionization was found to remain constant over this range of temperature. A sample series of results obtained for air is given in Table I.

Although the above readings are not absolutely constant, still any apparent variation is quite within the limits of experimental error and the agreement is really very good considering the variations which usually take place in the working of a Röntgen ray bulb. Each of the numbers given above represents the mean of several observations. The above results show without doubt that

the temperature of the air does not affect the amount of ionization produced in it if the density of the air does not change. It

TABLE I.—*Air.*

Rate of leak per sec. in scale divisions in cylinder AB	Rate of leak per sec. in scale divisions in standard apparatus	Temperature of air in cylinder AB
83·7	27·0	201° C.
86·5	30·3	152°
82·9	27·6	118°
85·7	31·5	87°
80·3	28·5	45°
83·2	28·5	14°

may be just mentioned here that these results are confirmed by some experiments which have not yet been published, and which were made with a different apparatus which was not air-tight where the air was allowed to expand freely. In that case when the proper correction was made for the change in density of the air the same result was obtained as in the present instance.

Experiments have also been made on carbon dioxide gas, and they give exactly the same results as those obtained for air. One of the series obtained is given in Table II.

TABLE II.—*Carbon Dioxide.*

Rate of leak per sec. in scale divisions in cylinder AB	Rate of leak per sec. in scale divisions in standard apparatus	Temperature of air in cylinder AB
80·1	24·9	222° C.
84·3	25·1	182°
82·2	25·2	146°
79·1	24·6	108°
83·8	26·2	69·6°
87·2	25·3	17·5°

The results show clearly that carbon dioxide acts in exactly the same way as air does in regard to the effect which the temperature

of the gas has on the amount of the ionization produced in it by the action of the Röntgen rays.

This investigation is still in progress, and the statement of a universal law with regard to the effect of temperature on ionization must therefore be reserved until further experiments are made. However, as far as the above experiments go, they show conclusively, within the limits investigated, that the amount of ionization in these gases is independent of the temperature of the gas when the density remains constant.

In conclusion, I wish to express my best thanks to Prof. Thomson for the kindly interest shown and the advice given during the course of this investigation.

A Coleopterous Insect embedded in the wall of the Human Intestine. By D. Sharp.

[*Read* 18 May 1903.]

Though no insects naturally inhabit the human alimentary canal, accounts frequently appear of specimens of this class of animals having been found alive in the situation alluded to. The evidence in such cases is however usually defective. The instance recorded in this paper is of some interest because (1) the specimen was found in its strange situation by a competent observer, and (2) because it was, when extracted, alive, and continued so for 24 hours or more afterwards.

The following abstract from the account of the post-mortem examination has been given me by Mr W. H. Ligertwood, Assistant Medical Officer, Somerset and Bath County Asylum.

B. C., Register No. 4901, was a patient here from June 16, 1900, to April 30, 1903, when he died of an extensive sub-dural hæmorrhage.

On admission he was suffering from melancholia, which gradually passed into dementia.

Age at death 73. Previous occupation, labourer.

His health had been very feeble for a year previous to his death, and he had taken little food but liquid nourishment (milk and eggs) for some months.

At the post-mortem examination the following appearances were found, May 1, 1903.

Intestines normal, with the exception of the following:

In the lower part of the ileum were two small oval, blackish lumps beneath the mucous membrane, the upper about 18 inches above the ileo-caecal valve and the other about six inches lower down. The peritoneal and mucous surfaces over these lumps were quite smooth and showed to the naked eye no appearance of any opening. On being cut into the upper nodule was found to contain a living beetle (since identified as Otiorhynchus tenebricosus). It was between the mucous membrane and the other coats of the intestine, and there were no signs of a cyst-wall.

The other nodule on being opened contained blackish, pultaceous material, with no trace of structure. There was an indefinite cyst-wall.

There were no inflammatory changes in the tissues round either nodule. The two were similar in external appearance and roughly circular, with a diameter of about half an inch. They projected towards the lumen of the intestine.

The living beetle was undoubtedly beneath the mucous membrane and not either (1) free in the lumen, or (2) between the sides of a fold in the wall.

The weevil Otiorhynchus tenebricosus occurs in this neighbourhood, but not very commonly.

A good specimen was found this afternoon in some turf in the garden of one of the out-buildings.

WALTER H. LIGERTWOOD,
Assistant Medical Officer,
Somerset and Bath County Asylum,
Wells, Somerset.

May 14, 1903.

The beetle extracted was at once sent to Mr C. G. Lamb, M.A., for identification, who for greater certainty brought it to me; it proved to be a female of Otiorhynchus tenebricosus, a weevil fully half-an-inch long. This beetle is not carnivorous, but in the larval state feeds on roots, and in the imago condition on the leaves of bushes.

The specimen is in a fairly good state of preservation, except that Mr Ligertwood accidentally severed one of its posterior legs in extracting it from its extraordinary prison.

Dr Thébault recently made experiments showing that the larva of Piophila casei—the common cheese-maggot—can traverse the whole length of the human alimentary canal without being killed*.

It appears therefore that accounts of the finding of living insects in the human alimentary canal must not be rejected on account of the inherent improbability of life being maintained in such a situation.

* *Arch. parasitol.* IV.

On the influence of Electrons on Colloidal solutions. By Mr W. B. HARDY.

[*Read* 18 May 1903.]

Specially purified globulin from blood was dissolved (*a*) in a trace of acetic acid, (*b*) in a trace of sodium hydrate. In presence of acetic acid the globulin was found to move in an electric field from anode to cathode, in presence of alkali it moved from cathode to anode. In the former case therefore the globulin particles carried a positive charge, in the latter a negative charge. These two solutions were exposed to the radiations from radium bromide and it was found that the electro-negative solution of globulin was turned into an opaque jelly in 3 minutes, while the electro-positive solution became more mobile and less opalescent.

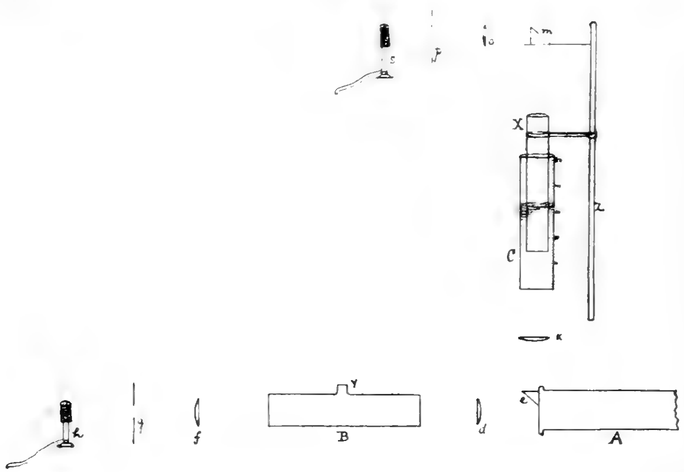
On a method of estimating the amounts of the oxides of Didymium and Erbium by means of the absorption bands of their solutions, and its application to other solutions. By J. E. Purvis, M.A., St John's College.

[*Read* 18 May 1903.]

The following paper describes a method for determining the amount of the oxides of Didymium and Erbium by means of the absorption bands of their solutions, without separating them by chemical methods from other elements with which they may be united.

Apparatus.

The apparatus consists of a spectroscope *A*, which may be a direct-vision one, or one arranged with a camera in order to take photographs of the spectral bands. In front of the slit is a glass tube *B*, of 6 inches length and of 1 inch internal diameter.



The diameter must be equal along the whole length of the cylinder, and the ends closed by flat glass firmly attached by screw joints

or even by Faraday wax. Between the end of the spectroscope and the end of this cylinder is a lens *d* of 3 inches focal length for condensing the light on the slit, and a similar one *f* placed at the further end of the cylinder. There is a screen *g*, having a hole of about $\frac{1}{8}$ of an inch diameter, to allow the light to pass through from a Welsbach incandescent lamp *h* placed a few inches away from it. There is also a tube *C* of exactly the same internal diameter as the tube *B*, and whose lower end is closed by a glass end in a similar way to those fixed at the ends of *B*. This tube *C* is held vertically by means of a clamp and stand *Z*, and it has a scale carefully marked on it or attached to the outside. In the tube *C* there is another tube *X* of about $\frac{3}{4}$ of an inch diameter, the upper end of which is open, and the lower end is also closed by plain glass. The tube *X* is so fixed to the stand that it can be easily raised or lowered either by a clamp or by a rack and pinion. There are lenses at *k* and *o* for condensing the ray of light in its passage through the circular hole in the screen *p*, and its reflection by means of the right-angled prism *m*. *s* is a Welsbach incandescent light similar to that at *h*; and at *e* there is a right-angled prism to reflect the ray of light into the spectroscope issuing from the solution in *C*. The incandescent lights at *s* and *h* are so arranged that their spectra are equally bright as seen through the spectroscope when the tubes *B* and *C* are removed.

Method of experiment.

The method is to have a definite volume of a definite strength of a pure salt of either of the earths of Didymia or Erbia placed in the perpendicular tube *C*. The chlorides are preferred because there is no change in the bands when their strong solutions are diluted. As the amount of the oxide contained in 1 c.c. of the solution is therefore known, the scale attached to *C* enables one to read off directly the amount dissolved in any fractional part of the volume of this solution.

The solution to be examined, or an aliquot portion of it, is placed in the tube *B*, whose volume is also exactly known, and so that its level at the lower end of the side tube *Y* is parallel with the inside wall of the cylinder. There will be a meniscus, of course, but it is so small that its effect may be neglected. The parallel spectra are then examined through the eye-piece of the spectroscope and the cylinder *C* is moved upwards or downwards until the intensities of the absorption bands of the two solutions are identical. As the intensities and widths of the bands depend upon the amount of the oxides dissolved, it follows that, if the bands in the solution in *C* measured from the bottom of the

movable cylinder X to the internal lower end of the cylinder C are exactly the same as the bands observed in the solution in B, the concentrations of the solutions must be inversely as the thickness producing the absorptions, and that between these limits there must be the same amount of the oxide dissolved in the one as in the other. Further, as the amount of the oxide dissolved in any aliquot portion of the solution in C is known, it is easy to read off by the scale the amount of the oxide dissolved in the solution in B.

It is desirable not to have the solutions too highly concentrated, because the variations in the intensities and widths of the various bands are not so easily distinguished when the solutions are too strong. It was found to be convenient to observe, in the Didymium solutions, the three bands in the blue part of the spectrum at about $\lambda 483$, $\lambda 476$ and $\lambda 469$; also the bands in the green at about $\lambda 519$ and $\lambda 522$; and particularly the bands in the yellow at about $\lambda 570$—$\lambda 577$. When the solutions are made of such strengths that these bands are distinctly and separately seen, the experimental results were found to be very exact; whereas when the solutions are very strong the bands in the green and yellow parts of the spectrum close up and appear very broad and black, causing the delicacy of the comparison to be very much lessened. In the solutions of Erbium, the bands at $\lambda 536$, $\lambda 541$, $\lambda 523$ and $\lambda 488$ were found the best to observe and compare and for the same reasons.

It is possible that, if photographs were taken, the adjustment of the movable cylinder X could be made much more accurately, and therefore an increased accuracy of the results obtained.

It is obvious that if both oxides of Didymium and Erbium are in solution in B, it is only necessary to place in the side tube C definite volumes of definite strengths of the solutions of each of these earths, and compare the bands which are recognized to belong to either Didymium or Erbium.

The method can, of course, be used for any solutions which give absorption spectra, and which are not altered on dilution.

Some results obtained by the method.

The following tables give some of the results obtained by this method. The tube B had an internal volume of 31·5 c.c., and was filled to the lower level of the side tube Y with the solution to be investigated. A definite volume of a definite strength of the solution of the salt was placed in the tube C, and the movable cylinder X adjusted until the bands of the two parallel spectra were identical. The scale on the tube C was read, and the

solution contained in B removed; the oxide was precipitated as oxalate, filtered, and ignited to obtain the oxide and weighed.

Didymium oxide. Di_2O_3.

Calculated directly by reading the scale on tube C	Found by precipitating the solution in B as oxalate, and obtaining the oxide
grams	grams
·23157	·2318
·12865	·1268
·10800	·1114
·11210	·1148
2·35000	2·3481
1·46500	1·3800
·13900	·1459
·09490	·0939
·35510	·4144
·73000	·7652

Erbium oxide. Er_2O_3.

2·5280	2·6117
1·4616	1·4433
·7508	·7373
·3818	·3765
·8690	·8788
·2896	·2856
·5800	·6030
·8960	·9283
·4937	·4975
·2237	·2115
·2633	·2715

It will be seen from these tables that, as before stated, the most accurate results are obtained when the solutions are not too highly concentrated. Therefore, if the solutions in the first instance have their absorption bands wide and intense, they should be diluted to a definite volume, and to such an extent that the separate bands above mentioned are clear and well defined.

Finally, I desire to thank Professor Liveing for his kindness now, as always, freely given.

On the influence of great dilution on the absorption spectra of highly concentrated solutions of the nitrates and chlorides of Didymium and Erbium. By J. E. PURVIS, M.A., St John's College.

[*Read* 18 May 1903.]

In a paper read by Professor Liveing before the Society and published in the *Transactions,* Vol. XVIII. p. 298 et seq., the "Effects of dilution, temperature, and other circumstances on the absorption spectra of solutions of Didymium and Erbium salts" were described and discussed. The present paper is to communicate the results of observations, completed some time ago, when very highly diluted solutions were experimented with.

Apparatus.

The observations were made by the photographic plate, and as the part of the spectrum studied was not in the ultra-violet, no special lenses or prisms of calcite or quartz were used. The spectroscope had a flint-glass prism of 60°, and fixed on the stage at the minimum angle of deviation for the sodium yellow line. The image was focussed on the photographic plate by an achromatic lens, and its length was about 3 inches. The glass tubes which held the solutions were 610 mm., 1224 mm., and 3599 mm. long respectively, and each of 16 mm. internal diameter. The first and second were fitted at each end with glass plates in the manner described in Professor Liveing's paper. Each of these tubes had a short upper branch placed about the centre of its length through which it was filled. The long tube of 3599 mm. length was provided with screw caps by which the glass ends could be firmly held between rubber washers against the two ends of the tube. Duplicate tubes of exactly the same length and shape were used in the case of the two tubes of 610 mm. and 1224 mm. length. In all these tubes the circular openings at the ends had exactly the same diameter. Heavy wooden blocks were used as supports for the tubes, and they were rigidly held in position on the bench by means of right-angled stays which firmly gripped the lower portion of each block and were glued to the bench. Along the upper surface of each block a V-shaped groove was cut in which the tubes rested, and so that they could be easily and exactly replaced in their former position

The plates are reproductions of some of the original photographs of the spectra described in the paper. It has been found quite impossible to reproduce them so as to give all the details of the original negatives, and, consequently, much of the sharper and finer differences do not appear.

The figures at the top of each plate give the wave lengths of the spectral bands photographed; the red end of the spectrum being on the left.

Plate I, No. 1. Absorption spectra of equivalent strengths of solutions of the nitrate and chloride of Didymium. The strongest solutions in a cell of 5 mm. thickness, and the diluted solution in a tube of 3599 mm. length; consequently the ratio of the dilution is 1 : 719·8.

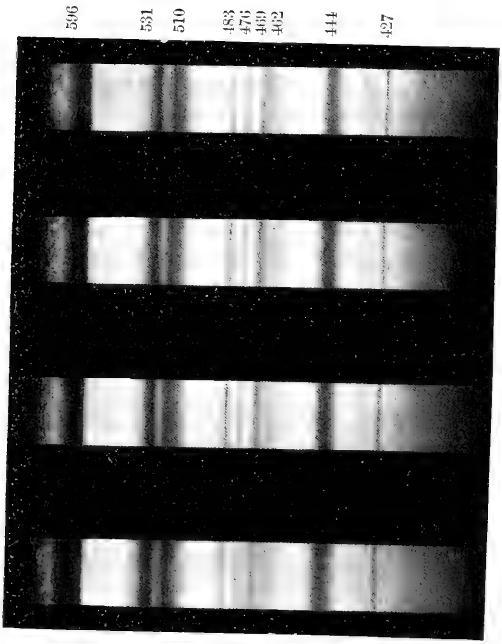
The chief differences in the strong solution of Didymium nitrate in the 5 mm. cell from the other three photographs are fairly well seen.

Plate I, No. 2. Absorption spectra of equivalent strengths of solutions of the nitrate and chloride of Didymium. The strongest solutions in a cell of 5 mm. thickness, and the diluted solution in a tube of 1224 mm. length; consequently the ratio of the dilution is 1 : 244·8.

The chief differences in the strong solution of Didymium nitrate can also be seen here, but they are not so well marked as in Plate I.

The photographs taken in tubes of 612 mm. length, and the cell of 5 mm. width, i.e. using solutions whose ratio was 1 : 122·4, shew similar differences.

PLATE I, No. 1.



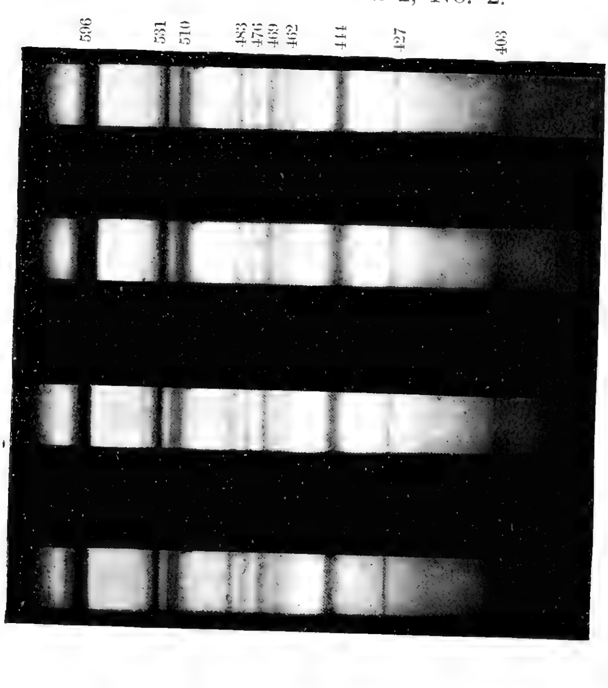
Diluted solution of Didymium chloride in 3599 mm. tube.

Diluted solution of Didymium nitrate in 3599 mm. tube.

Strong solution of Didymium chloride in 5 mm. cell.

Strong solution of Didymium nitrate in 5 mm. cell.

PLATE I, No. 2.



Diluted solution of Didymium nitrate in 1224 mm. tube.

Strong solution of Didymium nitrate in 5 mm. cell.

Diluted solution of Didymium chloride in 1224 mm. tube.

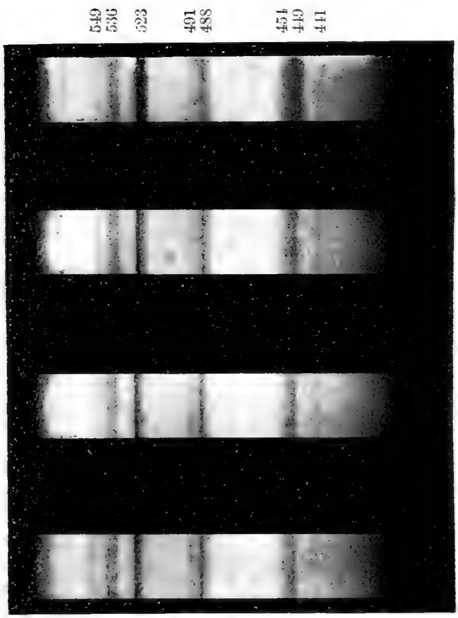
Strong solution of Didymium chloride in 5 mm. cell.

Plate II, No. 1. Absorption spectra of equivalent strengths of solutions of the nitrate and chloride of Erbium. The ratio of the dilutions is 1 : 719·8. Just as in the salts of Didymium in Plate I, No. 1, also here the chief differences are seen in the strong solution of the nitrate of Erbium.

Plate II, No. 2. Absorption spectra of equivalent strengths of solutions of the nitrate and chloride of Erbium; and the ratio of the dilutions is 1 : 244·8. The differences in the strong solution of the nitrate from the other solutions can be easily seen.

The photographs taken in tubes of 612 mm. length and the cell of 5 mm. width, i.e. the solutions being in the ratio of 1 : 122·4, shew similar differences.

Plate II, No. 1.



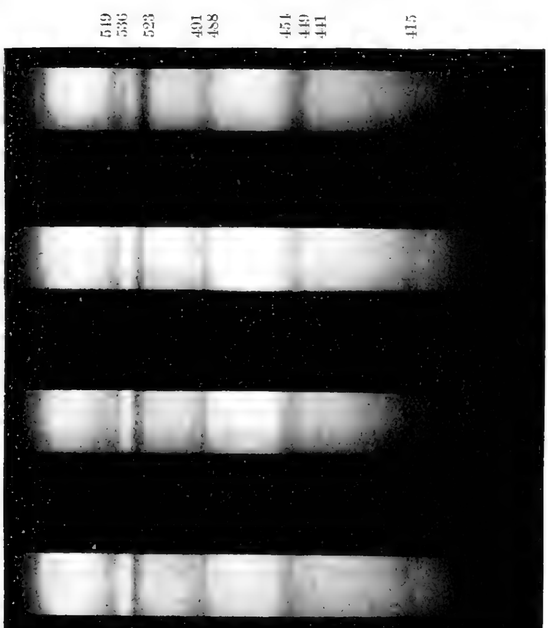
Strong solution of Erbium nitrate in 5 mm. cell.

Strong solution of Erbium chloride in 5 mm. cell.

Diluted solution of Erbium nitrate in 3599 mm. tube.

Diluted solution of Erbium chloride in 3599 mm. tube.

Plate II, No. 2.



Strong solution of Erbium nitrate in 5 mm. cell.

Diluted solution of Erbium nitrate in 1244 mm. tube.

Strong solution of Erbium chloride in 5 mm. cell.

Diluted solution of Erbium chloride in 1224 mm. tube.

whenever they were removed for any purpose. The strongest solution was contained in a thin glass cell of exactly 5 mm. internal width. The issuing light was concentrated by a lens of 3 in. focal length, which was fixed at that distance from the slit, whilst at the other end of the tube a lens of similar focal length was fixed, and beyond this again there was a fixed screen with a hole of about $\frac{1}{8}$ of an inch diameter, and still further beyond, was the source of light. The latter was a Welsbach incandescent light with no chimney, and it was placed several inches from the screen in order that the mantle of the burner should not be focussed in the slit. The photographs were taken on spectrum plates made by Messrs Neal and Cadett.

Method of experiment.

The solutions used were the nitrates and chlorides of Didymium and of Erbium. These two earths had been carefully separated from Gadolinite and Euxenite by a long and laborious series of operations, and the only impurity is a trace of Yttria. Weighed quantities of the oxides, Di_2O_3 and Er_2O_3, were dissolved in the two acids, and the solutions evaporated to drive away excess of the free acid. Each salt was then dissolved in a measured quantity of water. The most concentrated solutions of Didymium contained 606·3 grams of the anhydrous nitrate dissolved in a litre, and 459·7 grams of the anhydrous chloride in one litre. These numbers represent about 1·834 gram-molecules per litre. In the Erbium solutions the most concentrated solutions were 785·6 grams of the anhydrous nitrate to the litre, and 609·6 grams of the anhydrous chloride to the litre. These numbers represent about 2·213 gram-molecules per litre. The dilute solutions were made from these by taking measured quantities and adding water to the required amounts. As the width of the cell containing the strongest solutions was 5 mm. and as the lengths of the three tubes were 610 mm., 1224 mm., and 3599 mm. respectively, one c.c. of the strongest solutions was diluted 122, 245, and 719·8 times respectively.

The method of experiment was to fill one of the tubes with distilled water which had previously been boiled and filtered, and then to place it in position in the V-grooved blocks fixed in front of the slit of the spectroscope. The strongest solution in the 5 mm. cell was then firmly held by a clamp and stand against the end of the tube nearest the slit end of the spectroscope, and an exposure to the photographic plate of 10 or 15 minutes allowed. When this was completed, the tube and cell were replaced by one of the duplicate tubes of equal length containing the dilute solution, and exposed to the photographic plate for exactly

the same time. In the case of the tube of 3599 mm. length there was no duplicate tube used; but by means of the screw cap it was found quite easy to fill the tube first with distilled water and place the 5 mm. cell and its contents against one end as described above. After a photograph had been taken, the water was run out of the tube, and the latter was allowed to drain and dry, and then filled with the diluted solution of the salt, the screw cap being carefully replaced in its previous position.

It was found very essential to have the solutions absolutely free from dust and gas bubbles. Again and again the results were vitiated by these impediments; for, although the solutions appeared to the naked eye to be translucent and quite free from dust and bubbles, the sensitive plate readily detected the loss in the intensity of the light traversing them. These difficulties were considerably lessened by regularly filtering the solutions after several operations.

With regard to the tubes of 610 mm. and 1224 mm. length another arrangement was employed. It consisted in placing the tube containing the diluted solutions at right angles to that in front of the spectroscope, and so that the ray of light issuing from it was reflected into the slit of the spectroscope by a right-angled prism. The two sources of light were so arranged by the eye and photographically that the intensities of the light entering the spectroscope was exactly the same. By this arrangement the absorption spectra of both the concentrated and the diluted solutions were photographed at the same time.

The absorption bands photographed were, in the case of the Didymium solutions:—

1. A weak band at about λ 496.
2. A strong group of bands in the yellow at about λ590—λ570.
3. A weak band at λ531.
4. A strong group in the green at about λ528—520.
5. A less strong group of two diffuse bands in the green whose centre is about λ510.
6. Three well-marked bands in the blue of about λ483, λ476, and λ469.
7. A broad weak band whose centre is about λ462.
8. A broad band whose centre is about λ444.
9. A weak band at about λ433.
10. A narrow well-defined band at about λ427.
11. A weak band at about λ418.
12. A weak band at about λ415.
13. A weak band at about λ406.
14. A strong broad band at about λ403.

Beyond this, the light was absorbed and the photographic

plate was not affected to any sensible extent. It is interesting to note that by using a Rowland grating and a Nernst lamp as a source of light, the dispersion of the light in the third order of the spectrum showed that the narrow well-defined band at λ427 has another band adjoining it in the ultra-violet side of the spectrum of not quite the same width nor the same sharpness. It may be possible by using such very high dispersions to split up other bands.

With regard to the absorption bands of the solutions of Erbium, the following were noticed :—

1. A group of bands at about λ549 and λ536.
2. A very weak band at about λ527.
3. A very strong band at about λ523.
4. A weak one at about λ520.
5. A broad band at about λ491.
6. A narrow strong band at about λ488.
7. A weak narrow one at about λ486.
8. Two very weak and broad ones at about λ472 and λ467.
9. A broad band at about λ454.
10. A narrow sharp band at about λ449.
11. A weak band at about λ441.
12. A narrow one at about λ422.
13. A weak band at about λ418.
14. A broad band at about λ415.
15. Two almost equally strong bands at about λ404 and λ407.

A little beyond this there is no effect upon the photographic plate because the light beyond is absorbed.

Discussion of results.

The reproductions of the photographs in Plates I and II will show the differences and agreements perhaps better than any description, but it may be advisable to draw attention to several points in them.

Comparing the strong and dilute solutions of Didymium nitrate it is noticed that the broad band in the yellow from about λ590 to λ570 is much broader in the stronger solution. The same thing is to be observed in the band λ528—λ520. The group of two bands also in the green at about λ520 is much broader and the two are really fused into one large band. The bands in the blue at λ483 and λ469 are also slightly more diffuse but not so markedly as the middle of the triplet λ476, which is very faint and broad in the strong solution. The broad faint band at λ462 seems to be much broader, as is also the case with the broad well-defined band at λ444. The narrow faint band at λ433 almost disappears in the strong solution, whilst the well-defined narrow band at λ427 seems to be more diffuse. The

bands at λ418, λ415, and λ406 are faintly visible in both solutions and slightly wider in the strong solution, whilst the broad band at λ403 seems to be wider also.

The general effect appears to be that all the bands observed are more diffuse in the concentrated solution of Didymium nitrate than in the diluted solutions of equivalent thickness. On comparing the strong solution of Didymium chloride with that of the nitrate containing equivalent quantities of the salt it is noticed that the absorption bands resemble those of the *dilute* solution of the nitrate. There is no apparent difference between the strong solution of Didymium chloride and the diluted solution of Didymium nitrate. The only point of difference seems to be that there is an extension of the general absorption of the more refrangible end of the spectrum in the concentrated solution of Didymium chloride, and this is comparable with that in the strong solution of Didymium nitrate. If the concentrated and the diluted solutions of Didymium chloride are compared it will be noticed that there is no apparent difference in the various absorption bands, and the only difference is again that of a more general absorption of the more refrangible end by the concentrated solution. That is to say, in these experiments the concentrated solution of Didymium chloride, the dilute solution of Didymium chloride, and the dilute solution of Didymium nitrate are similar as regards their absorption spectra, and the only apparent difference is that mentioned above.

If the concentrated and diluted solutions of the nitrate of Erbium are similarly compared, it is noticed that there is the same general differences as in those of the Didymium nitrate. The bands in the concentrated solutions are much more diffuse than in the diluted solutions, whilst some of the very faint bands disappear and the more sharply defined bands appear wider. The band at about λ539 almost disappears. The two bands at about λ536 of almost equal intensity appear to be more diffuse, and the group of λ523 and λ527 merge into each other to form a single broad band. The groups λ491, λ488, and λ480 are more diffuse, and the bands λ449 and λ454 are almost completely diffused into each other, as is also the case with λ404 and λ407. The bands at λ418 and λ422 disappear completely in the concentrated solution, whilst the band at λ441 does not seem to suffer so marked a change. Again here, as in the case of the salts of Didymium, there is an extension of the general absorption of the more refrangible end in the concentrated solution.

Also, if the concentrated and diluted solutions of the chloride of Erbium are compared it is seen that there is no apparent difference in the various bands, and that these strictly correspond to those of the diluted solution of the nitrate of Erbium; the only marked difference being the greater extension of the general

absorption of the more refrangible rays by the concentrated solution. Again, just as in the case of the salts of Didymium, the concentrated solution of Erbium chloride, the dilute solution of Erbium chloride, and the dilute solution of Erbium nitrate are alike in the characters of their absorption bands, and the only apparent difference is that mentioned above.

With regard to the differences between the concentrated and diluted solutions of the nitrates of Didymium and Erbium it may be noticed that the photographs do not seem to show that each band is equally affected in diffuseness and intensity. Perhaps this is the most noticeable in the concentrated solutions of Didymium nitrate, where the band in the blue at $\lambda 476$ is so very diffuse that it is only just visible, and appears very well marked and strong in the diluted solutions, whilst the adjoining bands $\lambda 483$ and $\lambda 469$ do not seem to be affected to the same extent. Also the intensity of the band at about $\lambda 531$ is not so highly changed on dilution; the most apparent change being a shift towards the red end of the spectrum. The same remark applies to the faint band at about $\lambda 595$.

Also in the solutions of Erbium nitrate the two bands at about $\lambda 536$ and $\lambda 541$ do not seem to be affected on dilution to any great extent, and this applies to the faint band at about $\lambda 441$.

These experiments, therefore, seem to prove

(1) that the absorption bands of highly concentrated solutions of the chlorides of Didymium and Erbium are not altered when these solutions are highly diluted.

(2) that the absorption bands of highly concentrated solutions of the nitrates of Didymium and Erbium are considerably less diffuse when these solutions are highly diluted. It is noteworthy that this is an effect analogous to that produced in the spectra of some gases and vapours by diminishing the density of the gas or vapour.

(3) that the absorption bands of concentrated and diluted solutions of the chlorides of Didymium and Erbium are precisely similar to those observed in diluted solutions of the nitrates of these earths.

It is an obvious speculation to suggest that the various elements which compose the molecules of Didymium and Erbium form different complex groupings with nitric acid than with hydrochloric acid, and that this complexity is broken down when the concentrated solutions are diluted. The fact that these changes are gradual on gradual dilution may lend support to this view. It may also be mentioned that Praseodym and Neodym, two of the constituents of Didymium, are separated by repeated fusion and rapid cooling of the nitrates; whilst there is no such separation when the chlorides are similarly experimented with.

A Lecture Experiment to illustrate the effect of a straight current on a magnetic pole. By P. V. BEVAN, M.A., Trinity College.

[*Read* 18 May 1903.]

A simple form of apparatus is represented in fig. 1 for showing the continual rotation of a single magnetic pole round a straight wire carrying a current. The funnel A is attached to a tap leading to a tube B, which is drawn out at its lower end so that its bore is about ·5 mm. At about 1 cm. below the tube B is fixed a second tube C of diameter at the top about ·5 cm. and

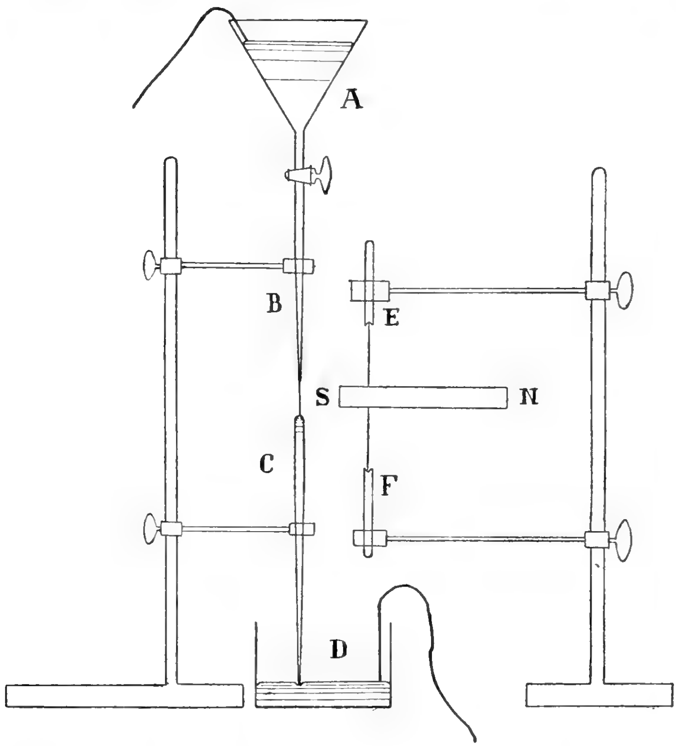


Fig. 1.

drawn out at the lower end so that its diameter is slightly less than that of the lower end of B. The lower end of C dips into mercury in a basin D. Leads from a battery dip into the mercury in the funnel A and the basin D. When the tap at the bottom of

the funnel is open, mercury having first been poured into the funnel, the mercury flows through B and soon fills C, running over its edge. Between B and C the jet of mercury in the air forms a continuous metallic connection so that we have a straight current flowing between A and D.

The magnet NS is formed of a thin strip of steel with two needles soldered at a point nearer one end than the other. The magnet is supported by the two needles resting in sockets E and F, so that the magnet can only turn about a vertical axis. These sockets can be easily made from pieces of glass rod. The sockets are held on a stand which can be moved independently of the stand carrying the glass tubes and mercury. The magnet is then adjusted so that the distance of the axis of support from the mercury jet is greater than its distance from one end of the magnet and less than its distance from the other. The height of the magnet is adjusted so that the longer arm of the magnet can pass through the gap between the tubes B and C, while the shorter arm never reaches as far as this gap. When the mercury flows and the tube C is filled, the current passes along the straight conductor BC. If now the magnet NS revolves once completely the North pole passes round the current while the South pole describes a circle not enclosing the current. The magnet can pass through the conductor, breaking the current for a very short time by interrupting the flow of mercury. If set revolving, the magnet will therefore continue to revolve, the energy required being obtained from the work done on the North pole.

With the magnet arranged as in this experiment there is no position in which it is in equilibrium, so that it will begin to rotate about the vertical axis whatever its position may be when the current begins to flow.

In fig. 2, let O be the section of the current which is supposed to be perpendicular to the plane of the paper, NS the magnet, A the axis on which the magnet revolves.

If i be the current, μ the pole-strength of the magnet, the force on the North pole is

$$\frac{2i\mu}{ON},$$

along NP where NP is perpendicular to ON.

Let AP be perpendicular to NP, then the moment of the force on this pole round A is

$$-2i\mu\frac{AP}{ON}.$$

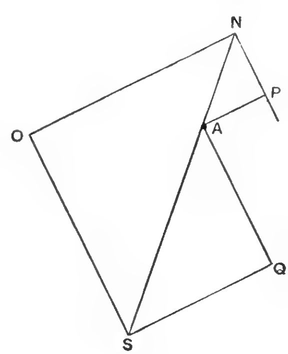


Fig. 2.

The moment of the force on the South pole is similarly

$$-2i\mu\frac{AQ}{OS}.$$

For equilibrium $$\frac{AP}{ON}=\frac{AQ}{OS},$$

so that NP and SQ meet on OA.

The centre of the circle ONS is therefore on the line OA, and for this to be possible OA must be greater or less than both AN and AS. There is therefore no equilibrium position with the magnet arranged as in the experiment.

Irreversible simultaneous linear reactions. By H. O. JONES, M.A., Clare College, and O. W. RICHARDSON, B.A., Trinity College.

[*Read* 18 May 1903.]

The authors have investigated the peculiar decomposition of the *phenylhydrazone of oxalacetic acid* when heated in aqueous and acid solutions, which was first observed by Fenton and Jones (*Trans. Chem. Soc.* 1901, LXXIX. 91). A complete account of the experiments has been published in the Journal of the Chemical Society (*Trans.* 1902, LXXXI. 1140). It was shown that the results obtained from a study of the rate of evolution of carbon-dioxide from solutions of the hydrazone in sulphuric acid of varying concentration at a temperature of about 80° C., and also of the amount of carbon-dioxide evolved from the same solutions at 100° C., could be explained by supposing that the hydrazone itself, when heated, lost carbon-dioxide and gave pyuric acid phenylhydrazone, the reaction being monomolecular, but that in presence of hydrogen ions the products of decomposition were pyrazolon carboxylic acid and water. In the latter case the reaction was bimolecular, the velocity of formation of the pyrazolon carboxylic acid being jointly proportional to the concentration of the oxalacetic acid phenylhydrazone and the concentration of the hydrogen ions in solution.

This reaction is an example of what may be called a *simultaneous linear reaction,* such a reaction involves the simultaneous production from one substance, of at least two others, whilst the rate of formation of each of these is directly proportional to the amount of the original substance present. In the case under consideration the reactions are irreversible and under these conditions the theory is considerably simplified.

The theory for the case when only two irreversible reactions occur simultaneously has been worked out at some length in the paper quoted above (*loc. cit.*); the theory for cases where any number of irreversible reactions take place simultaneously, however, presents no further difficulty. The authors anticipate that several other substances will be found to undergo this type of reaction, under certain conditions, and have therefore developed the theory for the general case, which is here presented.

Let us suppose that a substance, whose initial concentration* is C_0, is subject to any number m of irreversible simultaneous linear decompositions, *i.e.* decompositions in which the rate of formation of any product is proportional to the concentration of the original substance at the time considered. Let C_n be the concentration of the nth product at the time t, then the concentration of the original substance at the same time is $C_0 - \Sigma C_n$, Σ denoting summation over all the m reaction products. The rate of increase of C_n is thus

$$\frac{dC_n}{dt} = k_n (C_0 - \Sigma C_n),$$

where k_n is the velocity constant of the reaction which gives rise to the nth product. Since each of the other reactions satisfies a similar equation, we have

$$\frac{1}{k_n}\frac{dC_n}{dt} = \frac{1}{k_{n-1}}\frac{dC_{n-1}}{dt} = \ldots = C_0 - \Sigma C_n,$$

whence
$$\frac{C_n}{k_n} = \frac{C_{n-1}}{k_{n-1}} = \ldots = \frac{\Sigma C_n}{\Sigma k_n},$$

the integration constant vanishing, for

$$C_n = C_{n-1} = \ldots = C_1 = 0 \text{ when } t = 0,$$

since none of these products have been formed at the beginning of the reaction.

From the above we obtain

$$\Sigma C_n = \frac{C_n}{k_n} \Sigma k_n ;$$

whence
$$\frac{dC_n}{dt} = k_n C_0 - C_n \Sigma k_n ;$$

therefore
$$\log (k_n C_0 - C_n \Sigma k_n) = -\Sigma k_n t + A.$$

As before, when $t = 0$, $C_n = 0$, so that the integration constant $A = \log k_n C_0$, and

$$\log \left(1 - \frac{\Sigma k_n}{k_n} \frac{C_n}{C_0}\right) = -\Sigma k_n t \quad \text{.................(1)},$$

or
$$C_n = \frac{k_n C_0}{\Sigma k_n}(1 - e^{-\Sigma k_n t}) \text{.......................(2)},$$

with m similar equations for all the m products. This expression gives the concentration C_n of the nth product, at a time t, in terms of the initial concentration of the original substance and of the

* The concentrations are expressed in gram-molecules per litre.

velocity constants of the reactions involved. It will be seen that equation (2) is of the form $y = a(1 - e^{-bx})$, so that the amount of each product which has been formed varies with the time in the same manner as we found experimentally for the rate of evolution of carbon-dioxide from solutions containing oxalacetic acid phenylhydrazone.

By putting $t = \infty$ we find the final amount of C_n produced is

$$C_n^\infty = \frac{k_n C_0}{\Sigma k_n} \quad \text{.............................(3)};$$

it is thus equal to the initial concentration of the original substance multiplied by the ratio of the particular velocity constant considered to the sum of all the velocity constants.

We see therefore that by determining the final concentrations of the various products we can determine the values of the m fractions $\frac{k_n}{\Sigma k_n}$. If we substitute the values thus found in the equation (1), we obtain the absolute value of Σk_n in terms of the ratio of the concentration of the nth product at a time t, to the initial concentration of the original substance, and of t; so that we can obtain the absolute value of Σk_n from a series of observations of the variation of the amount of any one of the substances formed with the time. In this manner all the m quantities k_n may be completely determined. This method has been applied to the case of the phenylhydrazone of oxalacetic acid (*loc. cit.*, page 1157).

Preliminary experiments show that the decomposition of oxalacetic acid itself in aqueous solution is more complicated than that of its hydrazone; products other than carbon-dioxide and pyruvic acid are produced in increasing quantity as the concentration of the acid solution is diminished. The authors are attempting to apply this method to the investigation of this reaction.

The Estimation of Hydroxylamine (Preliminary note). By H. O. JONES, M.A., Clare College, and F. W. CARPENTER, Clare College.

[*Received* 16 June 1903.]

The authors found that the methods hitherto described for the estimation of hydroxylamine in solution were very unreliable, especially in presence of impurities such as metallic salts and carbon dioxide.

An attempt was then made to see if a method could be elaborated which was free from these objections, and it was found that hydroxylamine could be estimated with ease and accuracy by the method described by Wood and Berry (*Proc. Camb. Phil. Soc.* XII. ii. 97) for the estimation of reducing sugars.

Ten to twenty cubic centimetres of a solution of hydroxylamine and one of its salts (containing not more than 0·5 % NH_2OH) are run from a pipette into 30 c.c. of a boiling solution of copper in potassium bicarbonate (Soldani) or Fehling's solution, with stirring, the whole raised to the boiling-point, and the cuprous oxide immediately filtered off, washed, dissolved in some ferric sulphate solution in an atmosphere of carbon dioxide and the ferrous salt produced titrated with potassium permanganate.

In dilute solution (·5 %) the reaction between hydroxylamine and copper oxide may be expressed by the equation

$$2NH_2OH + 4CuO = N_2O + 2Cu_2O + 3H_2O,$$

so that in the titration with permanganate 4 gram molecules of permanganate correspond to 10 gram molecules of hydroxylamine, or 632 grams to 330 grams.

The effect of other substances, which themselves have no action on the copper solution, is negligible, *e.g.* that of salts, alcohol and ketoximes was tried.

On the differential equations of the hyperelliptic functions. By Dr H. F. BAKER, St John's College.

[*Received* 29 July 1903.]

The following note relates to some differential equations originally published in these *Proceedings* in the Autumn of 1898 (Vol. IX. Part ix. p. 513), of which a proof has recently been published in t. XXVII. of the *Acta Mathematica.* It deals with three points, (I) a group of differential operators which the differential equations allow, corresponding to the linear group by which the variable of the hyperelliptic integral may be transformed, (II) for $p=1$ or $p=2$, a set of modular relations which is deducible from the differential equations, which conversely would furnish a proof of the differential equations, (III) with the direct algebraical integration in the manner explained in the previous note in these *Proceedings* of the differential equations for the case $p=1$, and the degenerate cases for $p=2$, $p=3$; the general case $p=2$ was outlined in the previous note. The degenerate forms obtained correspond with those used for two variables by M. Painlevé in his recent paper in the *Acta Mathematica*, t. XXVII., on functions possessing an algebraic addition theorem; the present point of view emphasizes the existence of and gives the algebraic definition of the integral function from which the meromorphic functions are derivable. Comparing M. Painlevé's results in regard to the new transcendents found by him satisfying ordinary differential equations of the second order (*Acta Math.* XXV.) one is tempted to ponder on the existence of a theory including these transcendents and the hyperelliptic functions of one or two variables, to be found by a generalisation, perhaps only slight, in the forms of the differential equations satisfied by the latter. Such a generalisation, if not found by accident, requires a method of integrating the hyperelliptic equations founded on the general properties of functions, wholly different from the computative method followed here.

But such computations would appear to be often useful, for several reasons, and experience suggests that they are indispensable while the general theory is still incomplete. Nor probably need an apology be offered to English readers for adding as an

addendum a sketch of a proof of the theorem as to functions of one variable possessing an algebraical addition theorem. This theorem is that an algebraic simply transitive group of commutative transformations is necessarily expressible by Abelian functions; the group is therefore associable with an integral function of the canonical variables, this integral function satisfying in the case of one and two variables the differential equations now under discussion. The question suggests itself whether every simply transitive algebraic group is similarly derivable from a single function of the canonical variables of the group.

I.

The variables $u_1, u_2, \ldots$ being, as originally defined from the hyperelliptic construct $y^2 = f(x)$, expressible by sums of p integrals respectively of the forms

$$\int \frac{dx}{\sqrt{f(x)}}, \quad \int \frac{x\,dx}{\sqrt{f(x)}}, \ldots,$$

it is manifest that a linear transformation for x, say

$$x = (az + b)/(cz + d),$$

enables us to write $u_1, u_2, \ldots$ as linear functions with constant coefficients of variables $v_1, v_2, \ldots$ which have precisely similar expressions in terms of integrals in which x is replaced by z and the coefficients in the polynomial $\phi(z)$ which replaces $f(x)$ are certain linear functions of the coefficients in $f(x)$. The most general linear transformation for x is however capable of being built up from three particular transformations of the respective forms $x = mz$, $x = z^{-1}$, $x = z + h$. The first of these gives rise to the isobaric character of the differential equations; in any one of the differential equations the sum of the suffixes of the coefficients $\lambda_0, \lambda_1, \ldots$ and the functions $\wp_{ij}$ or $\wp_{ijkl}$ entering, is the same for each term. The second elementary transformation corresponds to the property of the differential equations that they can be arranged in pairs transformable into one another by altering λ_i, $\wp_{ij}$, $\wp_{ijkl}$ respectively into λ_{2p+2-i}, $\wp_{p-i+1, p-j+1}$, $\wp_{p-i+1, p-j+1, p-k+1, p-l+1}$. As to the third elementary transformation $x = z + h$, by taking h small we find at once that it gives the fact that the system of differential equations remains unaltered when acted on by a certain operator, to be presently put down, *provided that at the same time certain suitable constants are added to the functions $\wp_{ij}$ in the new forms of the differential equations.* In fact the simple forms found for the differential equations are partly due to a particular choice for a polynomial $f(x, z)$ (*Acta Math.* XXVII.

p. 146), variation in which is equivalent to alteration by additive constants of the functions $\wp_{ij}$; this $f(x, z)$ is however not a covariant of the polynomial

$$f(x) = \lambda_0 + \lambda_1 x + \ldots + \lambda_{2p+2} x^{2p+2},$$

whose square root appears in the hyperelliptic integrals. In order to avoid this complication it is therefore desirable in the original differential equations to define the functions $\wp_{ij}$ with the covariantive form $f(x, z) = 2a_x^{p+1} a_z^{p+1}$, where $f(x) = a_x^{2p+2}$. To be in unison with the usual notation we put therefore

$$\lambda_{2p+2} = C_0, \ \lambda_{2p+1} = \binom{2p+2}{1} C_1, \ldots, \ \lambda_i = \binom{2p+2}{i} C_{2p+2-i}, \ldots;$$

the changes in the functions $\wp_{ij}$ are easily found by calculating the difference between the covariantive $f(x, z)$ and the simplified Abelian form used

$$f(x, z) = \Sigma x^i z^i [2\lambda_{2i} + \lambda_{2i+1}(x + z)].$$

In terms of the coefficients C_i the operator is

$$P = \Sigma r C_{r-1} \frac{\partial}{\partial C_r} + \Sigma (\lambda \wp_{\lambda+1, \mu} + \mu \wp_{\lambda, \mu+1}) \frac{\partial}{\partial \wp_{\lambda\mu}} + \Sigma H_{\lambda\mu\nu\rho} \frac{\partial}{\partial \wp_{\lambda\mu\nu\rho}},$$

where

$$H_{\lambda\mu\nu\rho} = \lambda \wp_{\lambda+1, \mu, \nu, \rho} + \mu \wp_{\lambda, \mu+1, \nu, \rho} + \nu \wp_{\lambda, \mu, \nu+1, \rho} + \rho \wp_{\lambda, \mu, \nu, \rho+1};$$

the effect of this operator on the implicit forms of the differential equations, which are identical in regard to e_1, e_2, e_3, e_4 (*Acta Math.* XXVII. p. 144), is the same as the operator

$$\frac{\partial}{\partial e_1} + \frac{\partial}{\partial e_2} + \frac{\partial}{\partial e_3} + \frac{\partial}{\partial e_4}.$$

As in virtue of the second elementary transformation the differential equations also allow the operator

$$R = \sum_{r=0} (2p + 2 - r) C_{r+1} \frac{\partial}{\partial C_r} + \Sigma [(p - \lambda + 1) \wp_{\lambda-1, \mu}$$
$$+ (p - \mu + 1) \wp_{\lambda, \mu-1}] \frac{\partial}{\partial \wp_{\lambda, \mu}} + \Sigma K_{\lambda\mu\nu\rho} \frac{\partial}{\partial \wp_{\lambda\mu\nu\rho}},$$

where $$K = (p - \lambda + 1) \wp_{\lambda-1, \mu, \nu, \rho} + \ldots,$$

they must also allow the operator

$$Q = \tfrac{1}{2}(P, R) = \tfrac{1}{2}(PR - RP),$$

which is easily found to be

$$Q = \Sigma (p+1-r) C_r \frac{\partial}{\partial C_r} - \Sigma (p+1-\lambda-\mu) \wp_{\lambda\mu} \frac{\partial}{\partial \wp_{\lambda\mu}}$$
$$- \Sigma (2p+2-\lambda-\mu-\nu-\rho) \wp_{\lambda\mu\nu\rho} \frac{\partial}{\partial \wp_{\lambda\mu\nu\rho}},$$

while we easily find

$$(P, Q) = -P, \quad (Q, R) = -R,$$

and $(iP, -Q, iR)$ have the usual structure of the projective group in one variable.

In virtue of these transformations the differential equations are all deducible from several of them. Thus for $p=2$ the sigma function formed with the covariantive polynomial $f(x, z)$ is equal to that formed with the reduced Abelian polynomial multiplied by the factor

$$\exp\left[-\tfrac{1}{20}\left(\lambda_4 u_2^2 + \tfrac{1}{2}\lambda_3 u_2 u_1 + \lambda_2 u_1^2\right)\right],$$

and making the substitution to the coefficients C_i with binomial coefficients as above, and putting for brevity

$$Q_{\lambda\mu\nu\rho} = \wp_{\lambda\mu\nu\rho} - 2\left(\wp_{\mu\nu}\wp_{\lambda\rho} + \wp_{\nu\lambda}\wp_{\mu\rho} + \wp_{\lambda\mu}\wp_{\nu\rho}\right),$$

the five differential equations are, for $i = 0, 1, 2, 3, 4$ in turn,

$$-\tfrac{1}{3} Q^{(i)} = P_i + C_{i+2}\wp_{22} - 2C_{i+1}\wp_{21} + C_i\wp_{11},$$

where $Q^{(i)}$ denotes in turn Q_{2222}, Q_{2221}, Q_{2211}, Q_{2111}, Q_{1111}, and

$$P_0 = C_0C_4 - 4C_1C_3 + 3C_2^2, \quad P_1 = \tfrac{1}{2}(C_0C_5 - 3C_1C_4 + 2C_2C_3),$$
$$P_2 = \tfrac{1}{6}(C_0C_6 - 9C_2C_4 + 8C_3^2);$$
$$P_4 = C_6C_2 - 4C_5C_3 + 3C_4^2, \quad P_3 = \tfrac{1}{2}(C_6C_1 - 3C_5C_2 + 2C_4C_3).$$

These satisfy the condition of isobarism, are alike whether we count from the beginning or the end, and they are all deducible from the last of them by successive applications of the operator denoted above by P.

For $p=3$ we find that the sigma function formed with the covariantive form $f(x, z)$ is equal to that formed with the simplified Abelian form multiplied by the factor

$$\exp\left[-\tfrac{1}{2}\left(3C_2u_3^2 + 4C_3u_3u_2 + C_4u_3u_1 + 9C_4u_2^2 + 4C_5u_2u_1 + 3C_6u_1^2\right)\right]$$

and the fifteen differential equations become, if in the second parts of the right sides we abbreviate by writing down in order only the coefficients of

$$\wp_{33}, \quad \wp_{32}, \quad \wp_{31}, \quad \wp_{22}, \quad \wp_{21}, \quad \wp_{11}$$

and use $\qquad \Delta = \wp_{32}\wp_{21} - \wp_{31}\wp_{22} + \wp_{31}^2 - \wp_{33}\wp_{11},$

$$(1)\quad \begin{aligned} Q_{3333} &= -10C_0C_4 + 40C_1C_3 - 30C_2^2 \\ &\quad + (-8C_2,\ 8C_1,\ 4C_0,\ -3C_0,\ 0,\ 0), \end{aligned}$$

$$(2)\quad \begin{aligned} Q_{3332} &= -10C_0C_5 - 20C_2C_3 + 30C_1C_4 \\ &\quad + (-12C_3,\ 10C_2,\ 12C_1,\ -4C_1,\ -2C_0,\ 0), \end{aligned}$$

$$(3)\quad \begin{aligned} Q_{3331} &= -5C_2C_4 + 8C_1C_5 - 3C_0C_6 \\ &\quad + (-3C_4,\ 0,\ 10C_2,\ 0,\ -4C_1,\ C_0), \end{aligned}$$

$$(4)\quad \begin{aligned} Q_{3322} &= -40C_3^2 + 40C_2C_4 - 8C_0C_6 + 8C_1C_5 \\ &\quad + (-18C_4,\ 12C_3,\ 28C_2,\ -6C_2,\ -4C_1,\ -2C_0), \end{aligned}$$

$$(5)\quad \begin{aligned} Q_{3321} &= 12C_2C_5 - 10C_3C_4 - 2C_0C_7 \\ &\quad + (-4C_5,\ -2C_4,\ 20C_3,\ 0,\ -6C_2,\ 0), \end{aligned}$$

$$(6)\quad \begin{aligned} Q_{3311} &= 8C_3C_5 - \tfrac{15}{2}C_4^2 - \tfrac{1}{2}C_0C_8 \\ &\quad + (0,\ -4C_5,\ 12C_4,\ C_4,\ -4C_3,\ 0) + 2\Delta, \end{aligned}$$

$$(7)\quad \begin{aligned} Q_{3222} &= -60C_3C_4 - 20C_1C_6 + 84C_2C_5 - 4C_0C_7 \\ &\quad + (-28C_5,\ 16C_4,\ 56C_3,\ -12C_3,\ 0,\ -12C_1), \end{aligned}$$

$$(8)\quad \begin{aligned} Q_{3221} &= -\tfrac{35}{2}C_4^2 + 8C_3C_5 - 8C_1C_7 + 18C_2C_6 - \tfrac{1}{2}C_0C_8 \\ &\quad + (-6C_6,\ -4C_5,\ 36C_4,\ -2C_4,\ -4C_3,\ -6C_2) - 2\Delta, \end{aligned}$$

$$(9)\quad \begin{aligned} Q_{3211} &= -10C_4C_5 + 12C_3C_6 - 2C_1C_8 \\ &\quad + (0,\ -6C_6,\ 20C_5,\ 0,\ -2C_4,\ -4C_3), \end{aligned}$$

$$(10)\quad \begin{aligned} Q_{3111} &= 8C_3C_7 - 3C_2C_8 - 5C_4C_6 \\ &\quad + (C_8,\ -4C_7,\ 10C_6,\ 0,\ 0,\ -3C_4), \end{aligned}$$

$$(11)\quad \begin{aligned} Q_{2222} &= 216C_3C_5 + 4C_2C_6 - 195C_4^2 - 24C_1C_7 - C_0C_8 \\ &\quad + (-48C_6,\ 32C_5,\ 96C_4,\ -32C_4,\ 32C_3,\ -48C_2) + 12\Delta, \end{aligned}$$

$$(12)\quad \begin{aligned} Q_{2221} &= -60C_4C_5 + 84C_3C_6 - 20C_2C_7 - 4C_1C_8 \\ &\quad + (-12C_7,\ 0,\ 56C_5,\ -12C_5,\ 16C_4,\ -28C_3), \end{aligned}$$

$$(13)\quad \begin{aligned} Q_{2211} &= -40C_5^2 + 40C_4C_6 - 8C_2C_8 + 8C_3C_7 \\ &\quad + (-2C_8,\ -4C_7,\ 28C_6,\ -6C_6,\ 12C_5,\ -18C_4), \end{aligned}$$

$$(14)\quad \begin{aligned} Q_{2111} &= -20C_5C_6 - 10C_3C_8 + 30C_4C_7 \\ &\quad + (0,\ -2C_8,\ 12C_7,\ -4C_7,\ 10C_6,\ -12C_5), \end{aligned}$$

$$(15)\quad \begin{aligned} Q_{1111} &= -30C_6^2 - 10C_4C_8 + 40C_5C_7 \\ &\quad + (0,\ 0,\ 4C_8,\ -3C_8,\ 8C_7,\ -8C_6). \end{aligned}$$

These are seen on inspection* to satisfy (i) the condition of

* The last term of the second equation on p. 155 *Acta Math.* XXVII. should be $-2\lambda_8\wp_{21}$ instead of $+2\lambda_8\wp_{21}$. On p. 519 *Camb. Phil. Proc.* Vol. IX. in the ninth equation $-\lambda_0$ should be printed for λ_0, in the eleventh equation the term $-\frac{3}{2}\lambda_1$ is omitted; these would not affect the results on p. 520.

isobarism, (ii) the condition of being alike in pairs when the suffixes near their upper and lower values are interchanged, (iii) the condition of allowing the operator above denoted by P; and in fact it is seen on examination that *all the* 15 *equations are derivable from two of them, namely from* Q_{2222} *and* Q_{1111}, *by combinations of the two conditions* (ii) *and* (iii).

It follows of course in the same way that the $\frac{1}{2}p(p-1)$ rational relations connecting the $\frac{1}{2}p(p+1)$ functions $\wp_{ij}$ allow the three operators P, Q, R described above. Thus for instance the sixteen-nodal quartic, for $p=2$,

$$\begin{vmatrix} C_3-2y, & C_2-\frac{2}{3}x, & C_1, & C_0 \\ C_4-\frac{2}{3}z, & C_3+\frac{2}{9}y, & C_2+\frac{4}{9}x, & C_1 \\ C_5, & C_4+\frac{4}{9}z, & C_3+\frac{2}{9}y, & C_2-\frac{2}{3}x \\ C_6, & C_5, & C_4-\frac{2}{3}z, & C_3-2y \end{vmatrix}=0$$

is isobaric, x, y, z, C_i being of respective weights 4, 3, 2, $6-i$, is also symmetric in regard to $x, y, z, C_0, C_1, \ldots$ and $z, y, x, C_6, C_5, \ldots$, and admits the operator

$$P=x\frac{\partial}{\partial y}+2y\frac{\partial}{\partial z}+C_0\frac{\partial}{\partial C_1}+2C_1\frac{\partial}{\partial C_2}$$
$$+3C_2\frac{\partial}{\partial C_3}+4C_3\frac{\partial}{\partial C_4}+5C_4\frac{\partial}{\partial C_5}+6C_5\frac{\partial}{\partial C_6}.$$

But on the other hand this surface also admits the group of two parameters whose equations are given by the expressions for $\wp_{22}(u+\alpha)$, $\wp_{21}(u+\alpha)$, $\wp_{11}(u+\alpha)$ in terms of $\wp_{22}(u)$, $\wp_{21}(u)$, $\wp_{11}(u)$, $\wp_{22}(\alpha)$, $\wp_{21}(\alpha)$, $\wp_{11}(\alpha)$. If a, b, c, d denote the minors of the elements of any row of the determinant just written down, the generating operators of this group are

$$a\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+c\frac{\partial}{\partial z}, \qquad b\frac{\partial}{\partial x}+c\frac{\partial}{\partial y}+d\frac{\partial}{\partial z},$$

and the finite equations could easily be found by differentiation from the formula given in the *American Journal* (Vol. XX. p. 381) for the function

$$\frac{\sigma(u+\alpha)\,\sigma(u-\alpha)}{\sigma^2(u)\,\sigma^2(\alpha)}$$

in terms of the functions $\wp_{22}(u), \ldots, \wp_{11}(\alpha)$.

II.

Passing now to the modular relations referred to in the introduction of this paper, we use the formula, easily proved by direct multiplication of the series (as in the writer's *Abelian Functions*, p. 472)

$$\sigma(u+t)\,\sigma(u-t)=\sum_{\epsilon}\vartheta_{\epsilon}(u)\,\vartheta_{\epsilon}(t),$$

where as usual

$$\sigma\left[u\,;\ \frac{1}{2}\binom{q'}{q}\right]=\sum_{n}e^{au^2+2\pi iv(n+\frac{1}{2}q')+i\pi\tau(n+\frac{1}{2}q')^2+\pi iq(n+\frac{1}{2}q')},$$

$\vartheta(u)$ differs from $\sigma(u)$ only in having $2au^2$ for au^2, $4\pi iv$ for $2\pi iv$, and $2\pi i\tau$ for $\pi i\tau$, and $\vartheta_{\epsilon}(u)$ denotes $\vartheta\left[u\,;\ \frac{1}{2}\binom{\epsilon}{0}\right]$, the summation in regard to ϵ extending to the 2^p terms obtained by giving to the p elements of ϵ the values 0 and 1. We have, if

$$D^{\lambda\mu\nu\rho}=\frac{\partial^4}{\partial t_\lambda\partial t_\mu\partial t_\nu\partial t_\rho},\qquad D^{ij}=\frac{\partial^2}{\partial t_i\partial t_j},$$

$$\wp_{ij}(u)=-\frac{1}{2\sigma^2(u)}\left[D^{ij}\sigma(u+t)\,\sigma(u-t)\right]_{t=0},$$

$$Q_{\lambda\mu\nu\rho}(u)=-\frac{1}{2\sigma^2(u)}\left[D^{\lambda\mu\nu\rho}\sigma(u+t)\,\sigma(u-t)\right]_{t=0}.$$

Consider then any one of our differential equations in which the right side is linear in the functions $\wp_{ij}$, say an equation

$$Q_{\lambda\mu\nu\rho}(u)=A_{\lambda\mu\nu\rho}+\Sigma A_{ij}\wp_{ij}(u);$$

this is equivalent to

$$D^{\lambda\mu\nu\rho}\sigma(u+t)\,\sigma(u-t)+2A_{\lambda\mu\nu\rho}\sigma(u+t)\,\sigma(u-t)$$
$$-\Sigma A_{ij}D^{ij}\sigma(u+t)\,\sigma(u-t)=0,$$

where after differentiation t is to be put zero; substituting the above expression for $\sigma(u+t)\,\sigma(u-t)$ and equating to zero the coefficients of each of the 2^p functions $\vartheta_{\epsilon}(u)$, we obtain the equations

$$\vartheta_{\epsilon}^{\lambda\mu\nu\rho}(0)+2A_{\lambda\mu\nu\rho}\vartheta_{\epsilon}(0)-\Sigma A_{ij}\vartheta_{\epsilon}^{ij}(0)=0,$$

which are the modular equations in question.

These relations can be reduced; we have

$$\vartheta_{\epsilon}(u)=\sum_{n}e^{2au^2+4\pi iv(n+\frac{1}{2}\epsilon)+2\pi i\tau(n+\frac{1}{2}\epsilon)^2},\quad =e^{2au^2}\phi(v),\ \text{say},$$

where $v = \omega^{-1}u$; the differentiations in regard to u are linearly obtainable from those in regard to v, and we clearly have

$$\frac{\partial^2\phi(v)}{\partial v_\lambda \partial v_\mu} = 4\pi i \frac{\partial\phi(v)}{\partial\tau_{\lambda\mu}}, \qquad \frac{\partial^2\phi(v)}{\partial v_\lambda{}^2} = 8\pi i \frac{\partial\phi(v)}{\partial\tau_{\lambda\lambda}};$$

that is the modular equations involve only differentiations of the first and second orders in regard to the quantities $\tau_{\lambda\mu}$.

Conversely the relations can be used to express the algebraic moduli, that is the coefficients in the binary form occurring in the hyperelliptic equation, in terms of transcendental constants, in various forms; there are for instance in case $p=2$ twenty such modular equations. For larger values of p, the results can, *presumably*, be used to express transcendental conditions that the theta functions should be hyperelliptic. It is clear that the original differential equations are a consequence of the modular equations.

One complication in working out the modular equations in detail arises from the presence of the factor e^{au^2} in the theta function. In case $p=1$ we may start from the equation of Weierstrass's theory

$$\wp''(u) = 6\wp^2 u - \tfrac{1}{2}g_2,$$

and use the known facts

$$a = \frac{\eta}{2\omega}, \quad \frac{\omega^4 g_2}{\pi^4} = \tfrac{1}{12} + 20 \sum_{n=1}^{\infty} \frac{n^3 q^{2n}}{1-q^{2n}}, \quad \frac{12\eta\omega}{\pi^2} = 1 - 24 \sum_{n=1}^{\infty} \frac{nq^{2n}}{1-q^{2n}},$$

$$q\frac{d}{dq}(\eta\omega) = \frac{2}{\pi^2}\left[\eta^2\omega^2 - \tfrac{1}{12}g_2\omega^4\right].$$

Then we eventually find, proceeding just as in the general case, that the linear differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y\left[\tfrac{1}{16} + 15\sum_{n=1}^{\infty}\frac{n^3x^{2n}}{1-x^{2n}}\right] = 0,$$

of Fuchsian type, existing only within a circle of radius unity, has the two integrals

$$y_1 = \frac{1 + 2\sum\limits_{n=1}^{\infty} x^{2n^2}}{\left[x^{\frac{1}{12}}\prod\limits_{1}^{\infty}(1-x^{2n})\right]^3}, \quad y_2 = \frac{2\sum\limits_{n=0}^{\infty} x^{2(n+\frac{1}{2})^2}}{\left[x^{\frac{1}{12}}\prod\limits_{1}^{\infty}(1-x^{2n})\right]^3},$$

III.

Taking for one variable the differential equation

$$\wp'' - 6\wp^2 = -\tfrac{1}{2}\lambda_0\lambda_4 + \tfrac{1}{8}\lambda_1\lambda_3 + \lambda_2\wp,$$

giving $$\wp'^2 - 4\wp^3 = -(\lambda_0\lambda_4 - \tfrac{1}{4}\lambda_1\lambda_3)\wp + \lambda_2\wp^2 + \text{constant};$$

or, if $\wp + \frac{1}{12}\lambda_2 = t,$

$$\left(\frac{dt}{du}\right)^2 = 4t^3 - g_2 t - g_3';$$

where $$g_2 = \lambda_0\lambda_4 - \tfrac{1}{4}\lambda_1\lambda_3 + \tfrac{1}{12}\lambda_2^2$$

is the quadrinvariant of the quartic form

$$\lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \lambda_4 x^4,$$

but g_3' differs by an arbitrary constant of integration from the cubinvariant

$$g_3 = \tfrac{1}{6}\lambda_0\lambda_2\lambda_4 + \tfrac{1}{48}\lambda_1\lambda_2\lambda_3 - \tfrac{1}{16}\lambda_1^2\lambda_4 - \tfrac{1}{16}\lambda_3^2\lambda_0 - \tfrac{1}{216}\lambda_2^3,$$

we infer that the general integral of the differential equation is

$$-\tfrac{1}{12}\lambda_2 + \wp(u + \alpha,\ g_2,\ g_3'),$$

where α, g_3' are arbitrary constants, $\wp$ denotes Weierstrass's particular function, and g_2 is, as above, defined by the coefficients in the differential equation.

There is, however, another form of integral naturally suggested by the following work for $p = 2$, which though not new seems worth recapitulation.

Let $\phi(u)$ be a definite single-valued function satisfying the equation

$$\left(\frac{dx}{du}\right)^2 = \mu_0 + \mu_1 x + \mu_2 x^2 + \mu_3 x^3 + \mu_4 x^4,$$

wherein in the first instance we suppose the coefficients on the right to have general values and μ_4 not to be zero. At a point at which $\phi(u)$ is infinite the limit of $\dfrac{\phi' u}{\phi^2 u}$ may have one of two values; denoting a definite square root of μ_4 by $\sqrt{\mu_4}$, let

$$\frac{\phi'(\alpha)}{\phi^2(\alpha)} = \sqrt{\mu_4}, \quad \frac{\phi'(\beta)}{\phi^2(\beta)} = -\sqrt{\mu_4}.$$

Then near α we find

$$\phi(u) = \frac{1}{v} - \frac{\mu_3}{4\mu_4} + \text{positive powers of } u,$$

where $$v = -\sqrt{\mu_4}(u - \alpha);$$

near β we have the same expression in terms of

$$w = +\sqrt{\mu_4}(u - \beta).$$

Hence the expression

$$H(u) = -\tfrac{1}{2}\sqrt{\mu_4}\int^u \phi(u)\,du - \int^u du \int^u du\,[\tfrac{1}{4}\mu_3\phi(u) + \tfrac{1}{2}\mu_4\phi^2(u)]$$

is infinite near α like $\log(u-\alpha)$, but is not infinite near $u=\beta$.

Thus the function

$$\Sigma(u) = e^{H(u)},$$

which is definite save for a factor e^{Au+B}, where A, B are arbitrary constants, is a single-valued integral function vanishing to the first order at $u=\alpha$ and the congruent points (in regard to the periods of $\phi(u)$). We find, however, at once if we put

$$P(u) = -\frac{\partial^2}{\partial u^2}\log\Sigma(u)$$

that $$P''(u) - 6P^2(u) = \tfrac{1}{8}\mu_3\mu_1 - \tfrac{1}{2}\mu_0\mu_4 + \mu_2 P(u).$$

Hence it follows that, in terms of Weierstrass's particular σ-function,

$$\Sigma(u) = \sigma(u-\alpha;\ g_2,\ g_3')\,e^{\frac{1}{24}\mu_2 u^2 + Au + B},$$

where A, B are arbitrary, $g_2 = \mu_0\mu_4 - \frac{1}{4}\mu_1\mu_3 + \frac{1}{12}\mu_2^2$ and g_3' is to be determined so that the periods may agree with those of $\phi(u)$, which can be shown to be the case if g_3 is the cubinvariant of the quartic.

Consider now the differential equations for $p=2$. By equating in pairs the partial derivatives of the left sides we obtain linear equations in the functions $\wp_{222}$, $\wp_{221}$, etc., leading by elimination to the determinantal equation for the sixteen-nodal surface given* on p. 516 of *Camb. Phil. Proc.* Vol. IX. We consider here the case when $\lambda_5 = 0$ and $\lambda_6 = 0$, to which the coincidence of two roots of the sextic can always be reduced. Then the expanded equation takes a form which may be written ($x = \wp_{22}$, $y = \wp_{21}$, $z = \wp_{11}$)

$$4\,[x(xz - y^2) + \tfrac{1}{4}\lambda_3 xy - \tfrac{1}{2}\lambda_4 y^2]^2$$
$$= (\lambda_0 x^4 - \lambda_1 x^3 y + \lambda_2 x^2 y^2 - \lambda_3 x y^3 + \lambda_4 y^4)(4x + \lambda_4).$$

Following the method of integration sketched in the earlier part of the note just referred to we find from the first three of the homogeneous linear equations for $\wp_{222}$, $\wp_{221}$, etc., the equations

$$\wp_{222} = \mu x,\quad \wp_{221} = \mu y,\quad \wp_{211} = \mu\,\frac{zx + y^2 + \frac{1}{4}\lambda_3 y}{2x + \frac{1}{2}\lambda_4},$$

$$\wp_{111} = \mu\left[-\tfrac{1}{4}\lambda_1 + \frac{y(4z + \lambda_2)}{2x} - \frac{(2y + \frac{1}{2}\lambda_3)(zx + y^2 + \frac{1}{4}\lambda_3 y)}{2x(2x + \frac{1}{2}\lambda_4)}\right];$$

* Where in the third element of the second row $-(4x+\lambda_4)$ should be printed instead of $-(4x+\frac{1}{2}\lambda_4)$.

from the first two of these

$$\wp_{2221} = x \frac{\partial \mu}{\partial \mu_1} + \mu^2 y,$$

$$\wp_{2211} = y \frac{\partial \mu}{\partial \mu_1} + \mu^2 \frac{zx + y^2 + \frac{1}{4}\lambda_3 y}{2x + \frac{1}{2}\lambda_4},$$

and hence

$$y\wp_{2221} - x\wp_{2211} = \mu^2 \left\{ y^2 - x \frac{zx + y^2 + \frac{1}{4}\lambda_3 y}{2x + \frac{1}{2}\lambda_4} \right\};$$

substituting from the differential equations the expressions for $\wp_{2221}$ and $\wp_{2211}$ we find that there is a common factor

$$x(xz - y^2) + \tfrac{1}{4}\lambda_3 xy - \tfrac{1}{2}\lambda y^2$$

on both sides, this being that occurring on the left in the equation of the quartic; assuming this not zero we get

$$\mu^2 = 4x + \lambda_4.$$

Then $$dx = \wp_{222} du_2 + \wp_{221} du_1 = \mu (x du_2 + y du_1)$$

and $$dy = \mu \left[y du_2 + \frac{zx + y^2 + \frac{1}{4}\lambda_3 y}{2x + \frac{1}{2}\lambda_4} du_1 \right]$$

give $$x dy - y dx = \mu du_1 \frac{x(zx - y^2) + \frac{1}{4}\lambda_3 xy - \frac{1}{2}\lambda_4 y^2}{2x + \frac{1}{2}\lambda_4},$$

and

$$\frac{zx + y^2 + \frac{1}{4}\lambda_3 y}{2x + \frac{1}{2}\lambda_4} dx - y dy = \mu du_2 \frac{x(zx - y^2) + \frac{1}{4}\lambda_3 xy - \frac{1}{2}\lambda_4 y^2}{2x + \frac{1}{2}\lambda_4},$$

of which the left side is

$$\frac{x(zx - y^2) + \frac{1}{4}\lambda_3 xy - \frac{1}{2}\lambda_4 y^2}{x(2x + \frac{1}{2}\lambda_4)} dx - \frac{y}{x}(x dy - y dx);$$

hence from the fundamental quartic relation connecting x, y, z

$$du_1 = \frac{x dy - y dx}{\sqrt{Q}}, \quad du_2 = \frac{dx}{x\sqrt{4x + \lambda_4}} - \frac{y}{x}\frac{x dy - y dx}{\sqrt{Q}},$$

where $$Q = \lambda_0 x^4 - \lambda_1 x^3 y + \lambda_2 x^2 y^2 - \lambda_3 x y^3 + \lambda_4 y^4,$$

and we have put

$$\mu = \sqrt{4x + \lambda_4}, \quad \frac{x(xz - y^2) + \frac{1}{4}\lambda_3 xy - \frac{1}{2}\lambda_4 y^2}{4x + \lambda_4} = \tfrac{1}{2}\sqrt{Q}.$$

Choosing a particular single-valued function to satisfy

$$\left(\frac{d\phi(w)}{dw}\right)^2 = \lambda_0 - \lambda_1\phi + \lambda_2\phi^2 - \lambda_3\phi^3 + \lambda_4\phi^4,$$

and putting $$\int^w \phi(w)\,dw = \psi(w) + \text{const.},$$

this gives

$$\frac{y}{x} = \phi(u_1), \qquad u_2 + \psi(u_1) = \int^x \frac{dx}{x\sqrt{4x+\lambda_4}},$$

where u_1, u_2 stand for $\pm u_1 + \text{const.}$, $\pm u_2 + \text{const.}$, and hence

$$x = \tfrac{1}{4}\lambda_4 \operatorname{cosech}^2 [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))],$$

$$y = \tfrac{1}{4}\lambda_4 \phi(u_1) \operatorname{cosech}^2 [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))],$$

from which z follows without ambiguity by means of

$$x(xz - y^2) + \tfrac{1}{4}\lambda_3 xy - \tfrac{1}{2}\lambda_4 y^2$$
$$= -\tfrac{1}{2}x^2\sqrt{\lambda_4} \coth [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))] \frac{d\phi(u_1)}{du_1}.$$

From these the functions $\zeta_2(u_1, u_2)$, $\zeta_1(u_1, u_2)$ given by

$$-d\zeta_2 = x\,du_2 + y\,du_1, \quad -d\zeta_1 = y\,du_2 + z\,du_1$$

are at once found to be, save for additive constants,

$$\zeta_2 = \tfrac{1}{2}\sqrt{\lambda_4} \coth [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))],$$

$$\zeta_1 = \tfrac{1}{2}\sqrt{\lambda_4} \coth [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))]\,\phi(u_1) + \tfrac{1}{4}\lambda_3\psi(u_1) - \tfrac{1}{2}\lambda_4\chi(u_1)$$

where $$\chi(w) = \int^w (\phi(w))^2\,dw + \text{const.}$$

Then the integral function

$$\sigma(u_1, u_2) = \exp\left(\int \zeta_2\,du_2 + \zeta_1\,du_1\right)$$

is seen to be

$$\sigma(u_1, u_2) = \frac{\sinh [\tfrac{1}{2}\sqrt{\lambda_4}(u_2 + \psi(u_1))]}{\tfrac{1}{2}\sqrt{\lambda_4}}\, e^{-\int^{u_1} du_1 \int^{u_1} du_1 [-\frac{1}{4}\lambda_3\phi(u_1) + \frac{1}{2}\lambda_4\phi^2(u_1)] + Au_1 + Bu_2 + C},$$

where A, B, C are arbitrary constants, and u_1, u_2 are used for $\pm u_1 + \text{const.}$, $\pm u_2 + \text{const.}$

We take particular cases of this.

(i) When λ_4 is not zero and the other coefficients are general the function is, save for a factor of the form $e^{Au_1+Bu_2+C}$,

$$\sigma(u_1, u_2) = e^{\frac{1}{2}\sqrt{\lambda_4}u_2} H(u_1) + e^{-\frac{1}{2}\sqrt{\lambda_4}u_2} K(u_1),$$

where

$$\log H(u_1) = \tfrac{1}{2}\sqrt{\lambda_4}\int^{u_1}\phi(u_1)\,du_1$$
$$-\int^{u_1}du_1\int^{u_1}du_1\,[-\tfrac{1}{4}\lambda_3\phi(u_1)+\tfrac{1}{2}\lambda_4\phi^2(u_1)],$$

and $\log K(u_1)$ differs from this in the sign attached to $\sqrt{\lambda_4}$. Hence if α, β be incongruent poles of $\phi(u)$, such that

$$\phi'(\alpha)/\phi^2(\alpha) = \sqrt{\lambda_4},$$
$$\phi'(\beta)/\phi^2(\beta) = -\sqrt{\lambda_4},$$

it follows from what we have proved above, that

$$\sigma(u_1,\ u_2) = [e^{\frac{1}{2}u_2\sqrt{\lambda_4}}\sigma(u_1-\beta)+e^{-\frac{1}{2}u_2\sqrt{\lambda_4}}\sigma(u_1-\alpha)]\,e^{\frac{\lambda_2}{24}u_1^2+Au_1+Bu_2+C},$$

where $\sigma(u)$ is Weierstrass's particular function depending on the two invariants

$$g_2 = \lambda_0\lambda_4 - \tfrac{1}{4}\lambda_1\lambda_3 + \tfrac{1}{12}\lambda_2^2,$$
$$g_3 = \tfrac{1}{6}\lambda_0\lambda_2\lambda_4 + \tfrac{1}{48}\lambda_1\lambda_2\lambda_3 - \tfrac{1}{16}\lambda_1^2\lambda_4 - \tfrac{1}{16}\lambda_3^2\lambda_0 - \tfrac{1}{216}\lambda_2^3.$$

It is easy by differentiation of this expression to find the functions $\wp_{22}$, $\wp_{21}$, $\wp_{11}$ and to show that they are rational functions of the three

$$e^{u_2\sqrt{\lambda_4}}\frac{\sigma(u_1-\beta)}{\sigma(u_1-\alpha)},\quad \wp(u_1),\quad \wp'(u_1),$$

which are a set of triply periodic functions.

It may be remarked that the equation

$$\sigma(u_1,\ u_2) = 0,$$

regarded as an equation for determining u_2 in terms of u_1, gives

$$u_2 + \int^{u_1}\phi(u_1)\,du_1 = n\,\frac{2\pi i}{\sqrt{\lambda_4}},$$

where n is an arbitrary integer.

(ii) When $\lambda_4 = 0$ and the other coefficients are general

$$\sigma(u_1,\ u_2) = \left[u_2 + \int^{u_1}\phi(u_1)\,du_1\right] e^{\frac{1}{4}\lambda_3\int^{u_1}du_1\int^{u_1}\phi(u_1)\,du_1}$$

is easily found to be, save for a factor $e^{Au_1+Bu_2+C}$,

$$\sigma(u_1,\ u_2) = \left[u_2 - \frac{\lambda_2}{3\lambda_3}u_1 - \frac{1}{a^2}\zeta(au_1)\right]\sigma(au_1)\,e^{\frac{\lambda_2}{24}u_1^2},$$

where σ, ζ are Weierstrass's particular functions, $a^3 = -\frac{1}{4}\lambda_3$, and the invariants of the elliptic functions are

$$g_2 = \frac{1}{a}\left(\lambda_1 - \frac{\lambda_2^2}{3\lambda_3}\right), \quad g_3 = -\lambda_0 + \frac{\lambda_1\lambda_2}{3\lambda_3} - \frac{2\lambda_2^3}{27\lambda_3^2}.$$

In this case the functions $\wp_{22}$, $\wp_{21}$, $\wp_{11}$ are rationally expressible by

$$u_2 - \frac{\lambda_2}{3\lambda_3}u_1 - \frac{1}{a^2}\zeta(au_1), \quad \wp(au_1), \quad \wp'(au_1),$$

which are a doubly periodic set of functions, and the quartic surface reduces to

$$(xz - y^2 + \tfrac{1}{4}\lambda_3 y)^2 = \lambda_0 x^3 - \lambda_1 x^2 y + \lambda_2 x y^2 - \lambda_3 y^3.$$

(iii) In case $\lambda_4 = \lambda_3 = 0$, then, save for a factor $e^{Au_1 + Bu_2 + C}$,

$$\sigma(u_1, u_2) = u_2 + \int^{u_1} \phi(u_1)\, du_1,$$

where
$$u_1 = \int^{\phi(u_1)} \frac{dt}{\sqrt{\lambda_0 - \lambda_1 t + \lambda_2 t^2}},$$

so that in general

$$\sigma(u_1, u_2) = u_2 + \frac{\lambda_1}{2\lambda_2}u_1 + \frac{\sqrt{\lambda_1^2 - 4\lambda_0\lambda_2}}{2\lambda_2^{\frac{3}{2}}}\sinh(u_1\sqrt{\lambda_2});$$

but if $\lambda_2 = 0$, $\lambda_1 \neq 0$,

$$\sigma(u_1, u_2) = u_2 + \frac{\lambda_0}{\lambda_1}u_1 - \frac{\lambda_1}{8}u_1^3,$$

while if $\lambda_2 = 0$, $\lambda_1 = 0$,

$$\sigma(u_1, u_2) = u_2 + \tfrac{1}{2}\sqrt{\lambda_0}\, u_1^2,$$

and if $\lambda_2 \neq 0$, $\lambda_1^2 = 4\lambda_0\lambda_2$,

$$\sigma(u_1, u_2) = u_2 - \frac{\lambda_1}{2\lambda_2}u_1 + \frac{1}{\lambda_2}e^{u_1\sqrt{\lambda_2}}.$$

(iv) Another case, corresponding to that in which the original sextic has two repeated roots, is when $\lambda_0 = \lambda_1 = 0$, but

$$\lambda_2 \neq 0, \quad \lambda_3 \neq 0, \quad \lambda_4 \neq 0.$$

Then $\sigma(u_1, u_2)$ may be taken in the original form, and

$$\phi(u_1) = \left[\frac{\lambda_3}{2\lambda_2} + \frac{\sqrt{\lambda_3^2 - 4\lambda_2\lambda_4}}{2\lambda_2}\cosh(u_1\sqrt{\lambda_2})\right]^{-1}.$$

(v) Of this again a particular case is when $\lambda_3^2 = 4\lambda_2\lambda_4$.

For the case of $p = 3$, I have similarly partly considered the consequences of supposing $\lambda_7 = 0$, $\lambda_8 = 0$; and append the results for reference. When we take all possible partial derivatives of the quantities $\wp_{3333}$, etc., occurring on the left side of the differential equations there are just thirty equations obtainable by equating the resulting identically equal partial derivatives of the fifth order. These are not all independent, nor are those which are given below; they will be found to give enough independent equations to express all the functions as algebraic functions of three of them, that is, as rational functions of four connected by a rational algebraic equation; these four are apparently conveniently taken to be those denoted by ξ, η, ζ, P. It is however to allow choice that all the equations not obviously deducible from one another obtained from the thirty spoken of above have been retained.

Let $\wp_{33} = \xi$, $\wp_{32} = \eta$, $\wp_{31} = \zeta$, $\wp_{22} = x$, $\wp_{21} = y$, $\wp_{11} = z$,

$$P = (2\xi x + \eta(2\eta + \tfrac{1}{2}\lambda_5) + \lambda_6\zeta) \div (4\xi + \lambda_6),$$
$$Q = (2\xi y + \zeta(2\eta + \tfrac{1}{2}\lambda_5)) \div (4\xi + \lambda_6),$$
$$R = (2y\eta - 2x\zeta + 4\zeta^2) \div (4\xi + \lambda_6),$$

then, proceeding just as in the case of two variables above, we find

$$\wp_{333} = \mu\xi, \quad \wp_{332} = \mu\eta, \quad \wp_{331} = \mu\zeta, \quad \wp_{322} = \mu P, \quad \wp_{321} = \mu Q, \quad \wp_{311} = \mu R,$$
$$\mu^2 = 4\xi + \lambda_6,$$
$$\zeta P - \eta Q + \xi R = \zeta^2,$$

and the following sixteen equations:

$$2\xi\frac{\wp_{222}}{\mu} = -\tfrac{1}{2}\lambda_3\xi + (4x + \lambda_4)\eta + \lambda_5\zeta - (2\eta + \tfrac{1}{2}\lambda_5)P - \lambda_6 Q,$$

$$2\eta\frac{\wp_{222}}{\mu} = \tfrac{1}{2}\lambda_3\eta - \xi(4z + \lambda_2) + (2x - 4\zeta)P + 4\eta Q + (4\xi + \lambda_6)R,$$

$$2\zeta\frac{\wp_{222}}{\mu} = -\tfrac{1}{2}\lambda_1\xi - 2z\eta - 4y\zeta + 4yP + (8\zeta - 2x)Q - (2\eta + \tfrac{1}{2}\lambda_5)R,$$

$$2\xi\frac{\wp_{221}}{\mu} = 2z\xi + 2y\eta + (2x - 4\zeta + \lambda_4)\zeta + 4\zeta P - (6\eta + \tfrac{1}{2}\lambda_5)Q + (2\xi - \lambda_6)R,$$

$$2\eta\frac{\wp_{221}}{\mu} = -\tfrac{1}{2}\lambda_1\xi + \tfrac{1}{2}\lambda_3\zeta + 2yP - \tfrac{1}{2}\lambda_5 R,$$

$$2\zeta\frac{\wp_{221}}{\mu} = -\lambda_0\xi - 2z\zeta + 2yQ + 2\zeta R,$$

$$2\xi \frac{\wp_{211}}{\mu} = 2\eta z + \zeta(2y + \tfrac{1}{2}\lambda_3) - (2\eta + \tfrac{1}{2}\lambda_5) R,$$

$$2\eta \frac{\wp_{211}}{\mu} = -\tfrac{1}{2}\lambda_1\eta + \zeta(4z + \lambda_2) + (2x - 4\zeta) R,$$

$$2\zeta \frac{\wp_{211}}{\mu} = -\lambda_0\eta + \tfrac{1}{2}\lambda_1\zeta + 2yR,$$

$$2\xi \frac{\wp_{111}}{\mu} = -2z\zeta + 2zP + (2y + \tfrac{1}{2}\lambda_3) Q + (4\zeta - 4x - \lambda_4) R,$$

$$2\eta \frac{\wp_{111}}{\mu} = \tfrac{1}{2}\lambda_1\zeta - \tfrac{1}{2}\lambda_1 P + (4z + \lambda_2) Q - (2y + \tfrac{1}{2}\lambda_3) R,$$

$$2\zeta \frac{\wp_{111}}{\mu} = \lambda_0\zeta - \lambda_0 P + \tfrac{1}{2}\lambda_1 Q + 2zR,$$

$$-\tfrac{1}{2}\lambda_1\eta + (4z + \lambda_2)\zeta - 4yQ + (4x - 8\zeta) R + 2y\frac{\wp_{222}}{\mu} + (4\zeta - 2x)\frac{\wp_{221}}{\mu}$$
$$- (4\eta + \tfrac{1}{2}\lambda_5)\frac{\wp_{211}}{\mu} + (4\xi + \lambda_6)\frac{\wp_{111}}{\mu} = 0,$$

$$-2\lambda_0\eta + \tfrac{3}{2}\lambda_1\zeta - \tfrac{1}{2}\lambda_1 P + \lambda_2 Q - \lambda_3 R + 2z\frac{\wp_{222}}{\mu} + (2y + \tfrac{1}{2}\lambda_3)\frac{\wp_{221}}{\mu}$$
$$- (4x + \lambda_4)\frac{\wp_{211}}{\mu} + \tfrac{1}{2}\lambda_5\frac{\wp_{111}}{\mu} = 0,$$

$$2\lambda_0\zeta - 2\lambda_0 P + 2\lambda_1 Q - \lambda_2 R - \tfrac{1}{2}\lambda_1\frac{\wp_{222}}{\mu} + (4z + \lambda_2)\frac{\wp_{221}}{\mu}$$
$$- (2y + \tfrac{1}{2}\lambda_3)\frac{\wp_{211}}{\mu} - 2x\frac{\wp_{111}}{\mu} = 0,$$

$$\lambda_0 Q - \tfrac{1}{4}\lambda_1 R - \tfrac{1}{2}\lambda_0\frac{\wp_{222}}{\mu} + \tfrac{1}{4}\lambda_1\frac{\wp_{221}}{\mu} + z\frac{\wp_{211}}{\mu} - y\frac{\wp_{111}}{\mu} = 0.$$

By the elimination of $\wp_{222}$ and z from the first three of these sixteen, x and y being expressed in terms of P, Q, ξ, η, ζ, we obtain a linear equation to express Q in terms of ξ, η, ζ, P. From another set of three we obtain an equation connecting P, Q, ξ, η, ζ; hence we obtain at once the fundamental equation connecting ξ, η, ζ, P. The algebraic expressions of u_3, u_2, u_1 are then found from the equations

$$d\xi = \mu(\xi du_3 + \eta du_2 + \zeta du_1), \qquad d\eta = \mu(\eta du_3 + P du_2 + Q du_1),$$
$$d\zeta = \mu(\zeta du_3 + Q du_2 + R du_1).$$

Addendum.

We give now a sketch of a proof of the theorem that an analytical function of one variable possessing an algebraic addition theorem is an elliptic function or a particular case of such. The ideas employed have, of course, no novelty*.

It is supposed here that there is a finite range of values for u within which the function $\phi(u)$ is representable by a succession of power series which are continuations of one another, so that the function is monogenic; and similarly for $\phi(u_0)$ and $\phi(u+u_0)$, the range within which $\phi(u+u_0)$ is so representable being a consequence of the ranges for u and u_0. And that for the values of $\phi(u)$, $\phi(u_0)$, $\phi(u+u_0)$ so obtainable for a given pair of values of u and u_0 there exists always the same algebraic relation

$$\phi(u+u_0) = A\,[\phi(u),\ \phi(u_0)],$$

provided a proper signification be given to ambiguous algebraic signs occurring on the right.

Putting here $x_0 = \phi(u_0)$ we deduce thence an equation

$$\phi^m(u+u_0) + H_1[\phi(u),\ x_0]\,\phi^{m-1}(u+u_0) + \dots \\ + H_m[\phi(u),\ x_0] = 0,$$

which is rational in x_0, so that each coefficient is of the form

$$H[\phi(u),\ x_0] = \frac{x_0^h f_0(u) + x_0^{h-1} f_1(u) + \dots}{x_0^k g_0(u) + x_0^{k-1} g_1(u) + \dots},$$

wherein $f_0(u), f_1(u), \dots, g_0(u), \dots$ are power series in u defining by their continuation within the u-region above spoken of a series of monogenic functions.

If now every one of these functions $f_0, f_1, \dots, g_0, g_1, \dots$ be continued as far as possible, any value of u in the neighbourhood of which the continuation circles of one at least of these functions have zero for lower limit may be a singular point of the coefficient $H[\phi(u), x_0]$ under consideration. In a range for u, within which no such singular point arises for any one of the coefficients H, the function $\phi(u+u_0)$ satisfying the equation can be continued, as a function of u, and will behave like an algebraic function of u. In this range too every root of the equation has the same property. But a value of u, which is a singular point of one at least of the functions $f_0, f_1, \dots, g_0, g_1, \dots$ in one at least of the coefficients H, is independent of x_0; if it is a singularity of one of the coefficients H, it is a singularity of one of the roots of the equation; say such a value is $u = c$; then $w = c + u_0$ is a singularity of one root $\phi(w)$ of the equation; this is so for the definite value c for arbitrary

* Cf. Schwarz, *Formeln u. Lehrsätze*, §§ 1—3; Phragmén, *Acta Math.* VII. pp. 33—42; Forsyth, *Theory of Functions*, pp. 326—350; Painlevé, *Acta Math.* XXVII. p. 1.

values of u_0 in a certain range—an impossible thing for a function $\phi(w)$ otherwise analytical, unless c be infinite. Thus the only singularities of the coefficients H, which arise by singularities of the subsidiary functions $f_0, f_1, \ldots, g_0, g_1, \ldots$, are for $u=\infty$. Singularities of these coefficients H are possible which are not singularities of any one of the subsidiary functions $f_0, f_1, \ldots, g_0, g_1, \ldots$; but such are necessarily poles.

Thus $\phi(u+u_0)$ satisfies an algebraic equation, whose coefficients are rational in $x_0=\phi(u_0)$, and have no finite singularities other than poles; in particular there is no point such that a circuit of u round it changes the value of a coefficient, namely, these coefficients are single-valued.

Hence it follows that to a given value of u belongs only a finite number of values of $\phi(u)$. For putting $u_0=0$, the previously discussed equation gives an algebraic equation satisfied by $\phi(u)$, with single-valued coefficients having essential singularities only for $u=\infty$. Let

$$\phi^n(u)+K_1(u)\,\phi^{n-1}(u)+\ldots+K_n(u)=0$$

be the equation of lowest order of this character satisfied by $\phi(u)$; we assume that all roots of this equation are obtainable by continuation from one root, so that the algebraic addition theorem applies to all; let $K(u)$ be a rational symmetric function of the roots of this equation, which is therefore also a single-valued function with essential singularity only for $u=\infty$. Then $K(u+u_0)$ is the same rational symmetric function of functions $\phi(u+u_0)$, each of which is algebraically expressible by $x=\phi(u)$, $x_0=\phi(u_0)$; thus $K(u+u_0)$ is an algebraic function of x and x_0. Between this function then and $\phi(u+u_0)=A(x, x_0)$ the quantity x_0 can be eliminated. There is therefore an algebraic equation

$$B[\phi(u+u_0), \quad K(u+u_0)]=0$$

wherein, since $u+u_0$ can be kept unaltered when u alters by suitably adjusting the variation of u_0, the coefficients are independent of u. In other words, putting u_0 now zero, the function $\phi(u)$ is an algebraic function of a single-valued function with no essential singularity save for $u=\infty$.

We have thus to show that such a single-valued meromorphic function which has an algebraic addition equation is an elliptic function or a particular case of such.

Putting $$x=\phi(u), \quad y=\phi'(u)=\frac{dx}{du},$$

the equation $$\phi(u+u_0)=A(x, x_0)$$

gives $$\phi'(u+u_0)=A_2(x, x_0)\frac{dx_0}{du_0},$$

and hence $$y = \left(A_2(x, x_0)\frac{dx_0}{du_0}\right)_{u_0=0} = B(x),$$
an algebraic function of x, so that x, y are connected by a rational algebraic equation
$$f(y, x) = 0,$$
which we may suppose irreducible. To every finite value of u belongs a definite place $x = \phi(u)$, $y = \phi'(u)$ of this algebraic construct; conversely to every place (x, y) belongs a value of u given by
$$u = \int^{(x, y)} \frac{dx}{y},$$
save only that those definite places which are the infinities of this integral are limiting points in the neighbourhood of which u increases indefinitely. There is thus a finite number of definite places upon the construct not actually given by the representation $x = \phi(u)$, $y = \phi'(u)$ for any assignable finite value of u, these being the infinities of the integral; only in the case when $u = \infty$ is not an essential singularity of $\phi(u)$ can $u = \infty$ be assigned to these places and no other places. For example in the simple case $y = x$, the two places $(0, 0)$, (∞, ∞) of the construct are limiting points.

Consider now the algebraic function of x occurring in the fundamental equation
$$\phi(u + u_0) = A(x, x_0);$$
putting $h = \phi(0)$, $k = \phi'(0)$. To every ordinary place (x, y) belongs a definite value of u given by
$$u = \int_{(h, k)}^{(x, y)} \frac{dx}{y},$$
save for multiples of the periods of this integral, these periods being at most two in number since $\phi(u)$ is a single-valued meromorphic function; as these are also periods of $\phi(u + u_0)$ it follows that for every two places (x, y), (x_0, y_0), save the limiting places for which u becomes infinite, a definite value can be assigned to $A(x, x_0)$; and this is true however near (x, y), (x_0, y_0) may be to limiting places. Consider a limiting place; near it write, as usual, $x = a + t^m$, $y = b + P(t)$; either the function $A(x, x_0)$ is expressible by integral powers of t or not; if not, a circuit of t about $t = 0$ alters the value of the function without altering the place, contrary to hypothesis.

Thus the function $A(x, x_0)$ has a definite value even at the exceptional places, and is therefore a rational function of (x, y), and therefore also of (x_0, y_0). Thus we have
$$\phi(u + u_0) = R(x, y;\ x_0, y_0) = R[\phi(u), \phi'(u);\ \phi(u_0), \phi'(u_0)],$$

and from this $\phi'(u+u_0)$ is expressible in a similar form. These equations however are those of a birational transformation of the construct into itself involving an arbitrary parameter; we know that such a transformation is impossible unless the construct is of class unity or zero; and the proof of this result is entirely of an elementary character, depending on the existence in all cases when the class is greater than unity of a finite number of exceptional places upon the construct—as for instance the inflexions of a plane quartic curve (see Weierstrass, *Werke*, II. p. 235, and the writer's *Abelian Functions*, pp. 44 and 653).

When the construct is of class unity $\phi(u)$, $\phi'(u)$ can be expressed rationally by a pair of elliptic functions, and therefore by the elliptic functions $\wp(u)$, $\wp'(u)$, and conversely. In such case u is expressible in a form

$$u=\int_{\xi}^{\infty}\frac{d\xi}{\sqrt{4\xi^3-g_2\xi-g_3}},$$

and is without infinities, or the limiting places spoken of above are absent.

When the construct is of class zero, $x=\phi(u)$, $y=\phi'(u)$ can be expressed rationally by a parameter t, itself a rational function of x and y; so transformed let

$$u=\int R(t)\,dt,$$

and suppose t so chosen that $t=\infty$ does not make u infinite. In the neighbourhood of any finite value of t, other than the infinities of the integral, we have a form

$$u=A+A_1(t-a)+\ldots,$$

and, as t is a single-valued function of u, the coefficient A_1 cannot be zero; thus $\dfrac{du}{dt}$ is not zero for a finite value of t, other than those making the integral infinite; clearly $R(t)$ is not zero for finite values of t which do make the integral infinite. Hence $R(t)$ vanishes for no finite values of t, and $1/R(t)$ is a polynomial. Near $\xi=t^{-1}=0$,

$$u=-\int\frac{1}{\xi^2}R\left(\frac{1}{\xi}\right)d\xi$$

is not infinite, by hypothesis, and as before $t^2R(t)$ is not zero for $t=\infty$. Thus on the whole $1/R(t)$ is a polynomial of the second order. Putting then

$$u=\int\frac{dt}{A+Bt+Ct^2},$$

(i) if $A=0$, $B=0$, then $Ct=-u^{-1}$, and x, y are rational functions of u; while u is a rational function of x, y;

(ii) if $A=0$, $B\neq 0$, then $u=-\dfrac{1}{B}\log(C+B/t)$, $t=B/(e^{-Bu}-C)$, and $\phi(u)$ is a rational function of e^{Bu};

(iii) if $B^2\neq 4AC$, then $(t-\alpha)/(t-\beta)=e^{C(\alpha-\beta)u}$, and $\phi(u)$ is a rational function of $e^{C(\alpha-\beta)u}$;

(iv) if $B^2=4AC$, then $u=-1/C(t-\alpha)$, and $\phi(u)$ is a rational function of t.

The theorem is then proved. We notice that the integral u may be of the first, second, or third kind; it is in the last case that the limiting places spoken of come into consideration, and the theorem has then connection with Picard's theorem as to the values possibly not taken by an integral function. That the foregoing proof is capable of reduction to more fundamental considerations of a general character is sufficiently obvious; it suggests however the need of detailed investigation of algebraic constructs in several variables, and in particular the question of the existence of exceptional curves distinguishing constructs not expressible by Abelian functions to generalise the exceptional Weierstrass points of an algebraic curve.

On Bismuth. By R. H. ADIE, M.A., Lecturer of St John's College.

[*Read* 18 May 1903.]

1. In a previous paper on the sulphates of bismuth (*Chem. Soc. Proc.*, Dec. 1899) the author described the somewhat unusual formation of two different salts when a basic oxide Bi_2O_3 is dissolved in and crystallised from concentrated sulphuric acid. The salts are represented by Bi_2O_3, $4SO_3$, H_2O which crystallises out above 170° and Bi_2O_3, $4SO_3$, $10H_2O$ which crystallises out below 170° from the same solution as it cools.

In addition, the salt represented by Bi_2O_3, $4SO_3$, $3H_2O$ crystallises out from acids of concentration represented by H_2SO_4, H_2O and H_2SO_4, $2H_2O$ and also the salt Bi_2O_3, $4SO_3$, $7H_2O$ from acids of concentrations between and including those represented by H_2SO_4, $3H_2O$ and H_2SO_4, $5H_2O$.

During the time occupied by this work the properties of the pure oxide appeared to show slight differences from those usually described as normal, among which may be noted that it appeared to be increasingly difficult to precipitate the subnitrate by dilution alone. This impression of variability was confirmed by Mr P. MacEwan from the pharmaceutical point of view. Hence it appeared to be worth while to try to find out if there were an unidentified constituent in the oxide purified in the usual way or in the metal derived from it.

2. An examination of the various atomic weight determinations of bismuth confirmed this possibility.

Thus Schneider in 1851 after purifying his bismuth by precipitation as basic nitrate obtained values ranging from 207·95 to 208·6 (mean = 208). This value was confirmed by Marignac in 1883. Classen in 1890 after purifying by repeated precipitation as basic chloride followed by electrolysis of a solution of the nitrate in nitric acid obtained values ranging from 208·797 to 209·067 with a probable mean of 208·90, nearly one unit above

Schneider's value. Classen attributed the difference to the presence of lead in Schneider's pure metal and his method of purification was specially directed towards its removal.

Schneider in 1894 repeated his former work, without purification by electrolysis and confirmed the value he had previously obtained. He pointed out that the presence of lead would raise and not lower the atomic weight, and that it would require 0·25 per cent. of lead to raise the atomic weight by 0·17.

3. The determination of the equivalent of bismuth thus appeared to present a method of finding out at first whether any separation into different constituents could be effected in bismuth.

In the analysis of the sulphates the ratio Bi : SO_3 had been determined, and this was used to calculate an equivalent, with the following results:—

Salt	Equivalent ($SO_3=80$)	Atomic weight
Bi_2O_3, $4SO_3$, H_2O	69·4, 69·2	208·3, 207·5
,, ,, ,,	69·7, 68·7	209·1, 206·2
,, (recrystallised)	69·6	208·8
Bi_2O_3, $4SO_3$, $10H_2O$	68·8	206·4
,, (recrystallised)	69·7	209·1

The values for bismuth here given were obtained by reducing the sulphates by means of potassium cyanide in a porcelain crucible, and were compared with those obtained by oxidising the metal, similarly prepared, by means of nitric acid in a platinum crucible.

Metal from salt	Atomic weight
Bi_2O_3, $4SO_3$, $3H_2O$	208·2, 207·2
Bi_2O_3, $4SO_3$, $7H_2O$ (2nd crystallisation)	208·2
,, ,, ,, (3rd crystallisation)	208·5

4. The differences shown by these numbers were not consistent enough to be useful in deciding whether any separation had been effected, but it was noted that recrystallisation always tended to increase the atomic weight, and that a platinum crucible generally gave higher values than porcelain. In two cases the values 205·5, 205·7 in porcelain corresponded with 208·2, 208·7 in platinum.

5. It seemed advisable to localise the cause of this variation before proceeding further. About 2 gm. of pure bismuth oxide (B.P.) was fused alone in a porcelain crucible and it gained about 0·6 mg. After fusion, a residue of silica was left when the oxide

was dissolved in nitric acid. There was also a gain of 0·8 mg. when the same weight of the oxide was heated with silica and sodium silicate, but there was a loss on heating the oxide with sodium metaphosphate and with borax.

This peculiarity of behaviour in presence of silica only, suggested three explanations: (i) that the equivalent of bismuth was lowered by the presence of a substance which is only slowly oxidised when in the form of silicate (formed from the crucible), (ii) that the bismuth in the form of silicate absorbed more oxygen than corresponds with the oxide Bi_2O_3, (iii) that the gases from the flame diffusing through the platinum reduce some of the oxide. The metal so formed would alloy with the platinum and so raise the equivalent.

It was noted that the platinum became brittle and that a smell suggesting phosphoretted or siliciuretted hydrogen was developed on heating in platinum.

To test these hypotheses other methods of purifying bismuth were now examined.

6. The pure (B.P.) oxide was dissolved in pure nitric acid, and precipitated in four portions by water, followed by the addition of caustic potash. In one case 110 gm. of the oxide were dissolved in 300 c.c. of nitric acid (1·36) filtered through glass wool and precipitated by the successive addition of (i) 5 litres of distilled water, (ii) 20 gm. of caustic potash, (iii) 40 gm. of caustic potash, (iv) 40 gm. of caustic potash. Four fractions were obtained which were carefully washed and air-dried and were found to weigh 38 gm., 18 gm., 34 gm. and 19 gm. respectively. The first three of these fractions were white and crystalline, the fourth was yellowish and gelatinous.

At this stage a method of further purification was tried, based on the volatility of the chloride of bismuth, viz. the distillation of the oxide in a current of chlorine, by which lead chloride at any rate would be separated and left behind. A fluffy, light, non-volatile residue was left, but the greater part distilled over.

7. In order to combine the two processes as advantageously as possible, the four fractions precipitated as above from the nitrate solution were separately distilled in chlorine with the following results:

	Weight of precipitate taken	$BiCl_3$ distilled	Residue
(i)	38 gm.	37·6 gm.	0·05 gm.
(ii)	18 gm.	17·1 gm.	0·03 gm.
(iii)	34 gm.	29·3 gm.	1·26 gm.
(iv)	19 gm.	9·0 gm.	0·10 gm.

The non-volatile residue appeared to be concentrated in the fractions of the precipitate which came down last, and this observation was checked by distillation of four fractions precipitated by ammonia instead of potash. Thus in one case, the third fraction weighing 46 gm. yielded 0·01 gm., and the fourth fraction weighing 15·9 gm. yielded 0·36 gm. of residue.

The sulphates separating from concentrated sulphuric acid were also distilled. In one case the sulphates separating out from sulphuric acid saturated with bismuth oxide at 240° and allowed to cool (i) from 240° to 170°, (ii) from 170° to 100°, (iii) from 100° to 20°, were in the approximate ratio of 8 : 1 : 24. These fractions were dissolved in nitric acid, precipitated by ammonia and distilled in chlorine, but as they left residues this method was abandoned.

Lastly the beautifully crystalline sulphate Bi_2O_3, $4SO_3$, $7H_2O$ twice recrystallised, was treated as above and distilled in chlorine, but left a small residue*.

8. The method finally adopted was as follows:—

150 gm. of pure (B.P.) bismuth oxide were dissolved in 400 c.c. of pure redistilled nitric acid (1·42); to the clear filtered solution were added about 5 litres of distilled water and 300 c.c. of ammonia (·880). The precipitate was collected on linen, washed, and dried ($= R_1$), and represented nearly the first three fractions (i), (ii), and (iii) above; it contained about two-thirds of the bismuth taken. The filtrate was then rendered alkaline by ammonia, requiring about 100 c.c., and the precipitate ($= R_4$) was filtered, washed, and dried in the same way and contained about one-third of the bismuth taken.

The dried precipitates were then placed in one-half of a long wide combustion tube (about 1 metre long and of 2 cm. bore) with a loose plug of glass wool in the middle, after removing any loose fibres of glass. The tube was fitted with corks and single tubes, one of which was connected with an aspirating bottle, and the other with sulphuric acid driers. The tube was then gently heated in a combustion furnace, and a gentle current of air sucked through. Steam and nitrous fumes were given off and the bismuth oxide was left dry, porous and completely oxidised. When all nitrous fumes had been given off the temperature was raised, a chlorine Kipp attached to the sulphuric acid drier and a tube ending in a down draught to the other end. The current of chlorine was regulated so that nearly all the bismuth chloride condensed in the empty half of the tube which was not heated,

* Mr K. C. Browning also showed that it was impossible to obtain bismuth oxide free from this residue, by precipitation as sulphide.

and the gas was passed until no further trace of chloride distilled over.

On cooling, the tube was cut on both sides of the glass wool plug and the products separated.

By this method 50 gm. of the precipitate could be easily handled without any leakage of chlorine into the room.

9. The first fraction, R_1, yielded a colourless chloride and left very little residue, also colourless, the last fraction, R_4, yielded a yellow chloride and a bulky brownish residue.

The bismuth chloride from R_1 was found to leave no residue on redistillation in chlorine. It was dissolved in hydrochloric acid, precipitated by water to free it from any traces of iron, washed, dried, and reduced to the metal by fusion with potassium cyanide.

This pure bismuth showed no traces of impurity in quantity likely to affect its properties either on chemical analysis or in its spectrum. The oxide prepared from it was lemon-yellow, much less fusible than before purification, did not become coloured red so rapidly and did not change in weight on prolonged heating; it had also little or no action on a porcelain crucible in which it was fused.

The metal thus obtained was reserved for the atomic weight determination.

10. It appeared likely that any foreign element would be found concentrated in the residue. This resisted the common reagents except that it gave off hydrochloric acid when dropped into concentrated sulphuric acid, and was partly dissolved by caustic alkali.

On reduction by fusion with potassium cyanide a dark grey metallic powder was obtained, with a few small metallic beads if all traces of bismuth had not been distilled off.

When reduced in a current of hydrogen, hydrochloric acid and steam were given off but only in small quantity, thus in one case the residue gave on analysis

Chlorine	1·96
Oxygen	4·57
Residue	93·47
	100·00

It hardly seemed likely that this substance would be a definite compound, so some of its physical constants were determined.

Thus it was found to have a density of 4·78 gm. per c.c. and a specific heat of 0·1, while these constants for bismuth are 9·8 gm. per c.c. and 0·03 respectively. Both of these values point to a mixture.

After reduction the product was slightly soluble in nitric acid, giving a yellow solution which did not contain iron, uranium, gold, nor any recognised substance. This yellow substance was soluble in fused potassium cyanide, from which it could only be precipitated electrolytically. The mass of yellow substance was very small and was reserved for further investigation.

The residue was found, on testing, to be free from lead, tellurium and antimony, and was submitted to the spectroscope. Its spectrum showed the presence of silicon in large quantity, with traces of silver, copper, manganese, bismuth, and lead.

The proportion of silica in the residue was now determined by covering it with ammonia in a platinum basin and saturating it with hydrofluoric acid prepared by the action of sulphuric acid on pure ammonium fluoride; the liquid was then evaporated and ignited.

0·0986 gm. lost 0·0743 gm. which indicated that about 75 per cent. of the residue consisted of silica.

11. This tenacious adherence of silica, which dissolves with the oxide of bismuth, is precipitated with the basic salts and sulphide of bismuth, and is reduced and dissolved by bismuth in fused potassium cyanide, does not appear to have been noted, at any rate in connection with the atomic weight determination. The question now arises as to how far the presence of this silicon will affect the atomic weight. If the metallic bismuth contains only 0·07 per cent. the atomic weight will be lowered by 1 unit, and it was found (§ 7) that when only one-third of the bismuth was precipitated as basic nitrate, it left the smallest amount of residue and the precipitate contained nearly 0·15 per cent. of silica. This percentage was considerably increased by precipitating more of the basic salt.

It will thus be seen that purification by precipitation still leaves enough silica to account for the discrepancy between Classen and Schneider's numbers, and presumably the silica would not be reduced, and deposited, on electrolysis of the nitrate.

12. Some other evidence can be cited in favour of the view here brought forward, that the silica becomes reduced in presence of the fused bismuth and the resulting silicon or silicide of bismuth becomes oxidised, and follows the bismuth into solution.

Bismuth oxide when reduced by potassium cyanide always makes a bead and some black powder. This black powder becomes relatively more plentiful as the proportion of silica in the oxide and the residue left on chlorination increase, *e.g.* in R_4.

Bismuth nitrate prepared by dissolving bismuth in nitric acid is grey, and deposits a fine grey powder, which disappears on evaporation and concentration.

13. The above work leads to the conclusions

(i) Bismuth and bismuth oxide after passing through the usual methods of purification still contain silicon and silica respectively.

(ii) Precipitation from acid solution in the form of basic salts or sulphide does not free bismuth from silicon, though the amount is reduced.

(iii) Distillation of the oxide in chlorine does remove the silica.

(iv) Since the presence of silicon increases the absorption of oxygen, Classen's number for the atomic weight of bismuth is probably more correct than Schneider's. This is further supported by Dumas' (1859) value of 210, which has hitherto been rejected, based on the ratio of Cl : Bi in bismuth chloride.

I have much pleasure in acknowledging my indebtedness to Mr K. C. Browning who kindly undertook the qualitative work and most of the chlorination, to Mr J. W. Capstick for the specific heat determinations, and to Mr H. Ramage for the spectrum analysis.

NOTE. A preliminary determination of the atomic weight of bismuth using the pure metal gave the value about 208·8 (O = 16).

Note on the preparation of chlorine.

Chlorine is most conveniently prepared for a continuous supply by the action in a Kipp of strong hydrochloric acid on broken pieces of a mixture of 5 parts of roughly powdered potassium permanganate with 3 parts of plaster of Paris. The mixture is moistened, spread into a cake about $\frac{1}{2}$ in. thick on a piece of board and scored with cuts about $\frac{1}{2}$ in. apart. The permanganate in becoming reduced to a manganic salt liberates twice as much chlorine as an equal weight of bleaching powder, while the chlorine is free from oxides.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1902—1903.

ANNUAL GENERAL MEETING.

October 27th, 1902.

In the Optical Lecture Room.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected officers for the ensuing year:

President:

Dr H. F. Baker.

Vice-Presidents:

Professor Macalister.
Mr A. C. Seward.
Mr G. T. Walker.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Mr S. Skinner.
Mr H. M. Macdonald.

Other Members of the Council:

Professor Liveing.
Mr F. Darwin.
Dr E. W. Hobson.
Mr A. Hutchinson.
Mr C. T. R. Wilson.
Professor Thomson.
Mr H. J. H. Fenton.
Mr A. Berry.
The Master of Pembroke College.
Mr J. E. Marr.
Mr D. Sharp.
Mr J. Larmor.

The names of the Benefactors were recited.

The following was elected an Associate of the Society:

W. A. Cunnington, Christ's College.

The following Communications were made:

1. A case of extreme visceral dislocation: with remarks on the functional interpretation of the agminated glands of the Intestine. By E. BARCLAY-SMITH, M.D., Downing College.

2. Notes on the Genus Liparis. By J. J. Lister, M.A., St John's College.

3. Notes on the Anatomy of Macrozamia heteromera. By Miss A. Robertson. (Communicated by Mr A. C. Seward.)

4. Further experiments on radio-activity from rain. By C. T. R. Wilson, M.A., Sidney Sussex College.

November 10*th*, 1902.

In the Cavendish Laboratory.

Dr Baker, President, in the Chair.

The following Communications were made:

1. On a Vibration Magnetometer, and on the Ball-ended Magnets of Robison. By G. F. C. Searle, M.A., Peterhouse.

2. On cavitation in liquids, and its occurrence in lubrication. By S. Skinner, M.A., Christ's College.

3. On the Coral Reefs of Pemba Island and British East Africa. By C. Crossland, B.A., Clare College. (Communicated by Mr Stanley Gardiner.)

4. On the theory of Aggregates. By A. N. Whitehead, M.A., Trinity College.

November 24*th*, 1902.

In the Optical Lecture Room.

Dr Baker, President, in the Chair.

The following were elected Fellows of the Society:

P. V. Bevan, B.A., Trinity College.
W. F. Cooper, B.A., Clare College.
A. R. Hinks, M.A., Trinity College.
H. O. Jones, B.A., Clare College.
C. F. Mott, B.A., Trinity College.
C. S. Myers, M.D., Gonville and Caius College.

The following were elected Honorary Members of the Society:

Professor Isaac Bayley Balfour.
Professor Antoine Henri Becquerel.
Professor Emil Fischer.
Professor Richard Heymons.
Professor Jakob Heinrich van't Hoff.
Professor Camille Jordan.
Professor Henry Fairfield Osborn.
Professor Wilhelm Konrad von Röntgen.
Professor Corrado Segre.
Professor Hugo de Vries.

The following Communications were made:

1. The Origin of the Thoroughbred Horse. By W. RIDGEWAY, M.A., Gonville and Caius College, Disney Professor of Archæology. (Communicated by Mr A. E. Shipley.)

2. Note on the resolution of Compound characters by Cross-breeding. By W. BATESON, M.A., St John's College.

3. Notes on rearing the later stages of Echinoid Larvae. By L. DONCASTER, B.A., King's College.

4. (1) On the Galois theory of differential equations. (2) On the structure of continuous groups. By H. F. BAKER, Sc.D., St John's College.

5. On spontaneous ionization in air at different temperatures and pressures. By J. PATTERSON, B.A., Emmanuel College.

6. Note on the behaviour of a Potassium Amalgam Cathode in a vacuum tube. By THEODORE LYMAN.

January 19*th*, 1903.

In the Optical Lecture Room.

MR A. C. SEWARD, VICE-PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

E. S. Montagu, B.A., Trinity College.

The following Communications were made:

1. On the invariant factors of a determinant. By H. F. BAKER, Sc.D., St John's College.

2. On the variation with wave-length of the double refraction in strained glass. (Second paper.) By L. N. G. FILON, B.A., King's College.

3. On the Alimentary Canal of the Mosquito. By A. E. SHIPLEY, M.A., Christ's College.

4. On Integral Functions. (Second paper.) By E. W. BARNES, M.A., Trinity College.

5. On the theory of shadows. By H. M. MACDONALD, M.A., Clare College.

February 2nd, 1903.

In the Cavendish Laboratory.

DR BAKER, PRESIDENT, IN THE CHAIR.

The President proposed from the Chair,

"That the Cambridge Philosophical Society desires to express its sense of the great loss sustained by the University and the Society in the death of SIR GEORGE GABRIEL STOKES, to whom the Society was bound by so many ties of obligation and reverence."

This was seconded by Professor Thomson, and was carried unanimously.

The President then proposed,

"That the Society do now adjourn as a mark of respect to his memory."

The Society then adjourned.

February 16th, 1903.

In the Cavendish Laboratory.

MR A. C. SEWARD, VICE-PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

G. B. Mathews, M.A., St John's College.

The following Communications were made:

1. On the dynamics of the Electric field. By J. J. THOMSON, M.A., Trinity College, Cavendish Professor of Experimental Physics.

2. Rust-fungi and the Mycoplasm theory. By H. MARSHALL WARD, Sc.D., Sidney Sussex College, Professor of Botany.

3. On Radio-active Snow. By C. T. R. WILSON, M.A., Sidney Sussex College.

4. Note on the Slipperiness of Ice. By S. SKINNER, M.A., Christ's College.

5. On the Rise of a Spinning Top. By E. G. GALLOP, M.A., Gonville and Caius College.

6. On Automorphic Functions and the general theory of Algebraic Curves. By H. W. RICHMOND, M.A., King's College.

March 2nd, 1903.

In the Chemical Laboratory.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. On the probable presence in the Sun of the newly discovered gases of the Earth's atmosphere. By G. D. LIVEING, M.A., St John's College, Professor of Chemistry.

2. A Synthesis of Carboxy derivatives of Pyridine. By W. J. SELL, M.A., Christ's College, and F. W. DOOTSON, M.A., Trinity Hall.

3. Experiments illustrating new reactions for the identification of Urea and of Primary Amines. By H. J. H. FENTON, M.A., Christ's College.

4. (1) A rapid method of estimating sugars. (2) Selection of seeds by chemical methods. By T. B. WOOD, M.A., Gonville and Caius College, and R. A. BERRY.

5. Methods of preparation of Osones. By R. S. MORRELL, M.A., Gonville and Caius College.

6. Note on the stereochemistry of benzene. By H. O. JONES, M.A., Clare College, and J. KEWLEY, B.A., King's College.

7. A method of detecting Nickel and Cobalt in presence of each other. By F. W. DOOTSON, M.A., Trinity Hall.

8. On the Joule-Thomson effect. By P. V. BEVAN, M.A., Trinity College.

9. On a sensitive gold-leaf electrometer. By C. T. R. WILSON, M.A., Sidney Sussex College.

10. A new mineral from the Binnenthal. By R. H. SOLLY, M.A., Downing College.

May 4th, 1903.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

J. E. Purvis, M.A., St John's College.

The following Communications were made:

1. On Mendelian heredity of three characters allelomorphic to each other. By W. BATESON, M.A., St John's College.

2. On the Diathermancy of Antimonite. By A. HUTCHINSON, M.A., Pembroke College.

3. On the Potential difference between the terminals of a Vacuum tube. By W. A. D. RUDGE, M.A., St John's College.

4. The determination of curves satisfying given conditions. By H. BATEMAN. (Communicated by Mr H. M. Macdonald.)

5. On the Existence of a Radio-active Gas in the Cambridge tap-water. By J. J. THOMSON, M.A., Trinity College, Cavendish Professor of Experimental Physics.

6. On a continuous spectrum. By T. H. HAVELOCK. (Communicated by Professor Thomson.)

7. On the Thomson-effect in alloys of Bismuth and Tin. By S. C. LAWS. (Communicated by Professor Thomson.)

8. A preliminary account of an Investigation of the effect of Temperature on the Ionisation of gases by Röntgen Rays. By R. K. McCLUNG. (Communicated by Professor Thomson.)

May 18*th*, 1903.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

T. H. Middleton, M.A., St John's College, Professor of Agriculture.

The following Communications were made:

1. A Coleopterous insect embedded in the wall of the human intestine. By D. SHARP, M.A.

2. Exhibition of a rare Parasite. By A. E. SHIPLEY, M.A., Christ's College.

3. On the influence of Electrons on Colloidal solutions. By W. B. HARDY, M.A., Gonville and Caius College.

4. On Bismuth. By R. H. ADIE, M.A., Trinity College.

5. (1) On the influence of great dilution on the absorption spectra of highly concentrated solutions of the nitrates and chlorides of Didymium and Erbium. (2) On a method of estimating the amounts of the oxides of Didymium and Erbium by means of the absorption bands of their solutions. By J. E PURVIS, M.A., St John's College.

6. A lecture experiment to illustrate the rotation of a magnetic pole around a straight current. By P. V. BEVAN, M.A., Trinity College.

7. Irreversible simultaneous linear Reactions. By H. O. JONES, M.A., Clare College, and O. W. RICHARDSON, B.A., Trinity College.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Note on the Pulverization of "Nickel-grains" in Fuming Nitric Acid. By Dr W. A. HOLLIS, Trinity College.

[*Read* 26 October 1903.]

Commercial nickel is obtainable in three forms; namely, in rolled sheets, as wire, or again as polymorphous lumps, named technically "nickel-grains." The metal in the two former states is after purification submitted to various processes whereby it is rolled or drawn, as the case may be, into the form required; and it is placed upon the market with the well-known silvery sheen upon its surface. On the other hand nickel-grains are dull, dark gray, nodular masses of metal, offering unmistakable evidence of their crystalline character however, in an occasional solid angle which juts out beyond the roughened surface. The metal is obtained in this state directly from the smelting furnace. The grains vary in weight, from a few grains to a drachm or more each. Messrs Johnson, Matthey and Co., from whom I obtained the specimens I am here describing, inform me that they may be considered to consist of about 98 °/₀ of pure nickel. The chief impurities are cobalt and iron.

In common with the two other highly magnetisable metals, iron and cobalt, nickel in the presence of strong HNO_3, Aq, occasionally exhibits the phenomenon of "passivity." According to Morley and Muir (Watt's *Dict. of Chemistry*, 1892), Saint-Edme (*Compt. Rend.* 106, 1079) noted that commercial sheet nickel was passive in ordinary nitric acid. I can to some extent confirm and supplement this statement. Sheet nickel is only slightly attacked at ordinary temperatures by either strong (S.G. 1·42) or weaker

acids. After a while, however, the surface of the sheet loses its brilliancy, becomes etched in fact, and occasional bubbles form, especially upon the under surface of the metal, which show that some slight reaction takes place between the acid and the metal; particularly is this the case when the strong acid is used. As far as my experience carries me nickel wire shows the passive condition more markedly than even the sheet nickel does, whenever the former is immersed in the acid.

It is, however, with the behaviour of nickel-grains under similar conditions that this note is mainly concerned. If a few drops of fuming nitric acid are placed in a test-tube and a nickel-grain is dropped into it, one of three results follows. (1) Immediately, or after a short interval, violent ebullition occurs, with a considerable rise in the temperature of the mixture, and the copious disengagement of reddish brown vapours. In this case the ebullition continues and the metal is irregularly eaten away, drilled it may be into holes by the action of the acid, until a solid mass of green crystalline nitrate has replaced the liquid surrounding the metal core. (2) The nickel-grain upon immersion in the acid assumes the passive condition, and no chemical reaction is observable. (3) *A critical state of unstable equilibrium is set up apparently in the metal itself.* As a result a slow disintegration of the metal ensues and a powdery metallic precipitate falls to the bottom of the tube. At the same time some chemical reaction takes place,—in contrast to the physical changes above mentioned,—gaseous bubbles are more or less rapidly formed and the liquid assumes the green tint of the dissolved nitrate. If the chemical reaction becomes too vigorous, a sudden change to the ebullition stage takes place and (1) is initiated. In rare cases after a while (3) is succeeded by the passive state (2), and the experiment ends unsatisfactorily. As I was anxious to prolong the unstable stage so as to obtain more decided results than had been possible heretofore, some experiments were made with that object in view. It was eventually found that when a thin layer of fuming nitric acid was spread over a glass dish and a few nickel-grains so placed that their lowest surfaces were alone in contact with the acid, the greater number of them assumed state (3), and a considerable quantity of the powder was obtainable. Any objectionable grains, which refused to conform, if I may so put it, to the rules of the laboratory, were at once removed and the others allowed to remain so long as any reaction took place.

The powder so obtained is in colour a grayish white, with a dull metallic lustre. It is not pyrophorous, but is strongly magnetic. Its structure is presumably crystalline. At all events many of the grains when magnified are seen to be somewhat

angular in shape[1]. The powder dissolves in fuming nitric acid to some extent. Here again certain grains apparently assume the passive state and are not affected by the acid. The solution acquires the well-known green colour of nickel nitrate. The powder therefore so far behaves as the massive metal under like conditions.

The ductile and tenacious nickel observable in the wire and sheet of commerce suggests a fibrous structure. On the other hand nickel-grains, apart from other considerations, in their mere brittleness and in the appearance of the fractured surfaces indicate a crystalline texture. Now ductile fibrous nickel can be made into a permanent magnet, specially is this true of the wire; and in spite of its sheen the wire shows "passivity" in the presence of fuming HNO_3. On the other hand nickel-grains, although they are magnetic inasmuch as they attract the poles of a compass needle, do not show sufficient polarity in my experience to repel them. Moreover they do not constantly show passivity when introduced into fuming nitric acid, even when visibly coated with the oxide. In my opinion the passive (2) and the critical (3) states of this metal are intimately associated with its molecular arrangement.

Since writing the preceding note "On the Pulverization of 'Nickel-grains' in fuming Nitric Acid," I have had an opportunity of examining, through the courtesy of the Managing Director, Mr Carl Langer, two samples of nickel prepared by the Mond Nickel Company. One of these samples consisted of a massive variety of the metal, termed "shot"; the other specimen was made up of three thin-walled tubes; to be described hereafter.

Mond Nickel shot. The metal in this case appeared as somewhat irregularly shaped balls, having a brilliant silvery lustre, and weighing about a drachm each. Upon a close examination of the surface of a shot the irregularities were found to be due to an ill-defined polyhedral shape with outcropping solid angles. The shot, I understand, contains Ni 99·5 °/$_\circ$[2]. A recent sample from Johnson, Matthey and Company, in the form of nickel-grains, yielded 99 °/$_\circ$ of the pure metal; this gives the Mond nickel the advantage of half per cent. as regards purity over the ordinary commercial nickel. To my mind it is almost incredible that the differences observable in the physical characters of the

[1] I have roughly measured the angles of various nickel-grains. They are approximately 72°, *90°*, *120°*, 145°. Other angles are observable but these in my experience are the commonest. The angles italicized are frequently met with.

[2] Mr Langer kindly forwarded me the following analysis of the shot:—Ni 99·50; Cu 0·03; Fe 0·29; S 0·02; Insol. 0·11.

two samples detailed elsewhere should be entirely due to this small excess of impurity in the nickel-grains.

Nickel shot when submitted to the action of a thin layer of fuming nitric acid in the manner already described in my former note proved to be less readily affected than the nickel-grains. However by prolonging the exposure of the shot to the acid for a period of more than a week I succeeded in obtaining definite evidence of pulverization. The *modus operandi* was as follows:—Two glass dishes were provided and the shot were submitted to the action of the acid in one of these vessels until a dark green discoloration of the liquid had ensued; they were then transferred to the other vessel and the process repeated. This operation was continued at intervals so long as it was deemed necessary. Meanwhile the green liquid containing a small quantity of a black powdery precipitate was freely diluted and the powder allowed to subside. After numerous washings the powder (or what was left of it) mixed with a few drops of water was placed by means of a pipette upon a piece of thin sheet nickel and allowed to dry. If the powder is permitted to dry in a glass vessel it adheres to the surface so closely that its subsequent removal is rendered difficult.

The powder so obtained is about the colour and consistency of lampblack. It is not pyrophorous, but is slightly hygroscopic. Owing to its granular fineness probably, it smears readily on paper. Its greasy or tarry nature is not diminished by soakage in methylic alcohol and subsequently in ether. Whilst the specific gravity of the powder is undoubtedly higher than water, thin detached films often float upon the surface, and render the process of washing the precipitate in my hands a wasteful and tedious ordeal. When nickel shot have been exposed to the action of the acid for some hours minute flakes of bright metallic lustre are found commingled with the black powdery precipitate. If the surface of a shot is examined after removal from the acid bath in such a case, many superficial erosions are visible, mostly mapped out by thin lines of black powder; here and there the surface layers of the metal are so extensively undermined that a shot not infrequently resembles a miniature onion with its outer coats partly removed. The thinned free edges of these scales readily break off; and form the flakes visible in the washings above referred to. They are highly magnetic. The source of the black adhesive powder cannot be so easily traced. That this substance represents in some way the cement that welds the thin concentric metallic layers of a shot into a homogeneous whole seems probable. I must however leave its complete analysis to more competent hands than my own.

Thin Plates. The second sample forwarded me by the Mond

Nickel Company consisted of three thin-walled nickel tubes about 9 mm. in diameter, 12 or 13 cm. in length and each weighing between 3 and 4 gm. Although an absolute purity was not claimed for it, the metal was stated to be practically pure and free from copper and cobalt. The tubes had a brilliant metallic sheen over their convex surface, and a dull frosted appearance within. A careful examination showed that whilst there were many minute indentations externally, the small blister-shaped prominences were entirely confined to the inner aspect of the tube. When split lengthwise and flattened out a tube was readily converted into a thin plate of nickel. The plate (*a*) so formed, although less ductile and tenacious, was practically of the same thickness (*i.e.* 0·1 mm. approximately) as the sample of thin sheet nickel (*b*) obtained from Johnson, Matthey and Company. The two served well for comparison. When subject to the action of nitric acid in the manner already described they were both found to be very resistant, especially (*b*). After prolonged exposure the metal plates were reduced to grilles, but the openwork tracery of grille (*a*) was very unlike that of (*b*).

Final observations. To the present time I have dealt almost exclusively with descriptions more or less imperfectly worded of the structural changes which take place in different samples of commercial nickel when they are exposed to the action of fuming nitric acid. I wish before concluding to draw attention to a few phenomena observed during this investigation which may assist in elucidating the physical processes whereby these changes are occasioned.

Grain-nickel is sometimes met with in the form of a flattened plate. If one of these is subjected to the prolonged action of nitric acid, the plate under suitable conditions, when reduced to a thickness of 0·75 mm. approximately, becomes so friable that it can be crushed to powder between the thumb and finger. The powder so formed and that obtained from nickel-grains under the conditions mentioned in my former note seem to me to be alike in every respect. Apparently the disintegrating agent, whatever it may be, although doubtless most active over the surface of the metal can penetrate its substance to the depth of at least 0·375 mm. In the case of the grain-nickel plate I am now considering, this great change in cohesiveness was effected without any visible alteration in its contour or aspect beyond the general shrinkage due to persistent acid reaction.

If we examine the grilles of Mond nickel and of sheet nickel respectively under a low power objective we cannot fail to observe their great dissimilarity. Whilst the Mond nickel is perforated with, it may be many, circular or nearly circular holes, the sheet nickel is riven by narrow fissures which cross each other at various

Fig. 1. Mond Nickel Tube after acid reaction, × 50.

Fig. 2. Mond Nickel Tube after acid reaction, × 50.

angles. Direct perforations are few and more or less oval in shape. In other words the lines of least resistance in the two plates lie presumably in the one case in planes at right angles to the surface and in the other in a plane parallel to the surface. This conjecture is however only partly true, for if we examine the etched surface of Mond tube-nickel before perforation has taken place (and this can be conveniently done in the accompanying photomicrographs), we shall find that in the Mond nickel lines of least resistance, mapped out by furrows, also lie in planes parallel to the surface. These intricate figures (Figs. 1 and 2), so highly suggestive of wave and vortex motion, are mainly met with in my experience on the outer or convex surface of a Mond tube. They appear to indicate that in this highly purified metal the perforations take place as a rule in one direction, namely from without inwards.

I am indebted to Mr Caush, an amateur photographer of Brighton, for the accompanying photomicrographs.

Note on the action of Radium rays on Mercurous Salts. By S. SKINNER, M.A., Christ's College.

[*Read* 26 October 1903.]

When making Clark cells it is well known that the mercurous sulphate in the depolarising paste rapidly changes from white to a dark colour if a cell is left exposed to light. This change also occurs when dry mercurous sulphate is exposed to light. Some years ago I investigated the conditions of this change with the view of deciding whether it was sufficiently extensive to cause such an alteration in the chemical nature of the cell that the electromotive force might be affected; and among the experiments I then made some of the salt was submitted to the action of the Becquerel rays from uranium, but the result was negative. The much greater radio-activity of radium salts caused me to think their action would shew itself in some way. This has recently been tried, and an action similar to that of light was obtained.

As the interest of these experiments lies in the comparison of the effects produced by various agents I venture now to put together the earlier experiments with those made recently. The white mercurous sulphate when exposed to sunlight turns at first light brown, and with continued exposure passes through stages of gradually increasing darkness until it becomes dark brown or black. It was found that this occurred when the salt was exposed in daylight (*a*) in open air, (*b*) in a vacuum, (*c*) in shellac varnish, (*d*) in linseed oil. (*e*) under water, (*f*) gummed on paper. These experiments seem to indicate that the presence of air was not essential.

To find out which of the rays in sunlight caused the change an arc-light spectrum was cast by a quartz lens and prism on paper coated with the salt. Here it was found that the action was caused by the violet and ultra-violet rays. To further test this a vacuous tube containing some of the salt was taken to a height of 4,500 feet and exposed to the action of sunlight, and it was found that the action was much more intense in comparison with that at low altitudes. These results shewed that ultra-violet light was the chief agent in producing the action.

Now the question arises as to what the action is. Is it a conversion of the mercurous salt into a mercuric with the separation of mercury? This view has been put forward, specially by Haga[1], who has offered evidence that most mercurous salts undergo this change.

[1] *Chemical Society's Journal Trans.*, 1896, p. 1673.

To test this some mercurous sulphate was covered, both in a watch-glass and in a sealed tube, with sulphuric acid and the darkening occurred. Some was also covered with dilute nitric acid when again the darkening was observed. These results do not seem consistent with the view that free mercury, or an oxide, or a sulphide is set free.

The salt was exposed in a vacuum tube to a stream of cathode rays, and in another experiment to the blue discharge in a slightly exhausted tube. In both cases a darkening was rapidly produced.

Exposure to Röntgen rays had no effect, but I now think that the X-ray tube which was used at that time was one which did not give out a sufficiently powerful stream of rays, nor was the experiment continued long enough.

Becquerel rays from uranium nitrate gave no result, but recently I have found that the rays from 10 milligrammes of radium bromide, mounted as usual under mica, produce a visible result in 24 hours, whilst in 3 or 4 days they produce a very marked darkening. The mercurous salt phosphoresces very slightly in the presence of the rays. Mercurous bromide also phosphoresces, but the darkening was not visible in the time given for the exposure. This salt however is much less sensitive to light.

What the chemical action is, is a difficult point to settle as the coloration is limited to such a small mass. The facts seem to prevent an acceptance of the simple view of the separation of mercury with the formation of a mercuric salt. Two views seem still possible (*a*) that a dark sub-salt is formed or (*b*) that a polymeric modification is produced.

In regard to the first view we may compare with this action that of radium rays on sodium chloride which M. and Mme Curie[1] have shewn is coloured blue just as it is with cathode rays or by the vapour of sodium. This has been interpreted by supposing the formation of a coloured sub-salt. In accordance with this view we should postulate the formation of a mercury sub-sulphate.

With regard to the second view the polymerising action of the radium rays has been observed in the case of the production of ozone from oxygen[2] and of red phosphorus[3] from the yellow modification.

One of these views probably explains this action on mercurous salts. The action, however, is so small that its effect on the electromotive force of Clark cells cannot be large even if it is extensive enough for detection.

[1] *Soc. Franc. Phys. Bull.*, 142, p. 6, 1900.
[2] Demarçay, *C. R.*, 129, p. 823, 1899.
[3] Becquerel, *C. R.*, 133, p. 709, 1901.

On Nutrition and Sex-determination in Man. By R. C. PUNNETT, M.A., Gonville and Caius College.

[*Read* 26 October 1903.]

Of the various factors which have from time to time been brought forward to account for the determination of sex, that of nutrition appears to have found most favour. That favourable nutritive conditions tend to produce females, and unfavourable conditions males is a hypothesis which has received much support from observation and experiment on numerous forms of life[1]. In the case of the human species, however, strong evidence for or against this view is yet a desideratum. The idea of the writer of this paper was to consider the population of a large city divided up into areas differing in poverty or wealth (a factor greatly affecting nutrition), and to estimate the relative proportion of male and female infants in each of them. That out of the enquiry certain other points not devoid of interest have arisen, it is hoped will appear in what follows.

Though it is a matter of common knowledge that the food of a large proportion of the working classes is not always sufficient in quantity, more exact information has been difficult to come by. Quite recently, however, Rowntree[2] has published an invaluable work on the subject, where he deals very thoroughly with this question. The main conclusions at which he arrived may be best presented in his own words. He has in this case divided his population into three classes, *i.e.* (1) the servant-keeping class, (2) the artisan class where the family earnings are over 26/- a week, (3) the labouring class earning less than 26/- a week per family. The standard diet with which he compares the diets of these three classes contains 125 grams of proteids, and has a total energy value of 3500 calories. As the results of his enquiries Rowntree[3] concludes that :—

(1) The diet of the servant-keeping class is, upon the whole, in excess of that required for the maintenance of health.

(2) It is probably safe to assume that, in the case of average families of the artisan class, the food supply is adequate, although it is clear that in many cases it can only continue so by abstention from wasteful expenditure upon drink, &c.

[1] Cf. Geddes and Thomson, *Evolution of Sex*, 1st Edition, pp. 41—47.
[2] B. S. Rowntree, *Poverty, A Study of Town Life*, 2nd Edition, 1902.
[3] *loc. cit.* p. 259.

(3) On the other hand, the labouring classes, upon whom the bulk of the muscular work falls, and who form so large a proportion of the industrial population, are *seriously underfed.* The average energy value of the diet in the fourteen families selected for study being no less than 23 % below standard, while the average deficiency in proteids amounts to as much as 29 %. The enquiry, it is true, has shown that the money available for the purchase of food is not always spent in the most economical way, but the fact remains that unless an unreasonably stringent diet be adopted, the means to purchase a sufficient supply of nourishing food are not possessed by the labourers and their families.

Rowntree's investigations were carried out upon the city of York, but, as he himself points out (*loc. cit.* p. 299), there is no reason for considering that the conditions obtaining in London differ to any appreciable extent from those in the more northern city. Indeed, in either case, the proportion of the population living in poverty, and therefore seriously underfed, was about 30 %. It will be assumed, therefore, that the conclusions arrived at by Rowntree for York apply also to London.

In the official Census of the County of London for 1901, the metropolis is divided into 29 boroughs. Excluding the City of London, with its peculiar conditions, we are left with 28 boroughs, having a total population of just over 4,500,000. For each of these boroughs the proportion of male to female infants under one year has been reckoned. The servant-keeping capacity (*Census*, p. 154) has been taken as the guide to the relative wealth of the different boroughs, it being assumed that when a family is able to do so it will keep a servant. In Table I the 28 boroughs are arranged in order of wealth (on the servant-keeping capacity basis), whilst at the same time the relative proportion of male and female infants is given. At first sight there would appear to be no relation whatever between the two columns. This is probably because we are dealing with a long series and with small differences. Accordingly the 28 boroughs have been arranged in three groups: *A*, with a percentage of indoor servants less than 15 (per 100 families); *B*, with a percentage of over 15 but less than 30; and *C*, with a percentage of over 30. The relative proportion of male and female infants then works out to—

99·5 ♂s per 100 ♀s: for group *A*.
100·7 „ „ „ „ „ „ *B*.
102·2 „ „ „ „ „ „ *C*.

Hence in group *A* where there is a larger proportion of people living in poverty, and therefore under less favourable nutritive conditions, the relative number of ♀ infants is greater than for group *B*, and still greater than for group *C*. On the other hand

Table I.

	Percentage of indoor servants (per 100 families)	Infants ♂s per 100 ♀s	Infants ♂s per 100 ♀s (average per group)		No. of infants under 1 year per 1000 of population
Shoreditch	5·7	96·5			
Bethnal Green ...	5·8	100·8			
Bermondsey	6·6	101·3			
Southwark	7·8	92·3			
Poplar	8·1	102·0	Group A.	99·5	28·3
Finsbury	8·2	98·3	No. of ♂s	20,309	
Stepney	8·8	100·5	No. of ♀s	20,413	
Battersea	13·1	103·0	Total infants	40,722	
Woolwich	14·4	100·0			
Camberwell	15·3	100·5			
Deptford	15·4	103·8			
Islington	15·5	102·7			
St Pancras	16·9	96·8			
Hackney	17·9	97·3			
Lambeth	18·1	99·9	Group B.	100·7	24·2
Fulham	18·6	102·3	No. of ♂s	23,327	
Hammersmith ...	19·3	101·1	No. of ♀s	23,159	
Holborn	22·3	103·2	Total infants	46,486	
Greenwich.........	24·5	107·2			
Stoke Newington	27·8	98·8			
Wandsworth......	35·2	106·9			
Lewisham	36·2	103·0			
Paddington	50·2	99·4			
St Marylebone ...	51·4	97·4	Group C.	102·2	19·0
Chelsea	55·2	92·9	No. of ♂s	11,038	
Westminster......	65·8	103·2	No. of ♀s	10,803	
Kensington	80·0	101·5	Total infants	21,841	
Hampstead	81·4	108·1			
From *Burke's Peerage* (5225 children)				107·6	

in group *C* where the nutrition hypothesis of sex would lead us to expect the largest proportion of ♀s we find, on the contrary, the greatest proportion of ♂s. In other words we arrive at the conclusion that the more favourable the conditions of nutrition, the greater the proportion of ♂s produced. This conclusion is accentuated by the following piece of evidence. Records of 5225 births were made (for purposes referred to below) from *Burke's Peerage.* Of these 2708 were ♂s and 2517 ♀s, thus giving a proportion of 107·6 ♂s per 100 ♀s. And here one is justified in assuming the very best conditions as regards nourishment:—*i.e.* conditions considerably more favourable than in group *C* above. This evidence, therefore, considerably strengthens the conclusion arrived at just above. Before, however, such a conclusion can be regarded as having any value several points must be taken into consideration.

(1) *Comparative mortality in ♂ and ♀ infants.*

It is well known that during the first year or two of their existence ♂ infants are subject to a higher death-rate than ♀s. Now in the statistics dealt with here, there is given, not the birth-rate, nor the total number of infants born in a given year, but *the number of infants born in a given year, and which remain alive at the end of it.* The average age, therefore, of these infants will be about six months; and in order to obtain the relative numbers of the sexes at birth, the greater mortality among the ♂s during this period of six months must be taken into account. Reference to Tatham's *English Life-Table* (1897), shews that of 1,000,000 infants of each sex, 161,036 ♂s and 131,126 ♀s die during the first year, *i.e.* 29,910 more ♂ infants die than ♀s during this period. Hence for six months 14,955 more ♂s than ♀s will die, which is approximately 1·5 %. Consequently we may roughly emend the figures previously arrived at by adding 1·5 % to the percentage of males. In the case of the data from *Burke's Peerage* no such emendation is required.

TABLE II.

	Total no. of infants considered	No. of ♂ s per 100 ♀ s
Group *A*	40,722	101·0
,, *B*	46,486	102·2
,, *C*	21,841	103·7
From Burke ...	5,225	107·6

(2) *Infant mortality in general.*

The infant mortality in poorer districts is greater than that obtaining in wealthier. To gain some idea of the difference found on this head among the three groups, *A*, *B*, and *C*, the number of infants under 1 year has been in each case compared with those of between 1 and 2 years. The difference in each case, expressed in percentage of the total, will give the mortality in each group of infants between the *average* ages of 6 and 18 months.

TABLE III.

	No. of infants under 1 year (average age 6 months)	No. of infants of from 1—2 years (average age 18 months)	Difference (= mortality)	% Mortality
Group *A* ...	40,712	35,616	5096	12·5[1]
„ *B* ...	46,486	41,363	5123	11·0
„ *C* ...	21,841	20,190	1651	7·5

From this table it appears that the mortality during early infant life is very much higher in the poorer districts than in the better off ones. Should this higher mortality be more effective among ♂ infants, and it seems reasonable to suppose that this might well be so, it will tend to reduce the relative proportion of ♂s as compared with ♀s in groups *A* and *B*, whilst on the other hand the low mortality in group *C* will probably tend to raise the proportion of ♂s. Hence in Table I, where the effect of a different death-rate in the different groups has not been taken into account, the proportion of ♂s in groups *A* and *B* should probably be slightly raised compared with *C*. This, of course, is assuming that the higher death-rate affects the ♂ infants disproportionately as compared with the ♀s. It may be that the effect of such an increased death-rate would be proportional in the two sexes, in which case the figures on p. 264 would shew no alteration[2]. More, however, cannot be said until the influence of a fluctuating death-rate on the relative proportions of the sexes has been studied.

[1] The average mortality worked out from Tatham's Table amounts to a little under 10 % (for the same ages).

[2] It is greatly to be regretted that the Registrar-General cannot see his way to distinguishing between ♂ and ♀ births in the various returns.

(3) *Influence of birth-rate on proportion of sexes.*

To obtain some estimate of the action of this factor we must know (*a*) the relative birth-rates in the different groups which will afford some notion of the sizes of the families, and (*b*) the influence of curtailment of the size of the family on the proportions of the sexes.

(*a*) The relative birth-rate in groups *A*, *B*, and *C* is complicated by the fact that the mortality varies in the different groups. The Census statistics give comparatively few records of marriages before the age of 21. From 21—50 years has been taken roughly as the fertile period, and the proportion of the infants under 1 year compared to the number of adults of the fertile period has been taken as the relative birth-rate in the three groups. The following table shews the different proportions in

TABLE IV.

Age %	Under 1	1—20	21—50	51—	Total
Group *A*	2·83 %	45·3 %	39·3 %	12·6 %	1,440,227
„ *B*	2·42 „	39·3 „	44·1 „	14·2 „	1,917,162
„ *C*	1·90 „	34·5 „	48·4 „	15·2 „	1,152,229

the three groups of people at different ages. It will be noticed that the poorer the district the greater the proportion of infants and young people, and the smaller that of adults and old adults. This is no doubt due to the higher mortality in the poorer districts. It is, then, obvious that if we took as our birth-rate the proportion of infants under 1, to the total population, the lessened mortality in groups *B* and *C*, as compared with *A*, will tend to lower the birth-rates in these two groups more than in *A*. And similarly the birth-rate will be more greatly lowered in *C* than in *B*. On the other hand the proportion of youths from 1—20 is far greater in *A* than *B*, and in *B* than in *C*. This factor tends to lower the rate in *A* more than in *B* and in *B* again more than in *C*. On the whole it has seemed safest to compare the number of infants under 1 with the fecund portion of the population in each group—with the following results:—

TABLE V.

	No. of infants under 1 per 1000 of adults aged 21—50	Same expressed in terms of total population (cf. Table I)
Group *A*	68·6	30·2
,, *B*	55·0	24·2
,, *C*	39·0	17·2

These figures, then, give the proportional birth-rate in the three groups, subject however to the following source of error. Since the data are given in terms of infants under 1, and not of births, and since the mortality is greater in the poorer district (*i.e.* greater in *A* than in *B*, and in *B* than in *C*), it follows the birth-rate of *B* is a trifle low as compared with *C*, and that of *A* slightly more so. This, however, will not make much difference to the above figures. Hence we may conclude that the greater the servant-keeping capacity, the smaller becomes the birth-rate. It now remains to examine:—

(*b*) The influence of curtailment of the family. If the sexes are not equally spread over families, but if one sex tends to predominate over the other for the first few births, it is evident that any factor which affects the birth-rate will indirectly influence the proportion of the sexes. Thus if ♂ infants tend to make their appearance in a family before ♀ infants it is evident that any restriction of the family by artificial means will increase the relative proportion of ♂s, though the birth-rate will be diminished. Data for the consideration of this point were collected from *Burke's Peerage* (1894). They comprise 721 families with a total of 3046 children, of which 1618 were ♂s and 1428 ♀s. The size of the family varied from 1 to 16, and the succession of the sexes is known in each case (*vide* Table VI). The relative proportions of the sexes in the different parts of the family are given in the subjoined table. The families considered are those of the heads of houses. Consequently there is every reason to suppose that no artificial restriction has been placed on the number of the family.

This table brings out very clearly the fact that the chances are greatly in favour of the first child born of a marriage being a ♂ (*i.e.* 140 to 100). The chances of the second child being a male are also considerable (117 to 100). After this the chances of a male child appearing diminish until the ninth and subsequent births, when the chances of either sex are precisely equal. From

this it is evident that any artificial restriction of the family will tend to increase the proportion of male infants. And here a

TABLE VI.

	No. of ♀s	No. of ♂s	♂s to 100 ♀s	Males per 100 ♀s = 113·3.
1st child ...	300	420	140·0	
2nd " ...	284	333	117·2	
3rd " ...	245	255	104·1	
4th " ...	189	194	102·6	
5th & 6th " ...	238	241	101·2	
7th & 8th " ...	109	112	102·7	
9th " ...	63	63	100·0	
	1428	1618		

possible source of error must be noticed. The proportion of ♂s to ♀s in the table given is extremely high (*i.e.* 113·3 ♂s to 100 ♀s), even when allowance is made for a low death-rate of male infants owing to favourable conditions of existence here. In *Burke's Peerage* the dates of birth of each member are given in rather more than half the families. For comparison with these the children born of 360 marriages, whose dates of birth were omitted, have been considered. The table brings out the comparison between the two sets.

TABLE VII.

	No. of ♂s	No. of ♀s	♂s per 100 ♀s	Average no. per family
*Set 1, where dates of birth were given...	1561	1382	112·9	4·7
Set 2, where dates of birth were omitted	1090	1089	100·1	6·0

* These are the same as in the preceding table (Table VI) except that 103 single child families have been omitted. Set 1 and Set 2 contain only families with two or more children.

From this table it will be seen that Set 1 (with birth dates) differs from Set 2 (without birth dates) in shewing a greater proportion of ♂s and a much smaller average number per family. There would seem to be some relation between the large families with their greater proportion of ♀s in Set 2, and the unwillingness shewn here to return the dates of birth of the children. From what has gone before we should expect an increase in the size of the family to be correlated with an increased relative proportion of ♀s. This is actually the case, though to a greater extent than we should have been led to look for. If we possessed the birth dates in Set 2 it is not improbable that they might reduce somewhat the relative preponderance of ♂s among the first born, though it does not seem possible that such an effect would be very marked. For although ♀s tend to preponderate more in large families than in small ones, yet the proportion of ♂s is just as great in the first children of large families as of small. In 197 families taken from Set 1 and containing 6 or more children the first born was in 117 cases a ♂ and in 80 cases only a ♀, a proportion of ♂s (146 per 100 ♀s), which tallies closely with the general result given in Table VI.

As this feature of a more marked preponderance of ♂s for the first birth in a family is of considerable importance it may be worth while digressing for a moment in order to consider the evidence in favour of it. The phenomenon was familiar to Düsing[1], nearly twenty years ago when he stated that "die Erstgeburten überhaupt zeigen einen relativ grossen Knabenüberschuss," and gave statistics in support of his statement. More recently Orchansky[2] has brought forward additional evidence. He distinguishes two types of family—Type I where the first born is a ♂, and Type II where it is a ♀. From the data given by him I have been able to calculate that the proportion of ♂s to ♀s among first born (on a total of 2442 births) is 104·6 : 100, whilst the proportion of ♂s to ♀s for total births is 101·1 : 100. The excess of ♂ first births though quite distinct is not so well marked here owing probably to the heterogeneity[3] of the author's material. Such additional evidence as I have been able to obtain upon the subject all points in the same direction. In a total of 6938 first births from the records of Queen Charlotte's Lying-in

[1] C. Düsing, *Jen. Zeit.* 1884, p. 608.

[2] J. Orchansky, *Die Vererbung im gesunden und krankhaften Zustande u. s. w.* (Stuttgart, 1903), p. 103.

[3] Orchansky's material contained amongst others 542 Jewish families in which the proportion of ♂ to ♀ first births was only 81·9 : 100. If we consider the remainder of his material apart—1900 non-Jewish families—the proportion of ♂ to ♀ first births is 111·8 : 100, the proportion of total ♂ births to total ♀ births remaining nearly the same. This behaviour of Jewish infants is somewhat remarkable and is not supported by Düsing's evidence (*Jen. Zeit.* 1884, p. 625).

Hospital[1], between the years 1894—1902 (and inclusive of still births) there were 3625 ♂s and 3313 ♀s (see Table X, p. 273), giving a proportion of 109·4 ♂s : 100 ♀s—a proportion of ♂s markedly in excess of the normal. In 662 first births taken from the records of the Lambeth Lying-in Hospital[2] there occurred 343 ♂s and 319 ♀s, a proportion of 107·5 ♂s : 100 ♀s. Further, from the genealogical data collected by Dr Rivers[3] in Torres Straits I have been able to compile the following two Tables. Table VIII refers to the natives of the islands of Mabuiag and Badu, Table IX to those of Murray Island. Though the numbers are not large it is of interest to find the same phenomenon as well marked here as amongst Europeans.

TABLE VIII.

	No. of ♂s	No. of ♀s	No. of ♂s per 100 ♀s
1st birth ...	206	174	118·4
2nd " ...	143	129	110·8
3rd " ...	93	97	95·9
4th " ...	54	76	
5th " ...	48	45	
6th " ...	32	22	
7th " ...	15	11	93·3
8th " ...	2	8	
9th " ...	1	2	
10th " ...	1	0	
Total	595	564	105·5

Total no. of children = 1159, No. of families = 372.

[1] I would here take the opportunity of thanking the authorities of this Hospital for their courtesy in allowing my friend, Mr E. K. Williams, to have access to their records. To Mr Williams I am greatly indebted for the trouble and care expended in copying the particulars I required.

[2] For these figures I am indebted to Professor Karl Pearson. They form part of a series of 2060 births in which the number of the pregnancy was noted in each case. On the total series the proportion of ♂s to ♀s was 108, *i.e.* much the same as for first births only. This was due chiefly to the marked preponderance of ♂s after the third pregnancy (114·9 ♂s : 100 ♀s). For the second and third pregnancies the proportions are 100·9 ♂s : 100 ♀s, and 104·7 ♂s : 100 ♀s respectively.

[3] The genealogical tables referred to will shortly be published in the Reports of the Cambridge Anthropological Expedition to Torres Straits, Vol. v, and I would here express my thanks to Dr Rivers for allowing me to examine the material before its publication.

TABLE IX.

	No. of ♂s	No. of ♀s	No. of ♂s per 100 ♀s
1st birth ...	162	134	120·9
2nd ,, ...	126	110	114·5
3rd ,, ...	77	89	
4th ,, ...	52	51	102·2
5th, &c. ,, ...	54	39	
Total	471	423	111·1

Total no. of children = 894, No. of families = 296.

We may take it, therefore, that there are good grounds for stating that in the first one or two births in a family there tends to be a preponderance of ♂s, whilst in later births the ♀s are as numerous as the ♂s, even when the total number of ♂s is considerably greater than that of the ♀s. Consequently the larger the size of the family the greater the relative number of ♀s. And we are probably safe in expressing this otherwise as—the higher the birth-rate the greater the relative preponderance of ♀s. Now in our London groups the birth-rate in the poorest, *A*, is nearly double that of the wealthiest, *C* (*vide* Table V); whilst that of *B* is intermediate between the two. From this we should be led to expect a comparative preponderance of ♀s in *A* as compared with *B*, and again in *B* as compared with *C*. In Table I it has already been seen that this is actually the case. And it was there pointed out that such a result was not in accord with other observations on the influence of nutrition on sex. In the influence of the comparative birth-rate it would seem that we have some explanation of the preponderance of ♀s among the poorest nourished classes. The greater size of the family here means a relative preponderance of ♀s as compared with the better nourished districts where for some cause or other the size of the family is less, and the ♂s consequently are in the majority.

(4) *Influence of the age of the parents.*

Though I have not been able to collect any convincing evidence that the age of the father influences the determination of sex, the following data from Queen Charlotte's Lying-in Hospital would seem to shew that the age of the mother may have some effect. This material (which is tabulated in Table X) consists of 6938 cases of primiparas in which the age of the mother, together with

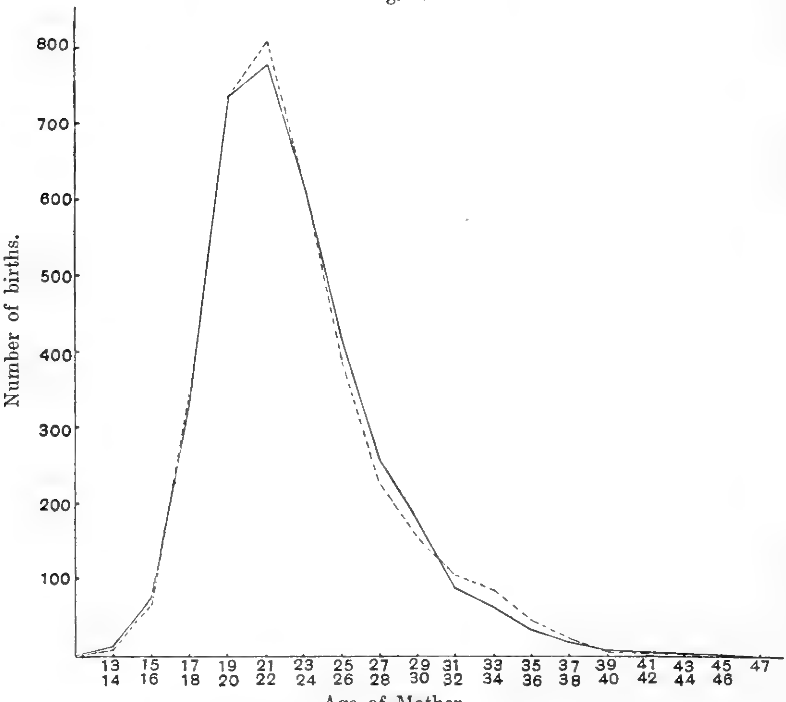
TABLE X.

	♂	*S. B.	♀	S. B.	♀ × $\frac{3625}{3313}$
13	2	—	1	—	1
14	10	1	7	—	8
15	23	—	15	1	16
16	55	2	38	1	42
17	117	2	119	2	130
18	213	10	194	8	212
19	374	17	301	6	329
20	360	13	368	12	403
21	420	14	365	6	399
22	356	14	375	12	410
23	327	12	300	12	328
24	288	11	262	5	287
25	244	8	191	7	209
26	174	7	165	4	181
27	134	7	115	—	126
28	125	4	94	3	103
29	91	2	67	2	73
30	87	3	77	2	84
31	44	2	48	3	55
32	46	2	47	5	51
33	35	1	45	3	49
34	29	1	36	3	39
35	19	2	31	3	34
36	15	3	13	—	14
37	12	2	14	1	15
38	7	—	7	—	8
39	5	—	5	—	5
40	4	—	2	—	2
41	2	1	2	—	2
42	3	1	4	—	4
43	1	—	3	—	3
44	1	—	2	—	2
45	1	—	—	—	—
46	—	—	—	—	—
47	1	—	—	—	—
	3625	142	3313	101	

* S. B. = still births which are included in the total numbers of each sex.

the sex and condition of birth (*i.e.* alive or dead) of the infants are known. From these data have been constructed the curves shewn in Fig. 1. The absolute number of ♂ births was used

Fig. 1.

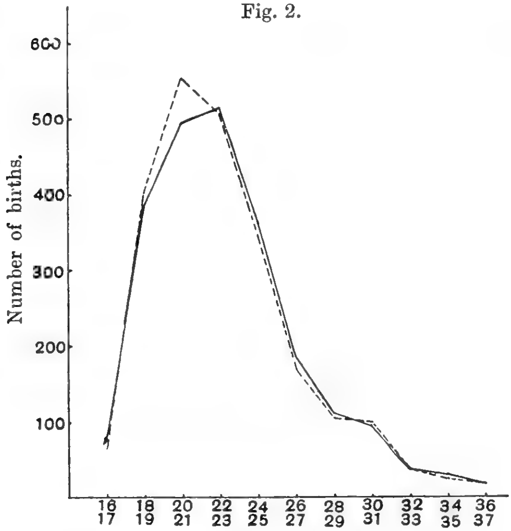


♂ curve represented by continuous, ♀ curve by broken line.

whilst the ♀ births were brought up to a comparable level by multiplying by the fraction $\frac{\text{total ♂ births}}{\text{total ♀ births}} \left(= \frac{3625}{3313} \text{ in this case}\right)$. The resultant curve indicates the *relative* number of ♀ births compared with the number of ♂ births at different ages of the mother. The noteworthy features of the curves are (1) the equality up to the age of 19, (2) the relative preponderance of ♀ births in mothers of 19—23, followed by (3) a relative preponderance of ♂ births. The differences are not very great, and if supported by this material alone might be objected to as accidental. I have therefore constructed curves in the same manner from the only other similar data known to me, viz. that given by Düsing[1] for two groups of first births containing respectively 4441 and 5756 cases. The resultant curves (Figs. 2 and 3) bear a close resemblance to Fig. 1 in the three points mentioned above.

[1] *loc. cit.* The data for Fig. 2 are given on p. 604, those for Fig. 3 on p. 605.

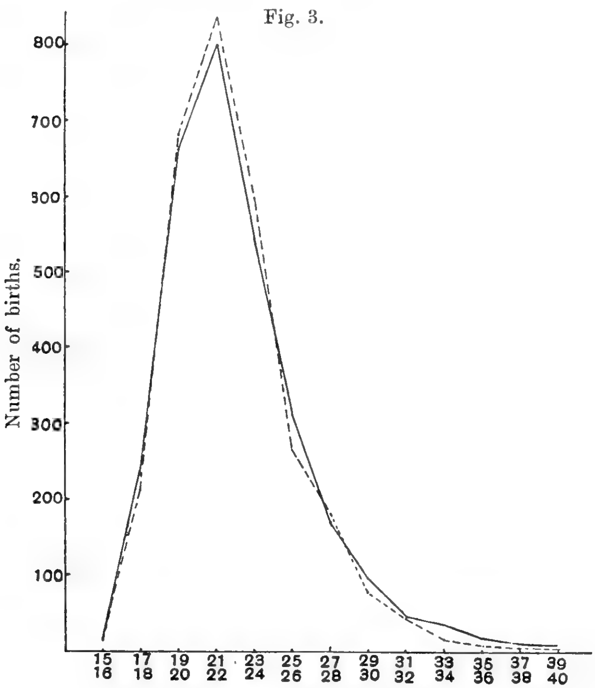
Fig. 2.



Age of Mother.

♂ curve represented by continuous, ♀ curve by broken line.

Fig. 3.



Age of Mother.

♂ curve represented by continuous, ♀ curve by broken line.

We have, therefore, good grounds for the conclusion that mothers between the ages of 19—23 tend to produce relatively more ♀ infants, whilst somewhat older mothers tend to produce relatively more ♂ infants. Consequently, early marriages would favour the production of a relatively greater number of ♀s, whereas marriages subsequent to the age of 23 would result in a production of ♂ infants relatively greater than the normal. Though I can give no figures on this head, I believe that there is reason to suppose that the largest proportion of early marriages occurs amongst the poorest classes, and that with better social conditions there is associated a tendency to defer the age of marriage. Should this supposition be well founded, it affords an additional explanation of the relative preponderance of ♀ births in group *A* as compared with group *B*, and again in group *B* as compared with group *C*.

Summary and Conclusion.

If the population of London be divided into three portions exhibiting graduated poverty it is found that the proportion of ♂ to ♀ infants produced is lowest in the poorest portion, highest in the wealthiest portion, and intermediate in the intermediate portion. The proportion of ♂s is highest of all in a number of births taken from *Burke's Peerage*, where the nutrition may be supposed to be of the best. From this the alternative conclusions may be drawn: that either more favourable conditions of nutrition (1) may result in a large proportion of ♂ births, or (2) may have no effect on the proportion of the sexes, or (3) may even result in a relative preponderance of ♀ births, but that in the last two cases the effect is masked by other factors which affect unequally the different strata of society. Such factors are shewn to exist in a differential infant mortality, a differential birth-rate, and probably also in a differential marriage-age. These factors all tend to diminish the proportion of ♂s in the poorer portions of the population, and consequently render the first of the above alternative conclusions improbable. Whether the second or third of the other possible conclusions is to be accepted must remain doubtful so long as we are not in a position to estimate the quantitative effect of the factors given above. From the necessarily rough estimate which he has been able to form, the writer's opinion is that their combined effect would not be sufficiently great to mask a preponderance of ♀ births due to better nutrition, and consequently he is inclined to believe that in man at any rate the determination of sex is independent of parental nutrition[1]. In any case its influence can be but small.

[1] Mention may here be made of some interesting experiments recently carried out on white mice by O. Schultze (*Archiv f. Mic. Anat.* 1903), who has arrived at the conclusion that in these creatures nutrition is without influence on sex-determination.

On some minerals from the Binnenthal, Switzerland. By R. H. SOLLY, M.A., Downing College.

[*Read* 26 October 1903.]

(*a*) A new mineral appearing in small crystals of a cochineal red colour, with a vermilion streak, crystallizing in the rhombic system, cleavage (100) highly perfect. Mr Hutchinson found As, S and Pb.

(*b*) A new sulpharsenite of lead closely allied to liveingite, crystallizing in the oblique system, the plane of symmetry is parallel to the cleavage plane.

(*c*) A new sulpharsenite of lead closely allied to rathite, crystallizing in the oblique system, the axis of symmetry is parallel to the zone axis of the principal prism which exhibits numerous fine oblique striations; the crystals are twinned about (100) the cleavage plane.

(*d*) A new mineral belonging to the mica group of an emerald green colour, but crystallizing in the anorthic system.

(*e*) Hyalophane, three new planes have been observed, viz. (380) ($\bar{2}$12) and (211).

(*f*) Albite in simple crystals. This mineral has not been found before at the Legenbach quarry.

All the above minerals were obtained from the Legenbach quarry.

(*g*) Anatase, some very fine crystals were found on the Ofenhorn this summer, the (313) plane is very large. Some highly modified crystals were also obtained from this mountain last summer. Five new forms were observed.

Exhibition of Living Gongylus gongyloides, a floral Mantis. By Captain C. E. WILLIAMS, Indian Medical Service. (Communicated by Mr D. SHARP.)

[*Read* 9 November 1903.]

The insects exhibited show the pupal or nymph stage in the development of this Mantis. They were hatched from the egg about January 18th of this year, and in the ordinary course should have attained the imago or winged state at the end of October. Development appears to have been arrested by the unsuitable climate of England, and by loss of appetite under confinement.

The floral mimicry is effected by the foliaceous expansion of the prothorax around the insertion of the front pair of legs. This expansion is roughly diamond or oval-shaped and on the under side is of a bright azure blue tipped with rose purple at the angles and margins; in the centre of this disc is a deeply pigmented black spot of triangular shape. The front pair of legs are held closely folded together in the front of the coloured disc. The azure coloured disc resembles a small flower and the black spot mimics the tube of a corolla. The attitude adopted by the insect when at rest and intent on catching its prey is an inverted position below a leaf or spray of leaves, the coloured side of the prothoracic disc being turned to the brightest light available. Insects, especially butterflies, are readily attracted by the floral simulation. The hinder part of the prothorax is drawn out into a long stalk and coloured a light green, enhancing the floral resemblance. The rest of the body is shaped and coloured to resemble a bunch of dead leaves and is practically undiscernible amid its natural surroundings. The insects exhibited were brought from Rangoon.

Wheat Breeding. By R. H. BIFFEN, M.A., Emmanuel College.

[*Read* 9 November 1903.]

During the summer of 1901 a large number of crosses were made between different sub-species and varieties of wheats, primarily with the object of raising improved varieties from the point of view of the farmer and miller, and also to ascertain to what extent Mendel's Laws of Inheritance hold for the distinctive characteristics of wheats.

For a study of heredity there are few plants as suitable, for with very rare exceptions the flowers are autogamous, and consequently there is no further trouble in self-fertilizing once the first crosses are made, and further, large numbers of plants can be raised on a small area—a matter of some importance in actual practice.

Whilst this work was in progress two papers of importance have appeared which deal with wheat-breeding. One by Spillman, working with no previous knowledge of Mendel's researches, shows clearly that certain pairs of characters conform to the now well-known laws, and the second by Tschermak serves to confirm his results. These workers have proved that the following characters are dominant: lax ears, beardless ears, velvet chaff, red chaff, while dense ears, bearded ears, smooth chaff and white chaff are the corresponding recessive characters. To this list I may now add a number of other pairs of characters such as grey and white-coloured chaff, keeled and rounded glumes, hollow and solid stems, rough and smooth leaf surface, short and long leaves, broad and narrow leaves, long and short grains, red and white grains, glutenous and starchy grains, late ripening and early ripening, etc., the dominant character being the first-mentioned of the pair in each case.

The majority of these pairs of characters have been investigated in the second generation, and the hybrids have been found to split up according to the usual proportions of three with dominant to one with recessive characters. By way of example we find

Beardless ears	: Bearded	::	273	:	91
Velvet chaff	: Smooth	::	63	:	23
Keeled glumes	: Round	::	201	:	68
Hollow stems	: Solid	::	170	:	56
Grey chaff	: White	::	161	:	56
Red chaff	: White	::	64	:	21
Red grain	: White	::	60	:	20

where a single pair are taken into account. Where two pairs are counted out the combinations agree with those predictable from a consideration of Mendel's laws. Thus a velvet-chaffed, white wheat was crossed with a smooth-chaffed, red wheat. The hybrid was velvet-chaffed and red-grained, and its progeny consisted of 45 velvet-chaffed red-grained, 15 velvet-chaffed white-grained, 16 smooth-chaffed red-grained, and 5 smooth-chaffed white-grained plants out of a total of 81—*i.e.* in the proportion of 9 : 3 : 3 : 1. As far as I have worked out the results for three and more pairs of characters, they also conform with the ratios deduced theoretically.

It may be taken as proved then, that, for the majority of characteristics at all events, wheats follow Mendel's laws. At the same time it is worthy of note that in some cases the hybrid does not show certain of the dominant characters as clearly as the parent, whilst in other cases the characters seem to be intensified. Generally speaking it is the most variable characters of the parents which give these results.

A typical case is afforded by the grey colouring of the chaff of Rivet wheat, which varies considerably in intensity, not only among individual plants but among the ears of the same plant. This wheat has been crossed with a number of white-chaffed and red-chaffed wheats, and in every case the hybrids obtained have been variable in colour, none being as dark as the original Rivet wheat, and none as pale as the other parent. In the following generation the extent of the variation is extraordinary, the colour ranging from almost black to isabelline white, the intermediate colours being the most numerous.

Apparently a similar phenomenon has occurred among Tschermak's hybrids between red and white wheats, and he has been able to show that two intermediates are found to one of red and one of white. It may be that the same type of splitting occurs among the grey and white hybrids, but the colour gradation is so subtle that I have not been able to decide into which classes the plants should be placed, especially as in some cases each ear would have required a separate class.

The characters lax and dense ears give the same sort of result. In the hybrid the ear is a little laxer than the lax parent (probably a correlation with increased height), and in the following generation a small number of very lax and very dense ears occur, the majority lying between these extremes.

Velvet and smooth chaff again offer a difficulty. Where Rough chaff wheat is the velvet parent, the velvet character is transmitted in full intensity to the hybrid and the splitting is pure. Where Rivet wheat is the velvet parent though the "felting" of the hybrid is variable and in its progeny still more variable—in fact

many ears would have been classed with the glabrous forms unless they had been examined with a powerful lens. On separating the velvet and smooth-chaffed plants, 223 of the former were found and 116 of the latter, *i.e.* roughly a ratio of 2 : 1. It is conceivable that the result is due to a partial shedding of the hairs as the ears ripen, but I have not been able to establish this so far.

This increased tendency for the characters to vary probably explains the fact that a number of typically spelt-like ears occur in the second generation of hybrids with Rivet wheat. The distinguishing marks of *T. spelta* varieties are the closed spikelets and the brittle rachis. Neither of these occur in Rivet wheat or the other parents, Red King, White Monarch, and Sunbrown. The glumes of Rivet wheat though are keeled and thicker than those of the varieties with which it was crossed, and if this characteristic should have become exaggerated the enclosed spikelets would only open with difficulty or not at all. The brittle rachis would then be a necessity for "seed" dispersal. The fact that some spelt-like ears, which will shed their grain on rubbing between the hands and having a tough rachis, are found, tends to give support to this view.

All of the well-known races of spelts are represented among the hybrids—bearded, beardless, grey, red and white. Should any of these breed true the presumption will be that *T. spelta* has arisen from *T. vulgare* and *T. turgidum.*

One point of special interest in Spillman's paper is that a cross between a lax-eared and dense-eared wheat gave rise to hybrid plants, with ears intermediate in length between the parents, and the succeeding generation consisted of long, intermediate, and short types, in the proportion of two intermediates to one long and one short. I have not met with similar results among my lax and dense crosses at present. One typical intermediate hybrid has been raised though by crossing Polish wheat (*T. Polonicum*) with Rivet wheat. The former is characterised by the enormous size of its glumes and grain when compared with our commonly cultivated varieties. In the hybrid the glumes and grain are intermediate between the parent forms, *e.g.*:—

	Polish wheat	Rivet wheat	Hybrid
Length of glumes ...	19·5	8	15·7 mm.
,, grain	8·5	5·4	6·8 ,,
Weight of 100 grains	6·2	4·8	5·6 ,,

In other words, although the influence of Polish wheat shows clearly in the hybrids there is no dominance of long glumes and

long grain over short glumes and short grain. The second generation of this cross has still to be grown.

It has also proved interesting in another respect. Polish wheat is noted for its habit of rapid growth and early maturation, while Rivet wheat grows and ripens slowly. To ripen both in August the former should be sown about the middle of March, the latter during the preceding autumn. The hybrid sown in the middle of March ripened its most vigorous ears by September 17th and the ears from the side tillers a month later. These dates correspond with those for the time of ripening of Rivet wheat sown in the middle of March, so that the late ripening habit is clearly a dominant character. The results of other crosses with Rivet wheat confirm this, both for the first and second generations.

We have thus one pair of "constitutional" as distinct from morphological characters obeying Mendel's laws. As many of these are of far greater importance to the agriculturist than the merely morphological characters, a number are being investigated in detail. The most important of these is the disposition to withstand the attacks of parasitic fungi such as those causing Rust and Mildew. From the preliminary experiments it appears that immunity from Rust is a definite recessive character.

On the Kinetic Theory of Matter. By H. C. POCKLINGTON, M.A., St John's College.

[*Read* 26 October 1903.]

1. In formulating a kinetic theory of matter we, at least tacitly, make the assumption (A) that matter forms, or, if the ether influences the phenomena, that matter and ether together form a mechanical system, *i.e.* a system the motion of which is given by Lagrange's equations. These equations of motion will contain undetermined multipliers if the number of coordinates chosen to specify the configuration of the system exceeds the number of independent velocities, and there must then be relations between the velocities of the coordinates chosen. Assumption A is equivalent to the assumption that the motion is deducible from the principle of Least Action, and also to the assumption that the ultimate particles of the system obey Newton's Laws of Motion, and that there are no forces of the nature of slipping friction.

It is usual to make the further assumption (B) that the number of independent coordinates required to specify the configuration of the system is equal to the number of independent coordinates required to specify its velocity. In §§ 2—5 this assumption is made, and by attacking the problem from a new standpoint the difficulties hitherto met with are avoided, with the result that we prove that we cannot explain the phenomena exhibited by an ordinary gas if Assumption B is accepted.

In §§ 6—11 Assumption B is accordingly rejected, and we suppose that the number of independent velocities is reduced by the existence of one or more equations of condition in the form of non-integrable linear differential equations of the first order. The class of system considered thus includes the simpler cases of bodies rolling on one another, some of which are chosen as illustrative examples.

In particular cases the Jacobian of the independent velocities (or of functions linear in them) and the coordinates with respect to their original values is unity, but this does not hold generally, and the theorems proved about the sharing of energy, etc., for the case where Assumption B is made require further examination.

A general treatment is impossible, and hence two particular cases are discussed. It is shown that the whole of the energy of the system can pass to a few (or even one) of its degrees of

freedom, and that a number of systems started from all possible configurations with all possible velocities can divide into discrete sets, in each of which some function of the coordinates and velocities has a definite value, different for the different sets. The relation of these facts to the known phenomena of matter is discussed in § 10, and the conclusion is drawn that if Assumption B is rejected, the kinetic theory of matter may probably be found capable of explaining them. No equations are suggested as forming the mathematical basis of the theory, but the kind of coordinates entering into the equation of condition is discussed, without however arriving at any definite result.

2. The system under investigation is taken to be a certain quantity of gas, with ether, contained in a bottle (the inner surface of which is not a surface of revolution) which is perfectly rigid and smooth, and perfectly reflecting to all disturbances of the ether. We first assume that the configuration of the system as to position and velocity (*i.e.* its phase) is defined by a certain number of independent coordinates $q_1 \ldots$ and an equal number of independent velocities. We define the momenta $p_1 \ldots$ as usual as the partial differential coefficients of the kinetic energy with respect to the velocities, and then prove that J, the Jacobian of the coordinates and momenta with respect to their initial values, is unity. This may be done either via Hamilton's Characteristic Function, or by evaluating $(dJ/dt)/J \equiv \Sigma d\dot{q}/dq + \Sigma d\dot{p}/dp$ by means of Hamilton's equations of motion.

We now suppose an infinite number of copies of the original system to be made, and that these are initially in all possible phases consistent with the total energy lying between certain limits, and with the condition that each phase can be reached from a given one without passing through a configuration in which the potential energy exceeds the total energy of the phase to be reached. Let the number of systems that lie between q_1, $q_1 + dq_1$, $\ldots p_1$, $p_1 + dp_1$, $\ldots$ be equal to $kdq_1dq_2 \ldots dp_1dp_2 \ldots$ initially, where k is a constant. Then by the Jacobian equation, this state is permanent.

3. Let new functions of the coordinates and momenta, linear in the latter, be found such that the kinetic energy is the sum of their squares. These functions have been called momentoids; an infinite number of sets of momentoids can be found. It can now be proved that at any moment the mean taken over the different systems (or say for the sake of brevity, the system-mean) of the square of any one of the momentoids is equal to $1/n$ of the system-mean of the kinetic energy, where n is the number of degrees of freedom.

We now select a particular set of momentoids. Since they are linear functions of the momenta, they are also linear functions

of the velocities, and we may find momentoids by resolving the kinetic energy into sums of squares of linear functions of the velocities. One such resolution is of the form

$$\begin{aligned} T = &(\alpha\dot{q}_1 + \beta\dot{q}_2 + \gamma\dot{q}_3 + \dots)^2 \\ &+ (\beta'\dot{q}_2 + \gamma'\dot{q}_3 + \dots)^2 \\ &+ (\gamma''\dot{q}_3 + \dots)^2 \\ &+ \text{etc.} \end{aligned}$$

where the first bracket contains all the velocities, and each succeeding one, one less. Hence the quantities in brackets are momentoids; call them u_1, u_2, etc.

The diminution of kinetic energy produced by putting $\dot{q}_1$ and $\dot{q}_2$ equal to zero without altering the other velocities is

$$u_1^2 + u_2^2 - (\gamma\dot{q}_3 + \dots)^2 - (\gamma'\dot{q}_3 + \dots)^2$$

and is not greater than the kinetic energy corresponding to the momentoids u_1, u_2. In general, the diminution of the kinetic energy produced by putting the first r velocities equal to zero is not greater than that corresponding to the r momentoids, and hence its system-mean is not greater than r/n of the system-mean of the original kinetic energy.

4. This theorem is true at any moment, and in particular after the lapse of a long time. Let us consider what the state of the systems then is. If an atom cannot be made, destroyed or altered in its fundamental properties without breaking the geometrical constraints of the system, and the gas chosen is monatomic, each system contains the same number of atoms of the same kind, and we conclude from the known physical properties of such gases that the systems are finally indistinguishable from each other by any physical test, excepting of course that their energies vary between the original limits.

Let us now take the r velocities mentioned at the end of § 3 to be the component velocities of translation of the atoms. Then r is three times the number of atoms. But n is the total number of degrees of freedom of ether and gas, and if not infinite is at least many times larger than r. Hence the system-mean of the change in kinetic energy produced by stopping the motion of translation of the atoms without altering any other velocity is at most a very small fraction of the system-mean of the kinetic energy. Since the systems are ultimately indistinguishable, the same is true of each system. This is entirely opposed to what is known of gases; in fact, on stopping the motion of translation of the atoms of a monatomic gas in its steady state nearly the whole of its energy disappears.

5. We must now critically examine the statement that the final states of all the systems are the same. This was deduced

from the results of physical experiments, but in such experiments the phases reached by a system may not include all those that are geometrically possible. For example, they certainly will not if each atom includes a gyrostat. There are two ways in which it is possible for the systems ultimately to differ, namely their physical properties may, on passing from one system to another, change either continuously, or in an essentially discontinuous manner.

If the change is continuous, we should expect to find evidence of its existence in nature. If, for example, a continuous change from the argon atom to the mercury atom is geometrically possible, each form being stable, we should expect to find the intermediate forms occurring (and those near to either limit should occur in nearly as great abundance as the limiting forms), even if we could not pass artificially from form to form. As they do not occur, we conclude that they cannot. The fact that the atomic weights of the known elements form an orderly series of numbers also suggests strongly that the various kinds of atom form an essentially discrete group.

On the other hand, it is not possible for the systems to differ ultimately in a discontinuous manner. For let each system be represented by a point in a $2n$-dimensional space where $q_1 \ldots p_1 \ldots$ are the coordinates of the point. Initially these points fill a $2n$-dimensional space (depending on the limits within which the energies of the systems lie) with uniform density. At any subsequent time the points lie within the same space, and, from the Jacobian equation, their density of distribution is the same as before. Hence they still fill the space, and the phases form a continuous group. The physical properties of the systems can only differ on account of differences in phase, and hence must vary continuously if at all.

6. We see then that the acceptance of Assumption B, even if we admit that the alteration of an atom is geometrically possible, involves consequences that directly contradict the observed phenomena, and we must now reject this assumption and investigate the case where there are one or more linear relations between the velocities. If these are integrable, they lead to relations between the coordinates in virtue of which the number of independent coordinates is reduced, and we fall back on the case already considered. We hence suppose the equations not to be integrable. Such equations occur in the simpler cases of rolling motion. One consequence of the existence of such equations is that the number of independent coordinates exceeds that of the independent velocities. In particular cases, it still happens that the Jacobian of the independent velocities (or of linear functions of them) and the coordinates with respect to their original values is unity. This

happens, for example, in the case of a sphere rolling on a cylinder under any forces, if we choose two coordinates of the centre of the sphere, the longitudinal and transverse velocities and the normal angular velocity as defining the phase, and we can prove the usual theorem about the distribution of energy between the degrees of freedom in the case of a set of such systems. In general, however, the Jacobian is not unity, and we may expect to find the distribution of energy obeying different laws. It will be useful to examine what happens in some particular cases, and these may conveniently be chosen to be such as can be approximated to by using actual bodies.

7. For a first case, let a body rest on a horizontal plane by two smooth contacts and a massless wheel, so that the problem is that of the plane motion of a body one point of which is constrained to move in the direction of a line fixed in the body. Let this line pass through the centre of gravity, and let the distance of the point of contact of the wheel from the centre of gravity be a. Let the position of the body be specified by x, y, the coordinates of the centre of gravity, and θ the angle that the line joining the centre of gravity to the point of contact of the wheel makes with the axis of x.

The Lagrangian function is

$$\tfrac{1}{2}(\dot{x}^2 + \dot{y}^2 + k^2\dot{\theta}^2),$$

and the condition is

$$\dot{x}\sin\theta - \dot{y}\cos\theta - a\dot{\theta} = 0 \quad \text{.................}(1).$$

Hence the equations of motion are

$$\ddot{x} + \lambda\sin\theta = 0,$$
$$\ddot{y} - \lambda\cos\theta = 0,$$
$$k^2\ddot{\theta} - \lambda a = 0,$$

or

$$\left.\begin{aligned} a\ddot{x} + k^2\ddot{\theta}\sin\theta = 0 \\ a\ddot{y} - k^2\ddot{\theta}\cos\theta = 0 \end{aligned}\right\} \quad \text{....................}(2).$$

On differentiating (1) and eliminating $\ddot{x}$, $\ddot{y}$ by (2), we have

$$\dot{x}\cos\theta + \dot{y}\sin\theta = \frac{k^2 + a^2}{a}\frac{\ddot{\theta}}{\dot{\theta}} \quad \text{.................}(3).$$

On differentiating this, and eliminating $\ddot{x}$, $\ddot{y}$ by (2) and $\dot{x}$, $\dot{y}$ by (1), we have

$$\frac{d}{dt}\left(\frac{\ddot{\theta}}{\dot{\theta}}\right) = -\frac{a^2}{k^2 + a^2}\dot{\theta}^2.$$

Putting a new variable for log $\dot{\theta}$, this becomes an equation of the second order, the integral[1] of which is

$$\dot{\theta} = \frac{b\sqrt{k^2 + a^2}}{a} \operatorname{sech} b\,(t - \epsilon),$$

where b, ϵ are the constants of integration.

Hence $\dot{\theta}$, after possibly increasing at first, tends ultimately to the limit zero, and the kinetic energy finally passes wholly into energy of translation.

We can prove from (3) and (1) that the body is then moving along the line joining the point of contact of the wheel with the centre of gravity, and in that sense. However the body is started, if we except an unstable steady motion, it ultimately moves as just described. We shall say that the motion tends to a final state, in which $\dot{\theta} = 0$.

The motion is of course reversible. If reversed when $\dot{\theta}$ has diminished nearly to zero, $\dot{\theta}$ increases, at first slowly, then more rapidly, reaches a maximum, and then decreases to the limit zero. The motion still tends to the same final state. The bearing of the results of this paragraph on the kinetic theory of matter is discussed in § 12.

8. It will be interesting to consider the Jacobian

$$J = \partial\,(x, y, \theta, \dot{x}, \dot{y}) / \partial\,(x_0, y_0, \theta_0, \dot{x}_0, \dot{y}_0).$$

It can be evaluated by means of the integral equations of motion by first finding the Jacobian relative to the constants of integration, but we prefer to use a method of somewhat more general application.

Differentiating, we find dJ/dt equal to a sum of five Jacobians, each of which differs from J by having an additional dot in the numerator. Dividing by J, and using the formula for change of independent variables,

$$\frac{1}{J}\frac{dJ}{dt} = \frac{\partial\,(\dot{x}, y, \theta, \dot{x}, \dot{y})}{\partial\,(x, y, \theta, \dot{x}, \dot{y})} + \text{etc.}$$

$$= \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{\theta}}{\partial \theta} + \frac{\partial \ddot{x}}{\partial \dot{x}} + \frac{\partial \ddot{y}}{\partial \dot{y}},$$

where the differentiations are partial, and the independent vari-

[1] This equation can be integrated again and, if $(k^2 + a^2)/a^2$ is the square of an integer, the values of x and y can be found in finite terms by integrating the values of $\dot{x}$ and $\dot{y}$ found from (1) and (3). The path of the point of contact of the wheel is somewhat simpler than that of the centre of gravity. Its intrinsic equation is

$$s = \frac{k^2 + a^2}{a} \log \sec \frac{a\phi}{\sqrt{k^2 + a^2}}.$$

ables are x, y, θ, $\dot{x}$, $\dot{y}$. The first two terms vanish. The third is found from (1) to be $(\dot{x}\cos\theta + \dot{y}\sin\theta)/a$, while from (2)

$$\frac{\partial \ddot{x}}{\partial \dot{x}} = -\frac{k^2}{a}\sin\theta\,\frac{\partial\ddot{\theta}}{\partial\dot{x}},$$

and from (3) and (1)

$$= -\frac{k^2}{a(k^2+a^2)}(\dot{x}\sin 2\theta - \dot{y}\cos 2\theta)\sin\theta.$$

Similarly

$$\frac{\partial \ddot{y}}{\partial \dot{y}} = -\frac{k^2}{a(k^2+a^2)}(\dot{y}\sin 2\theta + \dot{x}\cos 2\theta)\cos\theta,$$

and hence

$$\frac{1}{J}\frac{dJ}{dt} = \frac{a}{k^2+a^2}(\dot{x}\cos\theta + \dot{y}\sin\theta)$$

$$= \frac{\ddot{\theta}}{\dot{\theta}}$$

by (3), giving $J = \dot{\theta}/\dot{\theta}_0$.

In the final state $\dot{\theta}$ vanishes, and hence J is then zero.

This result can be generalised. For, representing an infinite number of systems by points in a m-dimensional space, if the systems tend to a final state, these points must ultimately collect on some surface or other locus of fewer dimensions than m. Their density there will be infinitely greater than it originally was, and J, which is proportional to the reciprocal of the density of distribution if the original distribution was uniform, will vanish on this locus, and therefore the final state must satisfy the equation $J = 0$.

The converse proposition, that if J is capable of vanishing, the system tends to a final state, given by $J = 0$, does not follow, and is possibly not always true. If J cannot vanish there is no tending to a final state. Using the last theorem, we can readily prove that there is no tending to a final state in the case of a circular disc rolling on a rough surface; the tendency found by experiment for it ultimately either to roll in a straight line or to spin with its plane vertical must be due to rolling friction or some other cause that is neglected in forming the ordinary equations of motion.

9. In the case considered in §§ 5, 6 the final state was given by $\dot{\theta} = 0$, an equation involving a velocity. It is possible for the final state to be given by an equation involving only coordinates. Let us consider the case of a tricycle. We suppose that the wheels and framework are massless, that a mechanical system, symmetrical about the steering axis, is mounted on it, and that the steering axis is connected to the framework by massless springs

such that when the axis is turned through an angle ϕ from the position for straight motion, there is a couple tending to move it back equal to $b \sin \phi$. Let the distance of the steering axis from the common geometrical axis of the other two wheels be a, and let the mass of the added system be M. Let v be the velocity of the steering axis, then the acceleration of the axis is v^2/ρ normal and dv/dt tangential to the steering wheel. The reactions exercised on the framework of the tricycle are equal and opposite to M times these accelerations, and, since the mass of the framework is zero, they must keep equilibrium with the couple $b \sin \phi$ exercised by the springs on the framework and the reactions at the wheels. The condition that this is so is found by the principle of virtual work to be $M dv/dt = b \sin \phi$, $\sin \phi / a = (b/a) \sin^2 \phi$. Hence unless ϕ is constantly zero, v increases, and as it cannot increase without limit, the motion is one that tends to a final state given[1] by $\phi = 0$. Excepting in special cases, absence of rotation of the steering axis implies absence of motion in the system mounted on it, and hence the final state is one in which the whole of the energy has passed into energy of translation. In the final state v is positive, and the triangle moves with its steering wheel foremost.

10. The case where the couple tending to restore the steering axis to its normal position is $b \sin n\phi$ is interesting. There are n stable positions of equilibrium of the axis. On investigating the small oscillations about the corresponding states of steady motion in the case where the added body is a simple flywheel, those steady motions are found to be stable in which the steering wheel is foremost, the vibrations being then affected with decrement. The general motion cannot be investigated, but we may perhaps assume that, however started, the system will be ultimately found to be moving in one of the stable steady motions.

An experiment on the laboratory scale shows all the phenomena mentioned above as well as might be expected. It is especially interesting with an arrangement that approximates to that considered in § 9 to notice how, when the system is started with the steering wheel hindmost, the oscillations of the steering wheel increase till they absorb all the energy, and the tricycle comes to rest. It then starts in the opposite direction, the oscillations die out, and the theoretical final state of rectilinear motion is closely approximated to.

11. In the case of § 9, the final state is definite only in that ϕ has a definite value; the tricycle is moving along an arbitrary line. It is possible however to have a perfectly definite final state. Let us add a massless arm to the front of the steering axis

[1] Hence in general the equation giving the final state of a system will contain both velocities and coordinates.

and place a massless particle on it which is attracted towards the axis of x with a force which is always normal to this axis, towards it, and small. Then to a first approximation the motion is unaffected (we assume that the initial kinetic energy is not small) and a final state of rectilinear motion is tended to. Making a closer approximation to find the subsequent motion, it is clear that the tricycle describes a sinuous curve about the axis of x. On account however of the tendency of the energy to pass into energy of translation, this motion will ultimately become motion of translation along the axis of x. The motion may be in either sense. However the body be started, its motion will tend to one of the two possible final states.

We can treat the case of §§ 5, 6 similarly, placing the particle at the centre of gravity of the body, and we then find that there are only two possible final motions.

12. We can of course explain Boyle's and Charles' laws for a gas by any form of kinetic theory that postulates atoms moving through the ether without producing much disturbance in it. The difficulties met with are those connected with reversibility and with the distribution of energy, especially the particular case of the relation between the energy held in the ether as radiant heat and that held as energy of translation of the molecules.

An actual gas when disturbed from its normal state always returns to it, hence the motion appears to be irreversible and hence non-mechanical. This difficulty does not exist if we reject Assumption B, for we have shown that reversibility is consistent with a tendency to pass to a definite state of motion, and to regain it if disturbed. Probably there is no real difficulty even if the assumption is accepted, for we can easily suppose that in order to obtain much increase of divergence from the normal state, the initial circumstances must be very accurately adjusted, and that even then the increase of divergence is only temporary, and is succeeded by a decrease to the limit zero.

The difficulty that each degree of freedom of the molecule, including the vibrational degrees of freedom corresponding to the emission or absorption spectra, should have an equal share of kinetic energy on the average, probably cannot be overcome if Assumption B is accepted. The more serious one that the ether should hold a share of the energy infinitely large compared with that held by the translational degrees of freedom of the molecules certainly cannot. On rejecting the assumption, we have however found cases where the whole of the energy passes into one degree of freedom, and there is no reason for thinking that the passage of nearly all the energy from the ether to the matter is irreconcilable with a mechanical theory of ether and matter, or for thinking that a mechanical theory of radiation necessarily leads to an infinite value for the radiation of a black body.

The result of § 10 shows that we may assume that the various kinds of atom are different final states of the same systems, in which case the possibility of transmutation of the elements is not wholly excluded. We may however explain them as conditioned by the various geometrically possible strain-systems in the ether (*cf.* the theory that the various kinds of atom are variously knotted vortex rings).

The simpler phenomena, such as Boyle's and Charles' laws, which depend on the collision of molecules, are given by the ordinary theory, for which Assumption B holds good. Again, the equations of motion of the ether are known, and contain no non-integrable equations of condition. There are only two places where such equations are to be expected, namely in the relations between the atom and its charge, and in the relations between the charge and the disturbance it produces in the ether. We do not know the phenomena connected with the motion of isolated charges or charged atoms well enough to say where the equations occur or to guess what they are.

Note on the proportion of the sexes in 'Carcinus maenas.'

By R. C. Punnett, M.A., Gonville and Caius College.

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It has been recently suggested by Bateson[1] that Mendel's Law may apply to the phenomenon of sex heredity. If such is the case Castle[2] has shewn that in the ♀s of most dioecious animals we must assume the existence of two kinds of ova, in which the ♂ and ♀ elements respectively are dominant; and a similar state of things must also hold good for the spermatozoa. If this segregation of characters (♂ and ♀) occurs in some cell division prior to the formation of the primary oocyte (or spermocyte), it follows that the number of ova exhibiting ♂ dominance produced by any ♀ must be numerically equal to the number of ova shewing ♀ dominance; and similarly for the spermatozoa produced by a ♂. Consequently we should expect to find the two sexes existing in equal numbers if the conditions of life are similar for each. Where the numbers are unequal we should most naturally turn for explanation to the possible existence of a different rate of sexual mortality. The evidence for or against the operation of this factor must depend upon our knowledge of the proportion of the sexes at different stages in the life-history of a species; and it was with a view to obtaining such evidence that the following statistics relating to *Carcinus maenas* were brought together.

The crabs here dealt with were collected on a small area of the coast of Guernsey at the southern extremity of Rocquaine Bay during the 2nd and 3rd weeks of June in the present year. They occurred here in great abundance, due doubtless to the fact that the area in question was that selected by a neighbouring hotel for the deposition of its garbage. The stones of the area (which lay in and rather above the *Fucus* zone) were systematically overturned, and the few inches of dirty sand found beneath most was carefully dug up. Practically every crab seen was captured and slain prior to the determination of its sex and maximum carapace breadth. The majority of the specimens collected on this area were adult, and few occurred whose carapace breadth did not exceed 10 mm. Smaller specimens were found much more abundantly nearer high-water-mark at a spot a few hundred yards along the coast in the direction of the lighthouse. These were preserved in formol and sorted later according to size, after which

[1] Bateson, W., and Saunders, E. R., *Reports to the Evolution Committee*, No. 1, London, 1902.

[2] 'The Heredity of Sex,' *Bulletin of Museum of Comp. Zoology* from Harvard College, 1903.

their sex was determined with the help of a dissecting microscope. The labour of collecting and measuring was materially lightened by the help of a friend, to whom also I am greatly indebted for a subsequent consignment of baby crabs forwarded to me shortly after leaving the island.

TABLE I. Shewing proportion of sexes for each increase of 1 millimetre in carapace breadth.

mm.	♂	♀	mm.	♂	♀
6	249	244	40	14	34
7	6	2	41	16	18
8	27	29	42	14	23
9	36	44	43	7	18
10	36	43	44	4	16
11	37	38	45	11	19
12	62	57	46	10	12
13	58	52	47	8	12
14	41	54	48	15	20
15	75	68	49	5	10
16	47	50	50	13	9
17	44	58	51	6	6
18	61	69	52	8	8
19	53	70	53	12	2
20	86	68	54	3	4
21	35	45	55	7	2
22	32	61	56	9	1
23	20	37	57	5	—
24	32	36	58	8	—
25	35	52	59	4	—
26	37	43	60	10	1
27	20	34	61	1	—
28	28	40	62	4	—
29	23	48	63	4	—
30	37	76	64	3	—
31	25	26	65	1	—
32	24	50	66	—	—
33	29	43	67	2	—
34	14	31	68	—	—
35	28	65	69	—	—
36	26	39	70	—	—
37	12	26	71	—	—
38	12	30	72	1	—
39	7	41			

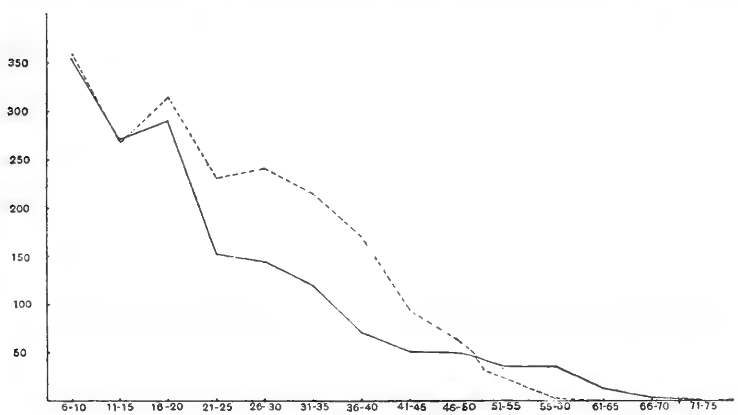
Total no. of ♂s = 1599, Total no. of ♀s = 1984, Total crabs = 3583.

TABLE II.

Max. carapace breadth in mm.	♂	♀
6—10	354	362
11—15	273	269
16—20	291	315
21—25	154	231
26—30	145	241
31—35	120	215
36—40	71	170
41—45	52	94
46—50	51	63
51—55	36	22
56—60	36	2
61—65	13	—
66—70	2	—
71—75	1	—
	Total 1599	Total 1984

♂s per 100 ♀s = 80·6.

Some 3583 crabs in all were examined and the results obtained are given in Table I. These data have been condensed in Table II, from which the curve below has been constructed. On this



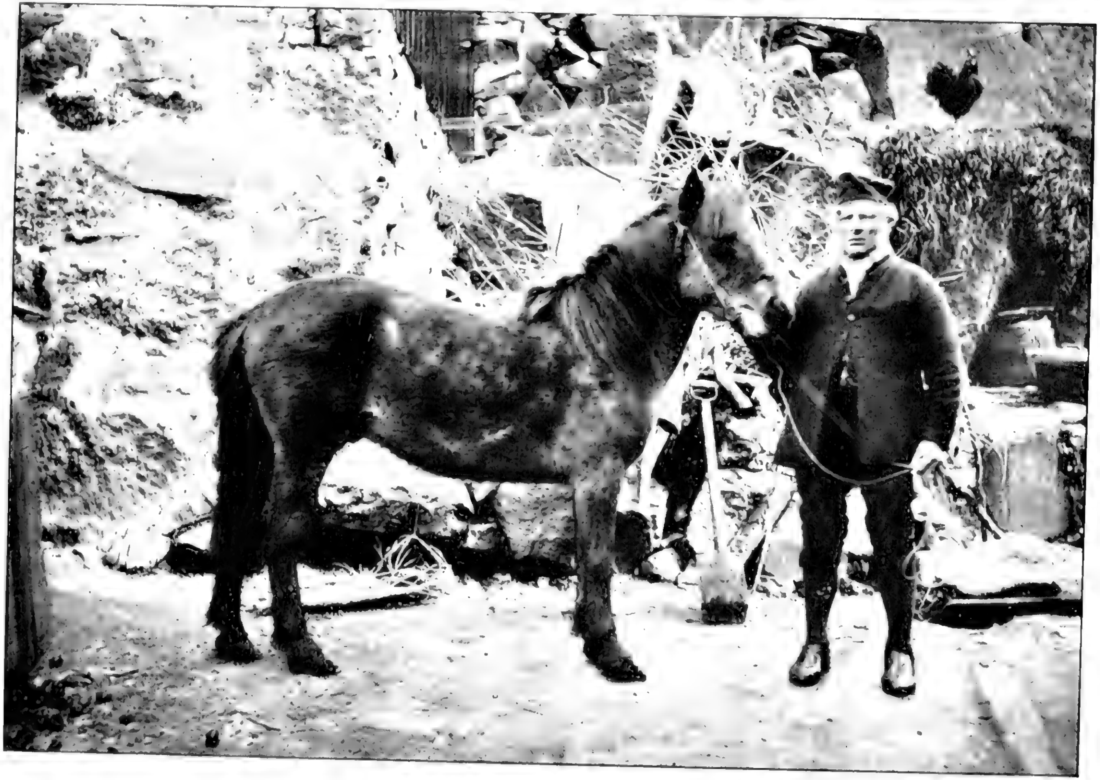
Carapace breadth in mm.

Fig. 1. Curve shewing relative proportion of the sexes at different stages of growth. The broken line denotes ♀s, the continuous line ♂s.

curve are shewn graphically the relative proportions of the two sexes at different stages of growth. Until a carapace breadth of 15 mm. is reached, ♂ and ♀ crabs exist in equal numbers. After this the ♀s begin to preponderate, until, for a carapace breadth of 36—40 mm., they are more than twice as numerous as the ♂s. When the carapace breadth exceeds 50 mm. the ♂s outnumber the other sex. This is due to the fact that the ♂ attains a somewhat greater size; but even when allowance is made for this the great preponderance of ♀s from 15 mm. onwards still remains. So it would appear that whilst in very young crabs the sexes exist in equal numbers, the mortality among the ♂s in subsequent stages is greater than that which occurs among the ♀s.

This increase of male mortality during the later stages of growth is probably due to the larger number of risks which they run owing to their more active habits. An observer stationed on the rocks will frequently notice crabs scurrying about near the edge of the incoming tide. In almost every case such crabs will turn out to be males. The ♀ on the other hand, burdened as she so frequently is with the weight of a future generation, is inclined to more sedentary habits. Whilst collecting I have noticed that the great majority of crabs which require to be dug out of the sand are members of this sex.

Consequently it may, I think, be fairly concluded that (1) during the early stages of growth the proportion of the sexes is equal, and that (2) the approach of sexual activity is accompanied by changes of habit and disposition which, by exposing the males to greater risks, lead to an increased mortality during later stages of growth in this sex as compared with the ♀s. If we assume that the death rate in the larval and early post-larval stages is equal for the two sexes, then the former of the above two conclusions is in accordance with the view that Mendel's law applies to sex heredity in the crab. The second conclusion suggests the danger of drawing any conclusion as to the relative numbers in which the sexes are produced from the proportion of the sexes at later stages of growth. It further hints at a possible connection between sexual dimorphism and a different sexual mortality rate. But until more cases of a similar nature have been investigated it would be unprofitable to dwell further on this point.



Faroe Pony.

The Horse in Iceland and the Faroes. By FRANCIS H. A. MARSHALL, B.A., Christ's College, Cambridge, and NELSON ANNANDALE, B.A., Balliol College, Oxford.

[*Read* 23 November 1903.]

In order to discuss the origin of the ponies of Iceland and the Faroe group it is first necessary to understand the ethnological history (1) of the islands, for the view that these ponies represent an indigenous race is quite untenable, unless we believe that the species *Equus caballus* has not only had a multiple origin but has sprung into being at diverse dates in diverse places. It will be necessary, therefore, to consider how the district under discussion was peopled and what intercourse there has been between it and other countries, before we enter upon the question that immediately concerns us.

The island of Iceland, which has an area roughly one-fifth greater than that of Ireland, was first colonized towards the end of the ninth century by adventurers mainly of Scandinavian origin, the only previous inhabitants having been occasional Irish or Hebridean anchorites, who contributed nothing to the subsequent population. It is important, however, to realize that although the first colonists called themselves Norsemen, over half of them, as is definitely proved by the *Landnámaboc* (2)—a very ancient record of the names, ancestry and estates of the first settlers—had been living in the British Isles before Iceland was discovered, and that only a minority came direct from Scandinavia. Moreover, those who came from the British Isles did not desert the rich English shires for so remote a home, but came for the most part from the Hebrides and the north of Ireland, and their records prove, as well as such names as Kalbann (3) and Njal, that they had freely intermarried with the Celtic[1] population, among which some at least of them had lived for two generations. It is reasonable to conclude that they brought with them to Iceland from the Hebrides and Ireland their domestic animals as well as their wives and families. About twenty years after the first settling, political events in Norway caused many of the more substantial chieftains of Scandinavia proper also to migrate to Iceland, taking with them all that they possessed. It appears, then, to be certain, as far as anything which happened a thousand years ago can be

[1] We do not attach any very precise ethnological significance to the term 'Celtic,' but apply it to those sections of the population of the British Isles in which an 'Iberian' (short, dark, dolichocephalic) element existed.

certain, that the original stock from which the Icelandic and Faroe ponies came was imported partly from Norway and partly from the Hebrides and Ireland, but the probable proportion from the first of these sources is lessened by the fact that many of the colonists of the second batch paid long visits to the Hebrides on their way north-west. The effective settlement of the Norse islands and districts of North-Western Europe only took place, except in the case of Iceland, after much bloodshed and confusion, but in Iceland it was peaceful; for Harald Fairhair, King of Norway, who fought and defeated his rebellious or fugitive vassals in the Hebrides, the Orkneys, the Shetlands, and perhaps the Faroes, went no further north, so that there was less chance of the breeds of domestic animals, including the horses, being exterminated or neglected during the colonisation of Iceland than was the case elsewhere in the Scandinavian settlements. Iceland has been more or less cut off from the rest of Europe since the twelfth or thirteenth century, and it is improbable that any foreign blood was introduced among the ponies until the second half of the nineteenth. During the interval they had been used merely as beasts of burden and as riding horses, and as late as 1806 (4) they do not appear to have been considered as possible objects of export trade. The following statistics, taken from Mackenzie's *Travels in the Island of Iceland*, show that they were formerly, as this author remarks, of less importance than horned cattle:—

Number of horses, horned cattle, and sheep at different dates in Iceland:

Date	Cattle	Horses	Sheep
1703	35,860	26,909	279,812
1783	21,457	36,408	—
1804	20,325	26,524	218,818

In 1896, according to Daniel Bruun (5), the number of horses was 43,235, giving an average of 400 for every thousand of the population. It is probable that they have increased considerably since that date.

In 1868 an English firm commenced to export ponies from Iceland to Great Britain, and for many years sent over from 800 to 1,000 annually. Attempts have lately been made to improve the breed by the introduction of Norwegian stallions, but it is probable that the majority of the animals used for stud purposes are still of pure Icelandic blood. In any case, it is not desired to depart greatly from the established type as regards size, as it is a small breed which is wanted in the Cumberland coal-mines, to which most of the Icelandic ponies imported into Great Britain, by far the largest market, are dispatched. The

larger and finer specimens of the breed are sometimes sent to Denmark, but there appears to be little demand for them, and it is probable that they have become much scarcer since horse-fighting, once a favourite amusement of the Icelanders, was abolished. From enquiries in Reykjavik (1900) it appeared that there was then little demand for them except among the comparatively rich people of that town.

The history of the Faroe ponies has been similar to that of the Icelandic breed, except that they have been even more neglected and have never had the same importance where the distances are so much shorter. They do not appear to be even mentioned in the *Faereyinga Saga* (3), an early Icelandic account of the history of the Faroes, and Grossman (6), who visited the islands in 1895, saw only two individuals. One of us saw few more in journeys to different parts of the island of Stromoe in 1896, 1897, and 1898, but they appeared to be distinctly less scarce in 1900 and 1903. We were told that there were more in the island of Sandoe than elsewhere in the group. Bruun gives the following statistics for the Faroes:—

Year	Cattle	Sheep	Horses
1893	99,657	3,896	631
1898	106,465	4,516	706

At least one cargo[1] of ponies is now exported every year from Thorshavn, the chief place on Stromoe, to Great Britain and Denmark. Recently Norwegian and Icelandic blood has been introduced, several stallions having been brought over from Scandinavia and at least one mare from Iceland, and we were told last summer that it was doubtful whether any specimens of pure-blooded Faroe ponies now exist, except possibly in certain remote villages.

So far as we have been able to discover, the chief, if not the only difference between the Icelandic and Faroe breeds, while they remained pure, was that of colour, for while the former was, and still is, typically either light dun, with a dark line down the centre of the back and often with dark transverse stripes on the legs; the Faroe ponies, according to Landt (7), a most trustworthy observer, were, at the beginning of last century, generally red and occasionally black, the skewbalds sometimes seen among them at the present day being possibly descended from Icelandic ancestors[2].

[1] In 1895, nine were exported; in 1899, sixty-two, and in 1901, eighteen. (Montagu Villiers, *The Trade of the Faroe Islands*, Consular and Diplomatic Report, No. 2984, 1903).

[2] Martin, writing at the end of the seventeenth century, stated that the eighteen ponies then on St Kilda were all of a red colour. (*A Voyage to Saint Kilda*, p. 17. Reprint: Glasgow, 1818). Red is the typical colour of the Hebridean ponies at the present day.

In a paper recently communicated to the Royal Society of Edinburgh and abstracted in *Nature* (vol. LXVII., 1903, p. 239), Professor Ewart has given an account of a 'new horse' from the Hebrides and the north of Ireland which he has provisionally named *Equus caballus celticus.* He described it as being a small-headed pony, with prominent eyes, small ears, slender limbs, small joints and narrow hoofs, and with short hairs in the upper part of the tail, as in mules. Moreover—and this was possibly the most important characteristic—there were no callosities on the hind limbs. The 'Celtic pony,' as thus described, was found to occur in Barra, Tiree and other of the Hebrides, as well as in Connemara, while the majority of the ponies imported into this country from Iceland and the Faroes were observed to possess the same characteristics.

Equus caballus celticus, then, is very distinct from the large-headed, thick-jointed Mongol[1] ponies, and also from those of Norway, both of these having well-marked hock-callosities, and belonging to what has been called the 'cart-horse type,' in spite of their comparatively small size. It is to this type, according to Professor Ewart (8), that Prjevalsky's horse is most closely related.

Professor Ewart suggested further that the two types of horse (the 'Celtic pony' and the 'cart-horse' types) were the present-day representatives of two distinct species or varieties of the genus *Equus*, which are believed, partly on archaeological and partly on palaeontological grounds, to have existed together in Europe in the Palaeolithic period. From these two forms all the domestic breeds of the present day, in Asia and Africa, as well as in Europe, may be supposed to be descended, the proportions of admixture being different in different breeds. Professor Ridgeway (9), however, in a paper read before this Society, has adduced a considerable amount of evidence in support of the view that all the fine horses of the world are derived from the Barb, which he suggested might have had an independent origin.

We have already mentioned that the ponies of Iceland and the Faroes have in recent years undergone improvement by the admixture of Norwegian blood. In the case of the Faroes the Norwegian stallions have only been introduced, apparently, within the last ten or twelve years, and in spite of this admixture, of which it does not appear that Professor Ewart was aware, the general characters of the majority of the Faroe and of the Icelandic ponies are those of the 'Celtic' type. This was made evident by an examination of a number of Faroe ponies in

[1] The similarity between the Norwegian and the Mongol ponies is an additional argument in favour of the theory that there is a Mongolian element, possibly derived from Central Asia, in the population of North-Western Europe. (MacRitchie, *Man*, 1903, No. 97, p. [171].)

Thorshavn this summer, for out of eleven individuals only one had large hock callosities, and this animal, though of small size, differed from the others in being exceptionally clumsy in build, with a large head strongly suggestive of a cart-horse. The remaining ten ponies either had the hock callosities much reduced in size, or, as in the case of two individuals, had no hock callosities. The height of these ponies varied from about eleven to about thirteen hands. A number of Icelandic ponies, averaging about thirteen hands high, were seen on board ship on their way from Reykjavik to Denmark, and of these six were examined and found to have no hock callosities; while another, which had been imported into the Faroes, had the same peculiarity.

About a dozen Faroe ponies have very recently been imported into this country, and of these fully one-third have no hock callosities, while the others have them very small. All these ponies were characterized by having short hairs in the upper part of the tail[1]. They were inferior in quality to, and somewhat smaller than, several of those we saw in Thorshavn. Of two Icelandic ponies, also recently arrived in Scotland, the hock callosities are absent in one and reduced in the other, while the tail characters are similar to those of the Faroe ponies.

The plate represents a Faroe pony, which (though evidently belonging to a better type[2]) closely resembles, so a native of the islands assured us, the animals which existed in the Faroes before the recent introduction of Norwegian blood. In this animal the 'Celtic' characters strongly predominate, the shorter hairs in the upper part of the tail being especially noteworthy, as this character is occasionally absent in the Norwegian cross-breeds.

In spite of what has been said above, we do not mean to assert that the ideal 'Celtic' type, absolutely pure-bred, ever existed in Iceland or the Faroes, for it is probably many thousand years since either this type or that represented by the cart-horse lived as a distinct form, and, as we have seen, the ancestors of the Icelandic and Faroe breeds did not come only from small islands, where it was possible that one of these two forms had been isolated, but also from Scandinavia proper, and it is only reasonable to conjecture that rovers like the Norse vikings brought home with them, to Scandinavia or to Iceland, horses from many different

[1] The presence of short hairs in the upper part of the tail is a character frequently present in Welsh and other ponies which are said invariably to have hock callosities. Such ponies, also, may be regarded as representing traces of the primitive short-headed variety of horse (*vide postea*), but as belonging to breeds which, owing to frequent intercrossing with the large-headed strains, have not maintained nearly the same constancy of type as the comparatively isolated Hebridean, Faroe and Icelandic ponies have done.

[2] Compare the photograph reproduced by Bruun, *t. c.*

countries, quite possibly from the Mediterranean coasts, as well as North-Western Europe. Indeed, the heads of the Icelandic horses in a mediaeval engraving[1] preserved in the National Library at Reykjavik and reproduced on a small scale by Bruun, are, in some cases, rather large and heavy, though the joints are small and the legs slender, while Landt, writing at the commencement of the nineteenth century, described the heads of the Faroe ponies as being 'thick and drooping.' All that we say on this point is that the characters of these breeds show an approximation to a certain ideal type, living representatives of which have been found in the Hebrides and north Ireland by Professor Ewart, but which probably no longer exists in a pure state, even as a breed, anywhere in the world.

This type, unlike the 'cart-horse type,' has no wild representative at the present day, though the researches of Capitan and Breuil (10), (11), and others show that horses at any rate closely approximating to it existed wild at the epoch geologically recent. These researches, however, are based on the rude sketches of contemporary man and on osseous remains, and it is therefore impossible that they should throw light on the question of the comparative development of the hock callosities, which are remarkably large in Prjevalsky's horse—the modern wild congener of the Palaeolithic heavy-headed, thick-jointed form—and are well developed in the modern Norwegian and Mongol ponies. Professor Ewart's investigations, on the other hand, would seem to prove that these callosities were reduced, if not altogether absent, in the ancestor of the pony type, for he has shown that it is in individuals which have the 'pony' characteristics most distinctly marked that these structures, which are vestigial in all known Equidae, have the greatest tendency to disappear. It is possible, of course, that the disappearance of the hock callosities has been approached more nearly in those breeds of mixed origin, in which the 'pony' type predominated, than in the pure-bred ancestor of this type, but no known factor in this case would lead us to believe that the result of cross-breeding has been to produce a character which was originally absent from both lines of descent.

Granted, then, that the modern domestic horse has been produced, at any rate in Western Europe, by the admixture of two forms which were distinct in Palaeolithic times, and granted, as we think Professor Ewart has shown, that one of these two forms has been the predominant partner in the ancestry of a proportion of the individuals now living in certain of the Outer Hebrides

[1] There is a large photograph of this engraving in the Pitt Rivers Museum at Oxford.

and in certain districts of north of Ireland, and also in that of the majority of Faroe and Icelandic ponies, it follows from the historic evidence we have adduced that those horses which the Scandinavian colonists of Iceland and the Faroes brought with them from the Western Isles and from Ireland were prepotent as compared with those which came from Scandinavia, that is to say, a larger proportion of their characters were dominant and so were perpetuated in the breeds that arose from the union of the two original stocks. As we have pointed out, Norwegian blood in considerable quantity has recently been introduced into the Faroes, and yet the 'Celtic' type is still closely approached in individuals at least half, in some cases three-quarters, Norwegian. It is worthy of note, however, that not all such individuals take after their island ancestors, and that in those which do so not all characters are dominant in equal degree, for it is not uncommon to find among the Faroe ponies, known to be of mixed Norwegian, Icelandic and native descent, individuals which have small hock callosities but otherwise resemble their Norwegian progenitors; in other words, which belong to the 'cart-horse' type of modern horses in the majority of their characters but approach the 'Celtic' or true pony type in one important feature[1]. If this be so, we have an instance where in-breeding has produced a race in which one character is more distinctly dominant than others when individuals of this race are crossed with individuals from the continent of Europe, and probably not in-bred to nearly so great an extent.

In conclusion we must acknowledge our obligations to Professor J. Cossar Ewart, but for whose researches our own would not have been undertaken.

BIBLIOGRAPHY.

(1) Annandale ....“The People of the Faroes,” *Proc. Roy. Soc. Edinburgh*, Vol. xxv., 1903.
“The Survival of Primitive Implements in the Faroes and Iceland,” *Journ. Anthrop. Inst.*, Vol. XXXIV., 1903.

(2) Dasent ....*The Story of Burnt Njal*, Edinburgh, 1861.

(3) York Powell ....*The Story of Thrond of Gate*, London, 1896.

[1] Professor Ewart informs us that he has noted complete absence of hock callosities in numerous ponies imported from Iceland, which were of heavy build and resembled the 'cart-horse' type in most of their characters.

(4) Mackenzie ....*Travels in the Island of Iceland, etc.*, Edinburgh, 1811.

(5) Bruun........................"Hesten i. Nordboernes Tjeneste paa Island," etc. Reprinted from the *Tidsskrift for Landökonomi*, Copenhagen, 1902.

(6) Grossman ...."The Færoes," *Geograph. Journ.*, Vol. VII., 1896.

(7) Landt ....*Description of the Feroe Islands*, London, 1810.

(8) Ewart ...."The Wild Horse, (*Equus prjevalskii*, Poliahoff)," *Proc. Roy. Soc. Edinburgh*, Vol. XXIV., 1903.

(9) Ridgeway ...."The Origin of the Thoroughbred," *Proc. Phil. Soc. Cambridge*, Vol. XII., 1903.

(10) Capitan and Breuil ......"Une nouvelle grotte avec figures peintes," etc., *C.R. de l'Acad. des Sciences*, Vol. CXXXIII., Paris, 1901.

(11) Munro........................"On the Prehistoric Horses of Europe, etc.," *Archaeol. Journ.* 1902, and *Proc. Roy. Phys. Soc. Edinburgh*, 1901—1902.

The Action of Ultra-Violet Light on Moist Air. By J. H. VINCENT, D.Sc., B.A., St John's College, Head of the Physics Department, Paddington Technical Institute.

[*Read* 23 November 1903.]

The experiments to be described in this paper were undertaken in the hope that they might lead to some satisfactory explanation of the genesis of the remarkable cloud formed in moist air by ultra-violet light. Although this hope was not gratified, some observations were made which increase the interest of the phenomenon.

C. T. R. Wilson[1] showed that when the light from a strong arc was concentrated by means of a quartz lens on to air saturated with moisture, a cloud made its appearance, and a similar result was obtained with oxygen and carbonic acid gas. It was found possible to produce such clouds even when the relative humidity was less than 90 %. The cloud was not produced in pure steam nor in hydrogen. The view taken by Wilson was that the droplets owed their origin and growth to the formation of hydrogen peroxide.

Lenard[2] subsequently showed that ultra-violet light produced ozone in air or oxygen and that it produced a volume ionisation in some gases.

The conductivity which Lenard observed is not necessarily connected with the presence of moisture, for I found that it occurred in air which had been standing over strong sulphuric acid for several weeks. For the leaking system I used a gold leaf and brass plate in a metal box provided with quartz windows. The box also had a plane glass window through which the gold leaf could be observed with a telescope. A beam of ultra-violet light from an aluminium spark was sent through the dry air in the box. It was admitted through a small quartz window and passed out through a large one so that no direct light fell on anything but the air in the box and the face of the large quartz window. The gold leaf and brass plate were well removed from the direct beam of light; the metal inside the box was not cleaned; it is thus very unlikely that any Hallwachs' effect could be present. The gold leaf lost electricity whether it was charged positively or negatively when the spark was on. The rate of leak was much greater in the former case, as was observed by

[1] C. T. R. Wilson, *Phil. Trans.* A, Vol. CXCII.
[2] Lenard, *Ann. d. Phys.* I. 4, and III. 4, 1900.

Lenard. The effect was stopped by placing even the thinnest sheet of mica or gelatine between the spark and the small quartz window, and also by a layer of coal-gas issuing from a fish-tail burner in the same position.

Thus ionisation is produced in dry dust-free air by ultra-violet light, and the production of the cloud may be due to some other effect in the presence of moisture.

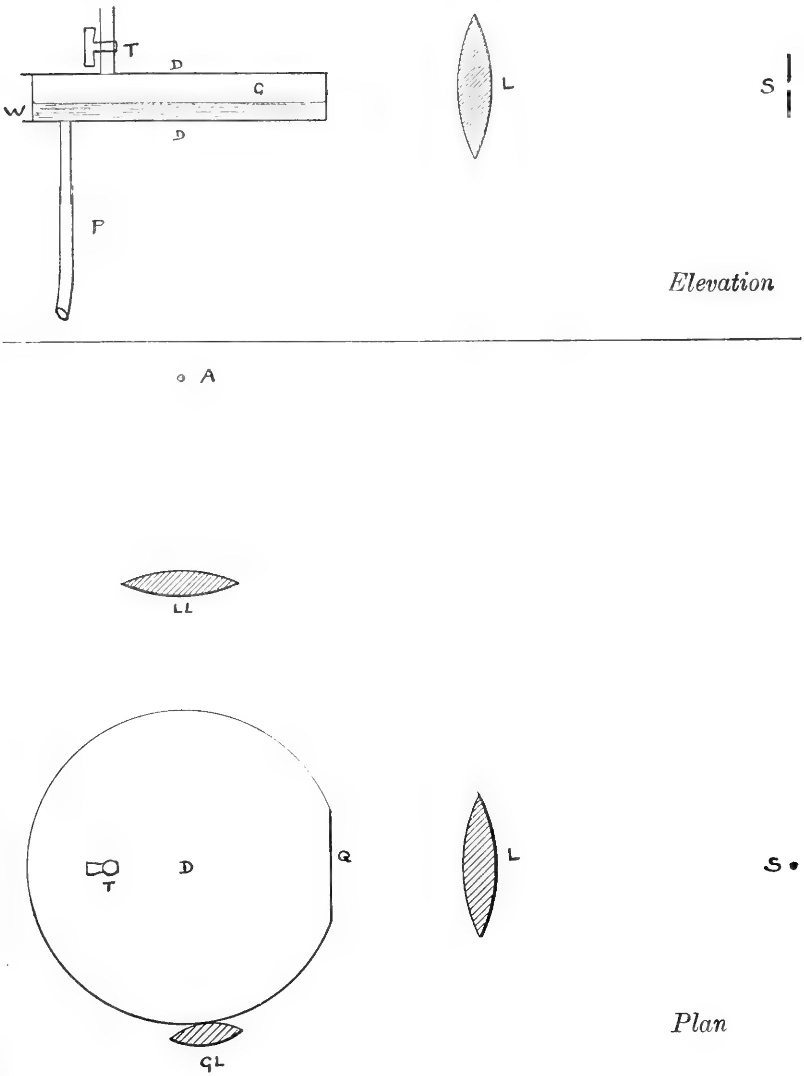
I next tried to prove the presence of hydrogen peroxide in air which had been subjected to the influence of ultra-violet light. In a previous paper[1] it was shown that hydrogen peroxide in the presence of ozone would produce a developable picture on an ordinary photographic dry plate. Air was saturated with moisture by passing it over water in a long horizontal glass tube. This method was used so as to avoid any complications that might be introduced by using air that had been bubbled through water. The air was then led into an airtight glass vessel provided with a thin quartz window, close to which the aluminium spark was placed. The air then passed into a brass box which contained the photographic plate. The plate was completely screened from light and a part of its surface was covered with a flat plate of quartz. On development any effect such as was sought for would have been evident by the appearance of a white picture of this bit of quartz on a black ground. No such effect was produced. The same experiment was tried under various conditions as to moisture of the air and length of exposure but without any result. It thus appears that if hydrogen peroxide is formed in moist air by ultra-violet light it must be in very small quantities. The same air which failed to give this test for hydrogen peroxide turned starch and potassium iodide blue in a few minutes; it also rendered india-rubber photographically active[2], thus confirming Lenard's observation of the formation of ozone.

In order to find out if the separate droplets in the cloud formed by ultra-violet light were charged the following form of apparatus was employed after the trial and rejection of some others.

Two stout circular discs D of brass about 14 cm. in diameter formed the top and bottom of the cloud chamber. These discs were fastened to a ring of glass G 2 cm. high by sealing-wax. This glass had a portion about 3 cm. long cut out of it, which was replaced by a sheet of quartz Q, which served to introduce the ultra-violet light into the chamber. The ultra-violet light was obtained from an aluminium spark S, three or four millimetres long, which was worked by a large induction

[1] Vincent, *Chem. News*, July, 1898.
[2] Vincent, *loc. cit.*

coil connected with the town mains (100 volts alternating). Several large Leyden jars were in parallel with the spark gap.



The light was brought to a focus in the middle of the cloud chamber by the quartz lens L, which was adjustable in position. The clouds were observed by means of a glass lens GL, which

was attached to the outside of the cloud chamber in the position indicated in the lower part of the figure. To render the cloud visible an arc light A was placed in the same horizontal plane as the spark and the cloud chamber, and a glass lens LL focussed the light where desired. The tap T served to introduce air or other gases into the chamber, and the rubber pipe P enabled the depth of the water W to be regulated. This pipe was connected to a water reservoir (not shown in the figure), which could be raised or lowered so as to subject the air in the chamber to successive expansions. In this way all dust particles could be readily removed from the air in the chamber. The effective height of the chamber could be varied by allowing different depths of water to stand in it. By connecting the top and the bottom plates D to the two terminals of a battery of small accumulators an electric field of known strength could be obtained.

An arc light was first employed as a source of ultra-violet rays, but this was not a satisfactory method. Only a very long arc taking a large current would produce the clouds. The introduction of aluminium into the arc lessened the time required to produce the cloud but no such effect followed the introduction of iron. The clouds were generally obtained in a few seconds using the aluminium spark, although this was not always so. In fact the production of the cloud seems to depend on some circumstances which are not under control. Sometimes the cloud could not be obtained at all. This was at first attributed to the formation of a film of dirt on the quartz window but the removal and cleaning of the window was not universally effective.

When the cloud is subjected to electric forces while the ultra-violet rays are still shining on it the droplets can be seen to move. A convenient intensity of field is 200 volts per cm. Under this force the particles exhibit a brisk motion, some moving in one direction, others in the other, while some remain stationary. If this be allowed to continue for a minute or so and then the field reversed, the whole cloud gathers towards the middle of the chamber then spreads out again, the upper and lower portions having crossed over the space occupied by the uncharged particles. There is no doubt that there are three sets of droplets charged positively, charged negatively, and uncharged.

If a similar test is applied to a small isolated patch of cloud from which the ultra-violet light has been cut off as soon as the cloud is visible, it can be split up into three parts and the two outer portions can be made to cross over the space occupied by the uncharged cloud. The field used in this experiment must be much weaker than before.

Attempts to determine the charges on the drops failed owing

to their velocities being widely different. This was so with or without the field.

A most curious phenomenon not apparently very directly concerned with this subject was observed while carrying out these experiments. Occasionally the ordinary ultra-violet light clouds were invaded by wisps and bursts of dense clouds which at times showed iridescent colours. These were produced without any ultra-violet light and were due to small sparks passing over the glass inside the chamber. These clouds were charged and behaved very similarly to the ultra-violet light clouds. They need further investigation.

When a cloud produced by ultra-violet light is allowed to remain in the chamber for some time without the light or the field being on, it loses its charge, and little motion can be observed if tested after a few minutes. An uncharged cloud can be readily obtained by keeping the field on after the light has been cut off. The charged droplets are simply removed and the uncharged ones remain behind.

Carbonic acid gas and oxygen give clouds like those in air. Contrary to the experience of C. T. R. Wilson I found that the cloud is formed more readily in carbonic acid gas than in air. Hydrogen does not give these clouds; the gas must however be fairly pure or they are formed readily.

A remarkable circumstance about the formation of these clouds is that they are generated even in a strong field. Thus I find in my notes, "The way in which cloud comes even with field on is very remarkable. Thus this morning with 840 volts on 9 cm. got cloud in one minute. The cloud thus formed is always a discrete cloud. It moved towards the negative plate but seemed finally to stop moving. On reversing the field it moved quickly towards the new negative plate." This does not mean that the cloud was composed entirely of positively charged drops. It means simply that the arc light was concentrated on the negative plate and that thus the positively charged particles were under observation. By a discrete cloud I mean one whose particles are big and far apart.

If ultra-violet light be allowed to act on a cloud for a long time (say 10 minutes) in the absence of any electric field, the cloud sometimes shows colours. This growth ceases after a certain time. Thinking that this was owing to the heavy drops falling out of the clouds and that this effect limited the life of any particular drop I tried to devise some way of keeping the drops in the ultra-violet light. This I did not succeed in doing. I arranged a glass tube 5 cm. in diameter and 25 cm. long as a cloud chamber. The tube was placed vertically and was closed at the top by a quartz lens and at the bottom by a

rubber bung through which tubes passed, which enabled the whole to be employed as a cloud chamber. The tube having been filled with saturated dust-free air, an aluminium spark was placed at the ultra-violet light focus of the lens. A dense cloud was formed throughout the chamber in a minute. The spark was kept on for an hour but no change in the cloud could be observed after the first 10 minutes. In this arrangement also the time that any drop is in the light is limited. As soon as a drop has fallen a centimetre or so in the tube the drops above it shield it from the light.

The light which produces these clouds is the light which Lenard has investigated with his steam-jet. In fact the production of clouds and the alteration in the appearance of the jet are practically the same phenomenon. The light to produce the clouds goes through quartz, but I could get no clouds when the light was filtered through Iceland spar or through 5 cm. of water. Iceland spar is opaque to light shorter than 2064 tenth metres, while quartz stops all light shorter than 1852 tenth metres[1], so that the effective light has a wave-length between these limits. Lenard found the wave-length of the light producing the steam-jet effect in air to be 1800 tenth-metres.

The explanation of the formation of these clouds is still to be found. The fact that their particles are charged some positively, some negatively and some not at all may be attributed to the gas in which the cloud is formed being ionised by the light which at the same time produces some chemical change which originates the drops. The ions in the gas will get on the drops and charge them. A similar thing happens with a cloud produced by a large expansion on the ions produced by X-rays, if the rays are kept on after the drops are formed[2].

But we need use no specific ionising agent and yet get charged droplets. The ions always present in the air are sufficient to charge some of the dust if this is present. On filling the cloud chamber with ordinary unfiltered air and condensing water on the dust by expansion a dense cloud is formed. The heavier particles soon fall and a few lighter ones are left floating in the chamber. These are seen as bright luminous points; on putting on a field some of these move up, others sink, while a few remain stationary. Reversing the field changes the direction of the motion.

The effect of ultra-violet light on air unfreed from dust is most marked. The cloud forms at once. A similar effect is produced by letting ultra-violet light act on air in which the

[1] Gifford, *B. A. Report*, 1898.
[2] H. A. Wilson, *Phil. Mag.* April, 1903.

ordinary dust cloud produced by expansion has not had time to completely settle.

These experiments have left many points of interest undetermined. Of these the relation of the very slow moving positive ions observed by Lenard to the cloud drops is one of the chief.

Conclusion. As far as these experiments go they seem to point to the effect of ultra-violet light on moist air being of a two-fold character.

It manufactures some unidentified substance which is less volatile than water and a solution of which in water forms the droplets. It ionises the air and the ions get on the cloud drops just as ions produced by any other process would be caught by any finely divided substance floating in the air.

I wish to thank Prof. J. J. Thomson, F.R.S., and Mr C. T. R. Wilson, F.R.S., who have kindly given me their help and advice during the course of these experiments.

Experiment to show that negative electricity is given off by a metal exposed to Röntgen Rays. By Professor J. J. THOMSON.

[*Read* 23 November 1903.]

Dorn as well as Curie and Sagnac have in different ways shown that a metal exposed to Röntgen rays gives out cathode rays: this I find can be shown very simply by mounting a small gold-leaf electroscope on a quartz support in a vessel in which a very good vacuum can be produced; when the vessel is exhausted and the gold leaves exposed to Röntgen rays they diverge and on testing they are found to have a charge of positive electricity. If before exposure to the rays the leaves are charged negatively then when the rays are applied the leaves at first collapse and then diverge, while if the initial charge is positive the divergence of the leaves increases from the time of putting on the rays. In this way we get a very direct proof that the gold leaves when exposed to the rays acquire positive and lose negative electricity.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the variation with the wave-length of the double refraction in strained glass. (Third paper.) By L. N. G. FILON, B.A., King's College.

[*Received* 23 December 1903.]

1. The investigations which form the subject of the present paper are a continuation of those described in my previous paper (*Camb. Phil. Soc. Proceedings*, Vol. XII. Part i. pp. 55—64). The apparatus employed was precisely the same and need not therefore be again described.

The glasses experimented upon were a series of glasses manufactured by Messrs Zeiss of Jena. They were selected as follows. The refractive index and mean dispersion were plotted to density from the numbers given in Zeiss's catalogue. A series of glasses were then taken, for which both refractive index and mean dispersion appeared to vary regularly with the density. The glasses thus selected were found from the same catalogue to be silicate flints.

Seven glasses in all were chosen, bearing the Jena catalogue numbers O 152, O 154, O 103, O 192, O 41, O 198, S 57. Blocks of dimensions 1 cm. × 4 cms. × 3 cms. were made for me by Messrs Zeiss, the faces 1 × 4 being polished. A rough description of the composition of each glass, and the approximate percentage of PbO were communicated to me through the courtesy of Messrs Zeiss to whom my obligations are due. The descriptions of the glasses, i.e. density, refractive index, mean dispersion and general chemical composition, are given in Table I. below.

TABLE I.

No.	Refractive Index D line	Medium Dispersion C to F	Specific Gravity	General Chemical Composition	Per cent. of PbO
O 152	1·5368	·01049	2·76	PbO, K_2O, Na_2O, [CaO], SiO_2	16
O 154	1·5710	·01327	3·16	PbO, K_2O, Na_2O, [ZnO], [BaO], SiO_2	31
O 103	1·6202	·01709	3·63	PbO, K_2O, SiO_2	47
O 192	1·6734	·02104	4·10	PbO, K_2O, SiO_2	56
O 41	1·7174	·02434	4·49	PbO, K_2O, SiO_2	63
O 198	1·7782	·02941	4·99	PbO, [K_2O], SiO_2	79*
S 57	1·9626	·04882	6·33	PbO, SiO_2	79

N.B. The materials in brackets are only used in very small quantities (1—2 per cent.).

2. The method of conducting the experiments was practically the same as the one described in the previous paper referred to. One or two alterations of detail, however, were made. Thus, after the first two sets of experiments on O 103 it was decided to remove one of the prisms in the spectroscope, as the additional accuracy in reading the circle was more than compensated by the greater diffuseness of the band, which practically rendered observations in the violet useless, the very broad hazy band being almost undistinguishable from the faint background. The removal of the one prism narrowed the band, and by decreasing the absorption due to the large thicknesses of glass traversed, considerably brightened the surrounding field in the violet.

With regard to the errors of measurement made under the new conditions, I took the probable errors on four different occasions, when the conditions were good, moderate, and poor. The probable errors were found by taking the means of differences of two readings which should have been the same and dividing by $\sqrt{2}$. They were found to be $0'{\cdot}6$, $1'{\cdot}1$, $1'{\cdot}1$, $1'{\cdot}4$ respectively, and these were found from observations in all parts of the spectrum, including the violet, in which the discordances are always greater. The accuracy in the brighter parts of the spectrum was probably better. Nevertheless, we may assume that, as far as the deter-

* I cannot help thinking there must be some error in the composition stated for this glass, as, comparing with the one next in the list, it seems unlikely that a *small* percentage of K_2O should cause such a large difference in specific gravity. This however does not affect the present paper, for no results, as will be seen, could be obtained with O 198.

mination of the true position of the fringe is concerned, the probable error is well represented by 1′. The range of observation was about 4°, representing a total change of wave-length of 2000. An error of 1′ will then correspond on an average to 8 tenth metres, so that the wave-length of extinction is determined with a proportional error of less than 1 in 600 (taking the mean value $\lambda = 5400$).

Now there exist discrepancies in the results from the same glass, taken on different days, which far exceed this amount. Even in the curves corresponding to a set of observations taken at one time, there are irregularities greater than these probable errors can account for, although they are usually less than the differences between observations of different sets. The only conclusion possible is that there must be considerable changes in the distribution of stress in the block from day to day. I had noticed these differences already in the earlier experiments (*Camb. Phil. Soc. Proceedings*, Vol. XII. Pt. i. p. 63), but I had supposed (*a*) that they were due to sudden jars or shake, so that, if they did occur in a set of observations taken in rapid succession, they would at once reveal themselves by an obvious discontinuity in the results; (*b*) that as they had not so revealed themselves, the observations of one set, although not comparable with the observations of another, so far as the absolute term was concerned, gave reliable results as regards the *relative variation* with the wave-length of the quantity C.

3. Later experiments, however, showed me convincingly, that although this might be true in several cases, there remained a much subtler and more dangerous source of error than this.

Even when the glass block has been so adjusted that it shows a uniform tint between crossed Nicols for any given total load (in my case I had usually chosen the load giving the tint of passage) it does not remain so for other loads. And this change is not an irregular one, but is, on the contrary, perfectly regular and *reversible*. The mode of bearing of the glass block upon the compressing planes has a definite dependence on the total load and, after many attempts, I found it practically impossible to ensure that the distribution of stress should remain uniform for any considerable range of load.

My attention was first called to this when, having observed the very definite black band for first extinction in O 152, I then doubled the load, to observe the band for second extinction. The band was practically invisible, and the cause of this turned out to be, on direct examination, that the stress was now no longer uniform. When the load was reduced, the stress again became uniform. When the block was adjusted so that the stress was

uniform for the neutral tint of second order, it was no longer uniform for the neutral tint of first order.

4. I then proceeded to investigate what the effect of such a want of uniformity must be. Near the middle of the block, at a distance from the compressing planes, the stress must be very nearly that due to a bending moment M (amount unknown) about a horizontal axis at right angles to the plane of loading, superimposed upon a uniform pressure T equal to the calculated mean pressure.

The actual stress at any point is therefore

$$T + \frac{My}{(abK^2)},$$

where a, b are the thickness and breadth of the block, K the radius of gyration of its horizontal section about the diameter parallel to the side a, y is the horizontal distance of the point considered from this diameter.

Denoting this by $T + Sy$, we have relative phase-retardation of the two components of the ray traversing the glass at distance y from the vertical mid-plane

$$= 2\pi \frac{Ca}{\lambda} (T + Sy).$$

The intensity of the light restored, due to a beam of light of cross-section $dx dy$, will be

$$A \sin^2 \left[\frac{\pi Ca (T + Sy)}{\lambda}\right] dx\, dy,$$

if the axes of the Nicols are inclined at 45° to the stress, A being the original intensity of the light per unit area of cross-section.

Hence the total intensity of the light restored, when the aperture is a rectangle of height $2h$ and breadth $2k$, is equal to

$$\int_{-h}^{+h} \int_{-k}^{+k} A \sin^2 \left\{\frac{\pi Ca (T + Sy)}{\lambda}\right\} dx dy,$$

T being now the stress corresponding to the centre of the rectangular aperture.

This integral is easily evaluated and reduces to

$$2Ahk \left[1 - \frac{\lambda}{2\pi CaSk} \sin\left(\frac{2\pi CaSk}{\lambda}\right) \cos\left(\frac{2\pi CaT}{\lambda}\right)\right].$$

The darkest part of the black band will then correspond to the

value of λ which makes the expression in square brackets a minimum. Obviously that expression is never actually zero, since

$$\frac{\lambda}{2\pi CaSk}\sin\left(\frac{2\pi CaSk}{\lambda}\right)$$

is always numerically less than unity. Accordingly a perfectly black band is not to be expected, and the pale character of the band may be taken as a good measure of the want of uniformity in the stress.

Let us write

$$\frac{2\pi CaT}{\lambda}=x,\quad \frac{Sk}{T}=p.$$

Then we have to find for what value of x the quantity

$$u=A\left(1-\frac{\sin px}{px}\cos x\right)$$

is a minimum.

$$\frac{du}{dx}=0, \text{ when}$$

$$\left(\frac{x}{A}\frac{dA}{dx}\right)(px-\cos x\sin px)$$
$$-\,px\cos px\cos x+\sin px\cos x+x\sin px\sin x=0.$$

Now p will in general be a small quantity, if the stress has been adjusted to be roughly uniform, so that we may suppose $\cos px$ and $\sin px$ expanded in powers of px. If we then neglect 5th powers, we find

$$\frac{1}{A}\frac{dA}{dx}\left\{1-\cos x\left(1-\frac{p^2x^2}{6}\right)\right\}+\cos x\left(\frac{p^2x}{3}\right)+\sin x\left(1-\frac{p^2x^2}{6}\right)=0.$$

If p were zero, the *minima* (and these are the only solutions of the above equation we are concerned with) would be given by

$$x=2n\pi,$$

n being an integer.

Let us assume $x=2n\pi-y$,

$$\frac{1}{A}\frac{dA}{dx}(1-\cos y)-\sin y\left(1-\frac{p^2x^2}{6}\right)+\cos y\left(\frac{p^2x}{3}+\frac{p^2x^2}{6}\frac{1}{A}\frac{dA}{dx}\right)=0,$$

y is here small. If we neglect its square, we find, for a first approximation

$$y=\frac{p^2x}{3}\left(1+\frac{x}{2A}\frac{dA}{dx}\right).$$

But here $\frac{dx}{x} = -\frac{d\lambda}{\lambda}$ (neglecting the small change in C), so that

$$y = \frac{2p^2n\pi}{3}\left(1 - \frac{\lambda}{2A}\frac{dA}{d\lambda}\right)$$

approximately.

Hence the proportional error in x is

$$\frac{p^2}{3}\left(1 - \frac{1}{2}\frac{\lambda}{A}\frac{dA}{d\lambda}\right).$$

Now $\frac{\lambda}{A}\frac{dA}{d\lambda}$ is the ratio of the proportional change in intensity to the corresponding proportional change in wave-length. Apart from the fact that, in estimating the blackness of a band, the eye compares with the intensity of the original field in that neighbourhood, rather than with an absolute standard, it seems unlikely that a change of 5 tenth-metres can anywhere (except perhaps right at the very ends of the range of visibility) imply more than 1 per cent. change in the intensity: so that about $\frac{1}{10}$ per cent. of λ to 1 per cent. of A, or $\frac{\lambda}{A}\frac{dA}{d\lambda} = 10$ numerically, appears an extreme estimate.

5. The following experiments were undertaken with the view of obtaining the order of the error introduced in this way. A fine vertical slit was placed, first at one edge of the block and then at the other, and readings of the black band were taken for these two positions of the slit.

They gave:

	reading	approximate λ
Slit at right-hand edge	48° 30′	4300
„ „ left „ „	46° 34′	5000

Distance between centres of slit in the two positions = 9 mm. roughly.

Assuming the stress-optical coefficient to be roughly constant over this range of λ, the extreme stresses are nearly in the ratio 43 : 50.

Therefore

$$\frac{\text{change in stress for 9 mm.}}{\text{stress in middle}} = \frac{7}{\left(\frac{43+50}{2}\right)} = \frac{14}{93},$$

$$\frac{\text{change in stress per mm.}}{\text{stress in middle}} = \frac{S}{T} = \frac{14}{837} = \frac{1}{60} \text{ nearly.}$$

Now the width of the slit used in the ordinary experiments was 2·5 mm., so that $k = 1·25$;

$$\therefore\ p = \frac{Sk}{T} = \frac{1·25}{60} = ·02 \text{ approximately,}$$

$$\tfrac{1}{3}p^2\left(1 - \frac{1}{2}\frac{\lambda}{A}\frac{dA}{d\lambda}\right) < \tfrac{1}{3}(·0004)\left(1 + \frac{10}{2}\right) < ·0008 \text{ (numerically),}$$

and is certainly negligible.

It is to be noted that this was a case where the stress was far from uniform. It was certainly less uniform than in the average observation. It seems reasonable therefore to assume that no sensible part of the error is contributed by this cause.

6. There remain two causes of error, (a) the possibility that the stress distribution near the middle of the block is not exactly of the form $T + Sy$; (b) the fact that the vertical centre-line of the slit used (which had to be shifted for each experiment) was not accurately opposite the vertical axis of the block.

Theoretical considerations, developed in another paper ("On an approximate solution for the bending of a beam of rectangular cross-section under any system of load," *Phil. Trans.* A. Vol. 201, pp. 63—155), make it unlikely that, at a distance from the points of application of stress equal to about twice the breadth, the influence of local perturbations should have been sensible. In the experiments under consideration, this condition was nearly satisfied for rays passing through the centre of the block, the height being four times the breadth. There is still the possibility that another type of stress may have made itself felt, for instance, torsion about a vertical axis. Such torsion might have been caused by friction against the compressing planes, owing to some want of symmetry in the application of the load. But such want of symmetry could hardly have caused a torsion couple sufficient to affect the observations.

(b) then remains as the probable cause of the discrepancies. It was impossible to adjust the diaphragm so that its axis was exactly central. If there was a small error ϵ in the position of the diaphragm, the stress corresponding to the observed black band was no longer T but $T + S\epsilon$. Now S was not simply proportional to the total load; this followed clearly from the experiments mentioned in § 3 above. Accordingly the stress varied according to some unknown law, and although this variation was probably small as a whole, yet it may account for the discrepancies observed in the curves deduced from observations of different sets.

7. In order to obviate as much as possible this source of error, in the later sets of experiments the slit was placed with its edge coincident with each of the two vertical edges of the block, in succession. This adjustment could be made with considerable exactness. The readings of the black band for these two extreme positions were noted, and the slit moved until the reading of the black band was the mean of the extreme readings. When possible, the position of the black band was measured without narrowing the beam by a diaphragm. This could be done if the stress was sufficiently uniform, and this should have given a result free from error. As a rule, however, this was only possible over a limited part of the range. The diaphragm was then introduced, and adjusted so that its effect was to increase the definition of the band without shifting its position.

In spite of these precautions, the results are not so consistent as could be wished. The method adopted has been to take several series of observations for each glass, at different times and under different conditions. Characteristic variations which are reproduced in such different sets may safely be taken as significant.

8. The results of the observations are given in Tables II.—VII. below.

In what follows C is expressed throughout in a unit equal to 10^{-7} sq. cms. per kilogramme weight.

TABLE II. O 152.

A. λ	A. c	B. λ	B. c	C. λ	C. c
4334	2·815	4463	2·820	4291	2·865
4600	2·824	4610	2·802	4451	2·853
4896	2·791	4753	2·783	4602	2·836
5188	2·761	4889	2·765	4739	2·816
5467	2·734	5000	2·751	4810	2·788
5653	2·711	5161	2·745	5343	2·733
5945	2·690	5295	2·725	5912	2·683
6255	2·679	5434	2·713		
6520	2·655	5555	2·713		
		5693	2·698		
		5828	2·682		
		5985	2·680		
		6098	2·679		
		6259	2·677		
		6387	2·659		
		6540	2·656		
		6615	2·642		

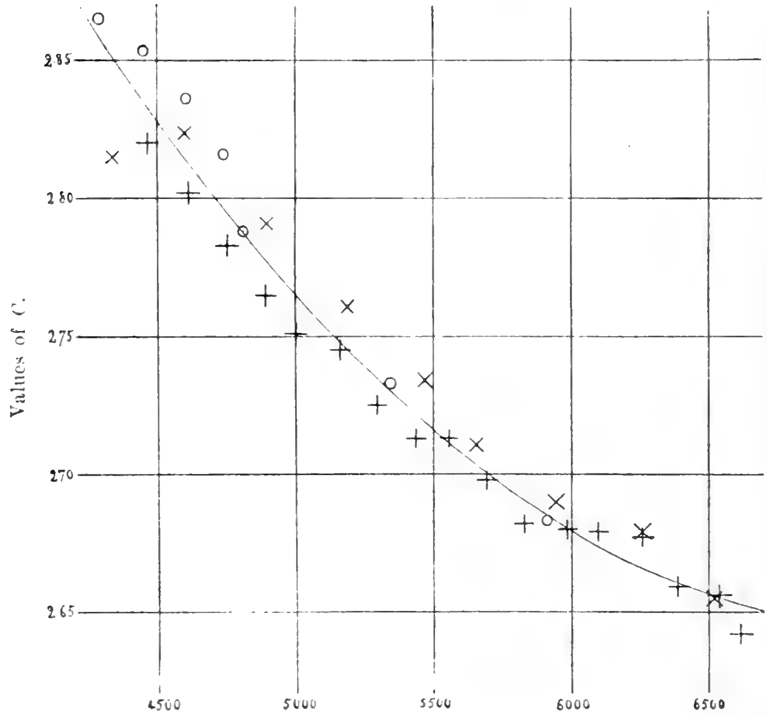
Remarks on O 152.

A. June 30. Central diaphragm used. Band observed for *first* extinction.

B. and C. June 27 and June 30. Central diaphragm. Band observed for *second* extinction.

Fig. 1. Glass O 152.

——— $C = 2{\cdot}725 - {\cdot}091\left(\frac{\lambda - 5400}{1000}\right) + {\cdot}026\left(\frac{\lambda - 5400}{1000}\right)^2.$



Wave-lengths in tenth-metres.

× Observations A. } Table II.
+ ,, B.
⊙ ,, C.

TABLE III. O 154.

A. λ	A. c	B. λ	B. c	C. λ	C. c
4492	2·917	4450	3·057	4308	2·959
4705	2·888	4560	3·043	4440	2·963
4965	2·830	4860	2·994	4595	2·944
5295	2·817	5024	2·911	4784	2·947
5600	2·800	5362	2·897	4910	2·916
5797	2·780	5623	2·876	4998	2·896
6100	2·760	5900	2·836	5156	2·883
6398	2·740	6211	2·818	5332	2·880
		6337	2·793	5487	2·871
		6630	2·769	5617	2·873
				5783	2·866
				5935	2·853
				6094	2·847
				6291	2·854
				6450	2·842
				6647	2·851
				6759	2·823

Remarks on O 154.

A. July 2. (Diaphragm central.) Band observed for first extinction. *Band very hazy and diffuse, even in the yellow.*

B. July 4. Diaphragm found afterwards to be not quite central. Band observed for second extinction. Band was far more difficult to observe than for the first extinction and was quite invisible when no diaphragm was interposed, even when in the ordinary test between crossed Nicols the glass showed a uniform tint.

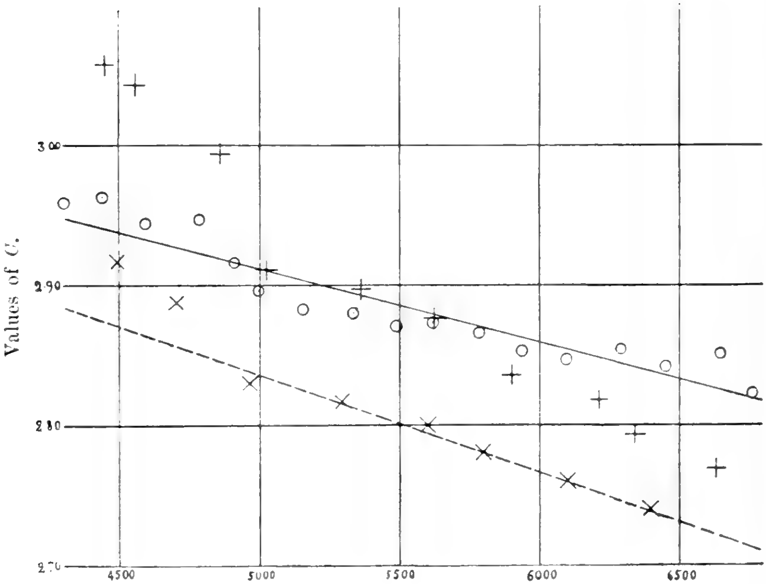
C. July 6. Experiments of July 4 repeated with the diaphragm more accurately central.

[The observations for this glass were troublesome and its behaviour appeared peculiar.]

Fig. 2. Glass O 154.

———— $C = 2{\cdot}891 - {\cdot}0525\left(\frac{\lambda - 5400}{1000}\right)$.

- - - - - - $C = 2{\cdot}808 - {\cdot}070\left(\frac{\lambda - 5400}{1000}\right)$.



Wave-lengths in tenth-metres.

× Observations A. } Table III.
\+ ,, B.
⊙ ,, C.

TABLE IV. O 103.

A.		B.		C.	
λ	c	λ	c	λ	c
4456	2·880	4521	2·922	4221	2·967
4657	2·870	4708	2·902	4422	2·950
4872	2·869	4885	2·876	4518	2·921
5065	2·854	5084	2·865	4718	2·908
5180	2·842	5195	2·851	4884	2·876
5360	2·824	5374	2·831	5070	2·857
5550	2·812	5555	2·814	5204	2·855
5736	2·798	5750	2·805	5395	2·842
5850	2·789	5851	2·789	5575	2·825
6034	2·776	6030	2·775	5788	2·823
6232	2·771	6220	2·766	5894	2·810
6433	2·766	6416	2·759	6083	2·799
6585	2·775	6517	2·746	6283	2·794
				6470	2·782
				6600	2·781

Remarks on O 103.

A. April 6. No diaphragm used. Double prism in spectroscope.

B. April 7. Narrow central diaphragm. Double prism.

C. April 8. Central diaphragm. Single prism.

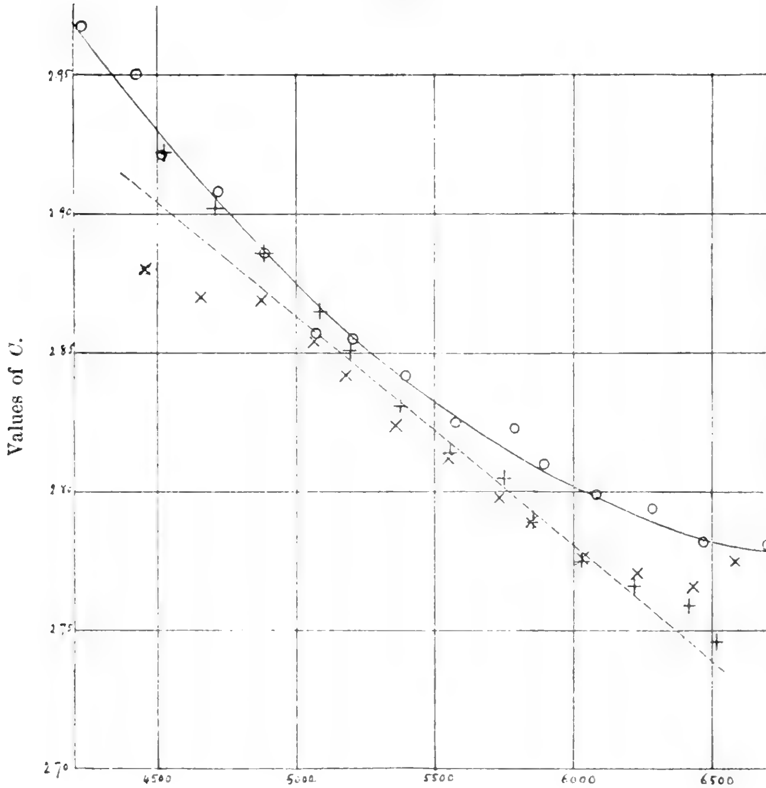
[A. and B. are the only two sets of observations for which the two prisms were used.

The band for first extinction was the only one observed.]

Fig. 3. Glass O 103.

——— $C = 2{\cdot}840 - {\cdot}0783\left(\frac{\lambda - 5400}{1000}\right) + {\cdot}0236\left(\frac{\lambda - 5400}{1000}\right)^2.$

- - - - - - $C = 2{\cdot}83 \quad - {\cdot}0825\left(\frac{\lambda - 5400}{1000}\right).$



Wave-lengths in tenth-metres.

× Observations A. ⎫
\+ ,, B. ⎬ Table IV.
⊙ ,, C. ⎭

TABLE V. . O 192.

A. λ	A. c	B. λ	B. c	C. λ	C. c	D. λ	D. c	E. λ	E. c
4354	2·271	4490	2·246	4330	2·258	4235	2·292	4461	2·284
4506	2·254	4790	2·261	4503	2·253	4363	2·290	4705	2·270
4768	2·250	5006	2·236	4760	2·247	4461	2·284	4917	2·263
5009	2·238	5241	2·227	5006	2·236	4717	2·276	5155	2·249
5241	2·227	5420	2·226	5238	2·226	4905	2·257	5354	2·239
5386	2·212	5661	2·216	5403	2·219	5145	2·244	5546	2 209
5639	2·207	5915	2·212	5651	2·212	5352	2·238	5803	2·224
5877	2·197	6134	2·199	5890	2·202	5577	2·221	6037	2·212
6108	2·189	6301	2·195	6112	2·191	5784	2·217		
6242	2·174	6546	2·189	6270	2·184	6001	2·199		
6500	2·173			6503	2·174	6194	2·191		
6667	2·144			6720	2·161	6410	2·175		
						6645	2·182		
						6830 ?	2·158 ?		

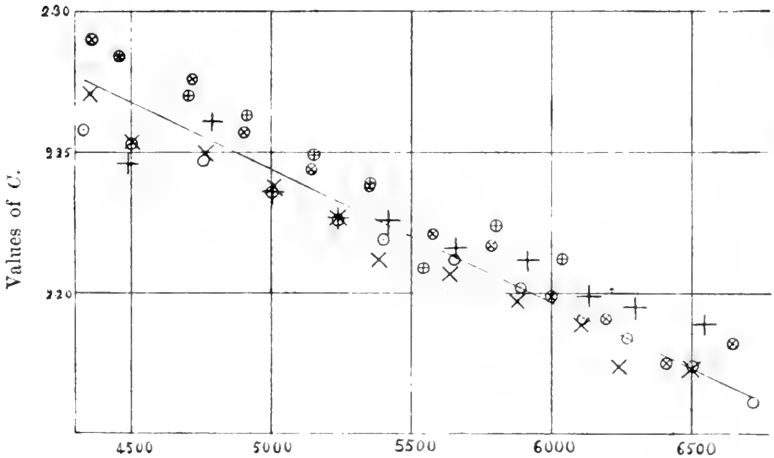
Remarks on O 192.

A. July 6. Diaphragm central. Band observed for first extinction.
B. July 7. No diaphragm used. „ „ „ „ „
C. July 7. Diaphragm central. „ „ „ „ „
D. July 8. „ „ „ „ „ second „
E. July 9. No diaphragm used. „ „ „ „ „

[Band very black and well defined, throughout this set of observations.]

Fig. 4. Glass O 192.

——— $C = 2{\cdot}225 - {\cdot}047\left(\frac{\lambda - 5400}{1000}\right)$.



Wave-lengths in tenth-metres.

× Observations A.
+ ,, B.
⊙ ,, C.
⊗ ,, D.
⊕ ,, E.
Table V.

TABLE VI. O 41.

A. λ	A. C	B. λ	B. C
4583	1·713	4585	1·715
4723	1·730	4787	1·737
4870	1·730	4924	1·734
5047	1·740	5075	1·735
5163	1·729	5175	1·738
5295	1·742	5335	1·743
5454	1·745	5482	1·744
5623	1·751	5640	1·747
5714	1·733	5740	1·750
5882	1·756	5876	1·747
6020	1·752	6025	1·748
6119	1·738	6150	1·741
6260	1·736	6267	1·749
6347	1·734	6402	1·746
6475	1·729	6540	1·743
6620	1·728	6683	1·743
6715	1·714		

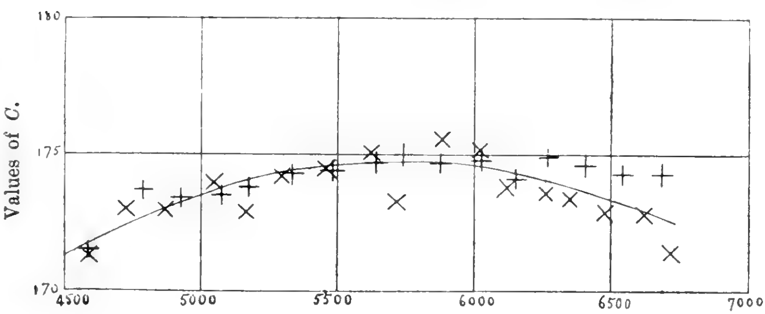
Remarks on O 41.

A. April 17 (morning). B. April 17 (afternoon).

Both taken with a narrow central diaphragm and both correspond to first extinction.

Fig. 5. Glass O 41.

——— $C = 1{\cdot}745 + {\cdot}0156\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}0236\left(\frac{\lambda - 5400}{1000}\right)^2$.



Wave-lengths in tenth-metres.

+ Observations B. } Table VI.
× ,, A. }

TABLE VII. S 57.

A.		B.		C.	
λ	c	λ	c	λ	c
4550	− 1·429	4544	− 1·427	4580	− 1·478
4677	− 1·362	4780	− 1·313	4784	− 1·354
4771	− 1·311	4984	− 1·216	4979	− 1·247
4878	− 1·254	5190	− 1·139	5200	− 1·171
4985	− 1·217	5388	− 1·074	5377	− 1·100
5090	− 1·170	5589	− 1·020	5579	− 1·046
5178	− 1·136	5812	− ·977	5772	− ·999
5300	− 1·102	6001	− ·938	5946	− ·955
5387	− 1·074	6204	− ·905	6147	− ·921
5500	− 1·044	6400	− ·870	6360	− ·888
5592	− 1·021				
5701	− ·995				
5819	− ·978	D.		E.	
5923	− ·955	4620	− 1·494	4538	− 1·464
6016	− ·941	4825	− 1·366	4746	− 1·344
6124	− ·921	4978	− 1·247	4935	− 1·236
6216	− ·907	5181	− 1·166	5142	− 1·158
6341	− ·892	5343	− 1·093	5335	− 1·091
6447	− ·876	5511	− 1·033	5517	− 1·034
		5690	− ·984	5695	− ·985
		5848	− ·939	5895	− ·947
		6010	− ·901	6092	− ·913
				6320	− ·883

Remarks on S 57.

A. June 23 (afternoon). Central diaphragm.

B. ,, ,, (morning). ,, ,,

C. July 9. Diaphragm adjusted to give same reading as that obtained when no diaphragm is put in.

D. July 10 (morning). No diaphragm used.

E. July 10 (afternoon). Central diaphragm.

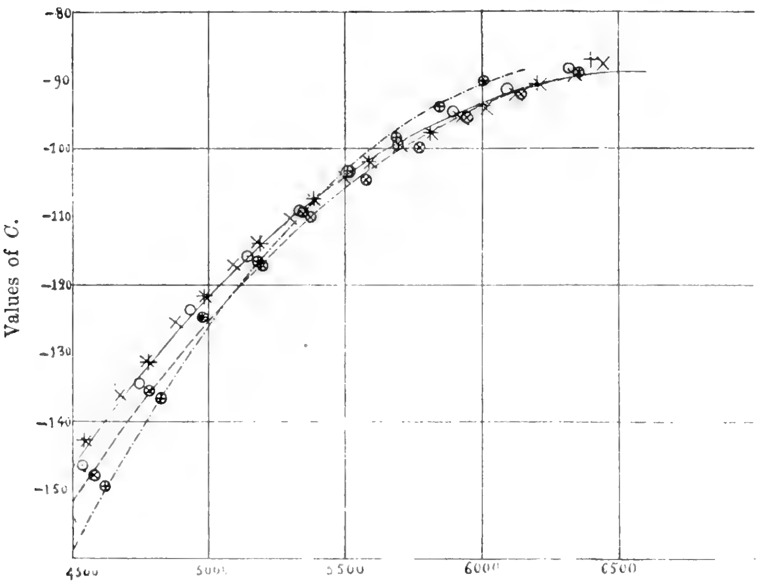
[Only the first extinction was observed.]

Fig. 6. Glass S 57.

$$\text{———}\quad C = -1{\cdot}070 + {\cdot}3156\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}1375\left(\frac{\lambda - 5400}{1000}\right)^2.$$

$$\text{------}\quad C = -1{\cdot}093 + {\cdot}3438\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}1438\left(\frac{\lambda - 5400}{1000}\right)^2.$$

$$\text{-·-·-·-}\quad C = -1{\cdot}071 + {\cdot}3967\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}2017\left(\frac{\lambda - 5400}{1000}\right)^2.$$



Wave-lengths in tenth-metres.

× Observations A.
+ ,, B.
⊗ ,, C. } Table VII.
⊕ ,, D.
⊙ ,, E.

No observations could be taken of the glass O 198 as the highest pressure which could be attained by means of the instrument, namely about 250 kilogrammes weight per sq. cm., was insufficient to reach the first extinction in this glass.

9. The results are exhibited graphically in Figs. 1—6. In calculating the constants of the curves to fit the observations the mean wave-length λ 5400 has been taken as a convenient origin.

In one case, namely S 57, the observations require a parabolic curve, without a shadow of doubt.

The observations, however, fall into three groups, which are best fitted by three different parabolas. One of these accounts fairly well for the *three* sets A., B. and E. of Table VII. If we take it as giving the best determination, then the stress-optical coefficient for this glass is given by the formula

$$C = -1{\cdot}070 + {\cdot}316\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}1375\left(\frac{\lambda - 5400}{1000}\right)^2.$$

As this glass has been examined by F. Pockels (*Annalen der Physik*, April, 1902) it was of interest to compare his values with mine. I therefore reduced his results so as to bring them into a form comparable with mine.

He determined the coefficient C for three wave-lengths only, namely those for lithium, sodium and thallium light. On examination of his reductions, I find that his assumed wave-lengths (although they are not explicitly stated in the paper referred to) were probably λ 6730 for lithium and λ 5360 for thallium.

From these I obtained the following

Glass S 57. Values of C.

Pockels	Present experiments	λ
$-2{\cdot}025$	$-1{\cdot}083$	5360
$-1{\cdot}89$	$-{\cdot}953$	5893
$-1{\cdot}76$	$-{\cdot}890$	6730

Thus, the absolute values found by Pockels for the stress optical coefficient are nearly twice my own. It may be that I have omitted a factor $\frac{1}{2}$ in the reduction of Pockels' observations, although, on further examination, I am unable to find any such omission and, in fact, the reduction, for this particular glass, has been practically done by Pockels himself, for in one place he gives the values of his $(\delta_x - \delta_z)\dfrac{B}{P}\dfrac{\lambda}{\lambda_{Na}}$ for S 57, and this, multiplied by the wave-length of sodium light, gives my C. The two sets of values agree fairly nearly, as they should.

On the other hand, I am quite satisfied that no error of this kind has entered into the reduction of my own observations.

The only possible conclusion, so far as I can see, is that the

composition of the two specimens supplied to Herr Pockels and to myself was not by any means identical. Indeed the curve given by Pockels in the paper referred to, showing the variation of C with the percentage of PbO, indicates that for glasses such as S 57 which contain near 80 per cent. of PbO the variation in C is extraordinarily rapid.

Thus the amount of PbO in my glass was stated by Messrs Zeiss to be about 79 per cent. The amount in the specimen examined by Pockels was 80·05 per cent. It seems therefore that this may quite well account for the difference, especially when we remember that C was found to vanish for a glass containing about 74 or 75 per cent. of PbO.

If we turn to the rate of change of C with the wave-length, we find

	Pockels	Present experiments
Mean change per tenth-metre between λ 5360 and λ 5893...	+ ·000253	+ ·000244
Mean change per tenth-metre between λ 5360 and λ 6730...	+ ·000193	+ ·000141

Here the agreement is considerably better, which would seem to suggest that the dispersion of double-refraction does not here vary quite so fast with the composition as the double-refraction itself. Again Pockels' results and my own agree in making C vary with λ far more rapidly in the green than in the red; a result still more marked when we take observations in the blue, as is shown in Fig. 6.

10. In the observations of the other glasses, the curvature is not so marked.

In the case of O 41 and O 152, more especially the latter, a straight line gave a decidedly poor fit, and the following formulae for C were adopted.

O 41,

$$C = 1{\cdot}745 + {\cdot}0156\left(\frac{\lambda - 5400}{1000}\right) - {\cdot}0236\left(\frac{\lambda - 5400}{1000}\right)^2.$$

O 152,

$$C = 2{\cdot}725 - {\cdot}091\left(\frac{\lambda - 5400}{1000}\right) + {\cdot}026\left(\frac{\lambda - 5400}{1000}\right)^2.$$

The case of O 103 is more doubtful. A set of good observations (C.) is well fitted by the parabola

$$C = 2{\cdot}840 - {\cdot}0783\left(\frac{\lambda - 5400}{1000}\right) + {\cdot}0236\left(\frac{\lambda - 5400}{1000}\right)^2.$$

The sets A. and B. are fairly accurately represented by a straight line

$$C = 2{\cdot}83 - {\cdot}0825\left(\frac{\lambda - 5400}{1000}\right),$$

except towards the ends of the range, where they seem to show a curvature, which may be significant.

Two of the glasses, O 192 and O 154, indicate an approximately linear variation.

For O 192,

$$C = 2{\cdot}225 - {\cdot}047\left(\frac{\lambda - 5400}{1000}\right)$$

is a good representation of the observations.

For O 154, the three groups agree very badly with one another. Group C. is fitted tolerably well by the formula

$$C = 2{\cdot}891 - {\cdot}0525\left(\frac{\lambda - 5400}{1000}\right).$$

Group A. suggests

$$C = 2{\cdot}808 - {\cdot}070\left(\frac{\lambda - 5400}{1000}\right),$$

between the green and the red. There are two outlying points, but no great significance should be attached to them.

Group B. is rather irregular. Altogether the observations of this glass are unsatisfactory.

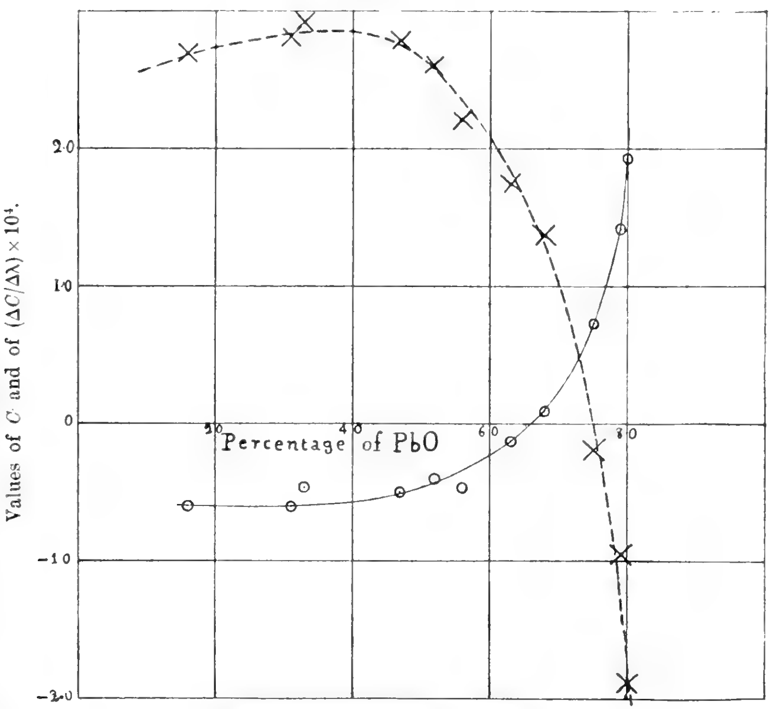
11. In order to be able to trace better the effect of the chemical composition upon the optical properties, I calculated, for a number of the glasses which have been examined by Pockels, my coefficient C. I also calculated in each case the mean change in C per tenth-metre, over the range from λ 5360 to λ 6730.

The results, grouped together, are given in Table VIII. below.

TABLE VIII.

Percentage of PbO	Number of Glass	C_{Na}	$\left[\frac{\Delta C}{\Delta\lambda}\right]_{Tl}^{Li}$	Remarks
0	O 428	2·768	+ ·000057	Borate
16	O 152	2·686	– ·000060	
25	O 658	3·776	+ ·000005	Borate
31	O 154	2·81	– ·000061	
33	O 2154	2·923	– ·000047	Borate (small quantity)
47	O 103	2·79	– ·000050	
52	O 1571	2·594	– ·000040	
56	O 192	2·202	– ·000047	
63	O 41	1·747	– ·000013	
68	O 500	1·375	+ ·000009	
75	O 2954	– ·190	+ ·000073	
79	S 57 (Filon)	– ·953	+ ·000141	
80	S 57 (Pockels)	– 1·890	+ ·000193	

Fig. 7.



× Points corresponding to values of C.

⊙ ,, ,, ,, ,, $\left[\frac{\Delta C}{\Delta\lambda}\right]_{Tl}^{Li}$.

In the above, to get C in sq. cms. per kilogramme weight, multiply by 10^7.

These results, with the exception of O 428 and O 658, are shown in Fig. 7. There are one or two points which are irregular, for example those due to O 192 and O 2154. These indicate an irregularity both in the curve of C and in that of $\frac{\Delta C}{\Delta\lambda}$, so that they are probably explained by the presence of some disturbing constituent.

In the case of O 2154 this may be due to the fact that this glass contains a small percentage (1·5) of B_2O_3, and the presence of this substance appears to have a very decided effect upon both the stress-optical coefficient and its variation with the wave-length. Thus both O 428 and O 658, which are borate glasses, show a comparatively high C. A glass, S 205, containing 69 per cent. of B_2O_3, examined by Pockels, gives $C = 4{\cdot}33$ for Na-light, and $\left[\frac{\Delta C}{\Delta\lambda}\right]_{Tl}^{Li} = -{\cdot}000084$. The other two borate glasses O 428 and O 658 show a *positive* $\frac{\Delta C}{\Delta\lambda}$ with a *high positive* C, quite an exceptional result.

The present series of experiments, however, was intended solely to deal with lead glasses and the consideration of borate glasses may here be postponed.

In conclusion, these experiments confirm, in every way, the results obtained by Pockels for the effect of lead on artificial double-refraction produced by stress for a given wave-length. They indicate further that the difference of the refractive indices (to which C is proportional) is greater in the violet than in the red for those lighter glasses which are not borates. That as the percentage of lead increases, this variation in the difference of the refractive indices decreases: it is probably zero, or Wertheim's law is obeyed, for a glass containing about 66 per cent. of PbO. After that, the sign of the variation is altered and it begins to increase rapidly numerically. For glasses nearly on the limit of high density C and its rate of change with the wave-length increase with great rapidity (the initial sign of both, however, being now changed). For such glasses the variation of C is parabolic, the difference of the refractive indices increasing much more rapidly towards the violet than it decreases towards the red. There are indications of a similar phenomenon in some of the other glasses, but in many cases, the variation appears, at all events to the first approximation, to follow a linear law.

I wish to renew my thanks to Professor Trouton, F.R.S., and to Assistant-Professor Porter, of University College, London, for their continued courtesy and kindness during the progress of these experiments, which were carried out in the Physical Laboratory of University College, London.

Note on the measurement of the dispersion of double refraction in strained glass. By L. N. G. FILON, B.A., King's College.

[*Received* 23 December 1903.]

In a note published in the *Annalen der Physik* (1903, Series 4, Vol. XI. part (3)), Herr F. Pockels takes objection to some statements made by myself in a paper "On the variation with the wave-length of the double refraction in strained glass" (*Camb. Phil. Soc. Proceedings,* Vol. XI. pp. 491—492).

He points out that for the lightest glass he has examined, viz., the Jena glass S 205, the change in his quantity $n(p-q)/\omega$[1] which is proportional to my stress-optical coefficient C amounts to 3 per cent. as we pass from red to green light, whereas in one of the heavier glasses, O 500, the dispersion of double refraction is sensibly nil.

Accordingly my statement (*loc. cit.* p. 491) that Herr Pockels had found no such dispersion for light glasses, but an appreciable one in heavier glasses, requires qualification, and I withdraw it therefore, with many apologies to Herr Pockels. I may, however, plead for my excuse that it was his own general statement (*Annalen der Physik,* April, 1902, p. 765) that the dispersion of double refraction was "bei den leichteren Gläsern sehr gering" which led me into this error.

With regard to my other criticism, concerning the significance of apparent dispersion of double refraction introduced by the correction applied for dispersion of double refraction in the quartz plates of the compensator, I quite agree with Herr Pockels that in his case the correction should be applied, since it is theoretically necessary. At the same time he himself (*loc. cit.* p. 751) admits a possible error of 1 or 2 per cent. in the observations, which is precisely the order of the correction applied.

Now it appears to me that the significance of numerically equal differences in the shift of the fringes is not the same when the difference is due to the correction, and when it is the outcome of direct observation. A difference observed directly may not be accurately measured, but it nearly always represents a *real* shift which the observer has noticed. A positive result of this kind has more weight than the negative result that there is *no* shift of the fringes *visible*; for an observer is more likely to overlook a shift which exists than to record one which does not. Of course this is, to a certain extent, a matter of opinion; for my own part I still feel that differences found in this way are less reliable than those directly observed.

[1] See F. Pockels, "Ueber die Aenderung des optischen Verhaltens verschiedener Gläser durch elastische Deformation," *Annalen der Physik*, April, 1902.

On Differences between the Spectra at Anode and Kathode in certain gases, and on the probable reasons for these differences. By G. D. LIVEING, M.A., Professor of Chemistry.

[*Read* 18 January 1904.]

It is well known that the spectrum of the glow about the kathode, when a continuous electric discharge passes through rarefied gas, is, in some cases, different from that of the light emitted by the gas in other parts of the tube.

In the case of nitrogen both these spectra were carefully described by Ångström and Thalén in 1875[1], and were distinguished by them from the, so-called, line spectrum, which is produced in nitrogen by a disruptive discharge. They noticed that the glow about the kathode increased, and became more brilliant, as the gas was more rarefied, but they attributed the light at the anode to an oxide of nitrogen. Another element well known to give two spectra in the same tube at reduced pressure is hydrogen[2], but I am not aware that anyone hitherto has pointed out that these two spectra bear to each other a relation similar to that which subsists between the two channelled spectra of nitrogen, namely, that one, the series of rays corresponding to the Fraunhofer lines *C*, *F*, *G'*, *h* and Huggins' ultra-violet series, is that of the kathode glow, while the second spectrum, also consisting of lines, but much more numerous, is that of the anode. It is as easy to shew this as to shew the corresponding facts with regard to the spectra of nitrogen. To shew it clearly it is necessary to have the hydrogen as pure as possible. I have used hydrogen occluded in palladium which was enclosed in a glass tube fitted with a stopcock, which may be called stopcock (*a*). This tube, beyond (*a*), was sealed by fusion to a long and wide tube loosely filled with phosphoric anhydride, also having a stopcock (*b*) at its farther end, beyond which it was sealed to the sparking tube, and this in turn to a mercury pump. Stopcock (*b*) being open and (*a*) shut, the air in the system was exhausted as completely as the pump would do it. Stopcock (*b*) was then closed and (*a*) opened to allow hydrogen from the palladium to fill the tube holding phosphoric anhydride, and then (*a*) was closed again. The whole was left in this state all night. Next morning on passing a spark the tube shewed the well known

[1] "Recherches sur les spectres des métalloids," *Nova Acta Reg. Soc. Sc. Ups.*, Ser. III.

[2] Plücker and Hittorf, *Trans. R. S.*, 1865.

Fig. 1.

Kathode glow in oxygen. Kathode terminated by two plane facets unequally inclined to the axis.

Fig. 2.

Kathode glow in oxygen. Kathode glow terminated by one plane facet inclined to the axis at an acute angle.

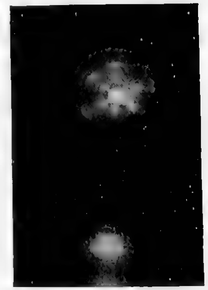


Fig. 3.

Kathode glow in hydrogen. Kathode a six sided prism, giving an end-on view.

In Fig. 3 the central spot is the luminous sheath of the kathode, whence six fine rays diverge, and after crossing the dark space spread out forming the kathode glow, beyond which the bright spots are due to fluorescence of the glass tube.

spectrum of carbon oxide, no doubt from some of that gas having escaped from the glass. This having been pumped out as much as possible, stopcock (*b*) was opened and well dried hydrogen from the phosphoric anhydride tube allowed to enter the sparking tube, and then the whole system, up to stopcock (*a*), again exhausted. Stopcock (*b*) was then closed and (*a*) opened for a short time and then closed again. The system was then left until evening, when the same series of operations were repeated. It was not until the fifth day that the CO^2 ceased to appear in the sparking tube, so that I could observe the behaviour of the hydrogen alone. Even after such treatment the tubes generally give out more carbon oxide if kept for many days. Much sparking of the tube in a highly exhausted state must be avoided, because the sparking is likely to decompose the glass. If the yellow* sodium light is seen at the point where the electrode enters the tube it is a sure sign of decomposition of the glass, and I could never get such tubes to give clean spectra.

With a fairly clean tube, still attached to the pump and to the other tubes, it is easy to trace the changes in the spectra of hydrogen at different pressures. At about 4 mm. pressure the negative glow is well seen about the kathode. It begins as a close-fitting pink sheath on the kathode, which expands as the pressure is reduced. The spectroscope shews that its light is chiefly that of the rays *C*, *F*, and *G′*. It maps out extremely well the course of the kathode rays. If the kathode be a cylindrical wire of one or two millimetres thickness, terminated by two plane facets inclined to the axis, the gas is lighted up equally all round the cylindrical part of the kathode, while from near the centre of each terminal facet proceeds a cone of luminosity exactly in the course of the kathode rays, see Figs. I, II, and when the density of the gas is so reduced that a sensible fraction of the kathode rays escapes absorption by the gas, the base of the cone, where it meets the glass, makes the tube fluoresce. If the kathode be a six-sided prism instead of a cylinder, and be looked at end on, six conical pencils of luminosity diverge from the plane faces of the prism and are easily traceable right across the Crookes dark space, which is only relatively dark; see Fig. III. It is well known that the luminosity is deflected by a magnet along with the kathode rays.

When the pressure of the gas is so far reduced that striations appear, the spectrum of the negative glow consists of the lines *C*, *F*, &c. only, without a trace of the second spectrum. In the light of the negative glow photographs shew nine of the ultra-violet lines of hydrogen. Turning the spectroscope on to the head of the first stria, the second spectrum is there easily visible, and so it is at the bright convex edge of succeeding striæ. In the

capillary both spectra are bright at first, but as the pressure is reduced the lines *C*, *F*, &c. become relatively brighter and the second spectrum weaker, until the latter entirely disappears from the capillary, and is seen only at the bright spot on the anode. At the higher pressures (7 mm.) the bright spot on the anode shews only the second spectrum without a trace of *C* or *F*. The behaviour of the two channelled spectra of nitrogen is precisely similar. When the dark space is sufficiently developed the kathode glow shews none of the brilliant channellings in the orange which are characteristic of the positive spectrum, but they appear plainly at the head of the first striation, and as the exhaustion proceeds are driven further and further from the kathode, until the capillary shews only the spectrum of the negative glow, and the orange and yellow channellings are seen only on the anode.

In order to see these appearances the discharge must be continuous in one direction, and therefore, if an induction coil be used, there must be sufficient resistance in the circuit to stop the passage of the indirect induced current. A short air gap will serve this purpose very well without making the discharge discontinuous. A condenser cannot be used in connexion with the secondary circuit, since it produces an oscillatory discharge.

Pure oxygen produces no luminous positive column and no striations, as was observed by Morren, and the whole of the light emitted by it in a tube at low pressure under the influence of a continuous electric discharge appears to be that of the negative glow. Schuster has carefully observed the spectrum of oxygen, but my observations, made with a continuous current from an induction coil giving a spark in air 7 cm. long, do not accord with his in all respects. In the first place I have failed to see the continuous spectrum and the narrow spark throughout the tube, when the exhaustion first reaches the stage at which the discharge will pass through the tube[1]. The first luminous indication of a discharge that I see is a thin sheath of luminosity around the kathode and nothing more. As the exhaustion proceeds the glow at the kathode spreads out, and in a darkened room a very faint orange-coloured light is seen in the capillary, but the rest of the tube is dark, and so is the anode if all carbon dioxide has been well removed. A trace of carbon dioxide shews itself by producing a slight whitish cloud on the anode, and is almost always seen after a time, so that it is only in a tube from which carbon dioxide has been well exhausted, and then only for a short time after being refilled with pure oxygen, that the anode shews no cloud. By further exhaustion the negative glow becomes more diffuse and in the capillary becomes quite bright.

[1] "Spectrum of Metalloids," *Phil. Trans. R. S.*, Part I., 1879, Exp. 8.

By still further exhaustion the negative glow is further diffused, the light of the capillary again pales, and the glass about the anode fluoresces under the action of the kathode rays which get through the capillary.

Schuster distinguishes the spectrum of the negative glow, which he describes as consisting of five bands in the green and orange, from the four lines, which he calls the compound line spectrum. But I am quite unable to make out that these are two independent spectra. I always see both together in the negative glow and have never seen either produced in any other way. When the glow is very faint the three green bands are more easily seen than the lines because they fill a larger space on the retina, not because they are intrinsically brighter. And when the density of the gas is such that the dark space extends but a short distance round the kathode, both the bands and lines are very bright, brighter than they are in the capillary at any stage of the exhaustion.

Schuster's elementary line spectrum is best seen when the discharge is taken in oxygen at atmospheric pressure, nevertheless it is very well seen in the negative glow along with the bands and lines above mentioned, and can be easily photographed from that source.

I have reason to think that sulphur resembles oxygen in giving no anode light. Hitherto I have not been able to examine the discharge in sulphur vapour, but in sulphurous acid gas the kathode glow gives a series of bands which are probably those of sulphur, and are entirely absent from the anode which appears to have no spectrum.

Chlorine, Bromine, and Iodine, closely resemble one another in their behaviour. Each gives a bright green negative glow, of which the spectrum consists of bright lines, described by Salet[1], of which the groups in the green are most characteristic. The light from the anode seems to be only a continuous spectrum, very bright in the case of iodine and bromine, less bright in chlorine. In chlorine I see no lines at all in this continuous spectrum, which extends through the orange and green. In bromine, and iodine, the disappearance of the green lines at the anode is as well marked a feature as in chlorine, but the orange lines and some others are visible. Nevertheless I do not think that they belong to the anode, for a reason which is peculiar to the halogens so far as I have observed. When the pressure is not much less than that at which the spark will just pass through the gas, the path of the discharge forms a thread through the tube from one electrode to the other and there is no definite kathode glow, but the whole thread of discharge seems to emit

[1] *Ann. Chimie et Physique* (4), XXVIII. 24, 1873.

kathode rays, for it makes the glass fluoresce exactly as if it were exposed to kathode rays. The spectrum of the thread of light also is that of the kathode glow superposed on the continuous spectrum of the anode. The fluorescence of the tube falls away when the pressure is reduced, and so does the brilliance of the light emitted, especially in the wide part of the tube, while the glow at the kathode becomes plainly developed; nevertheless I think the thread of discharge continues to emit kathode rays to some extent, and that the lines observed at the anode are really due to such rays. It is in accord with this supposition that bromine and iodine, which produce the strongest fluorescence, are also those which shew the lines at the anode.

It is difficult to manipulate the halogens because the mercury pump cannot be used directly, so that I had to reduce the pressure by connecting the sparking tube with a globe previously exhausted, and this involved the use of stopcocks which had to be lubricated; and further the electrodes are rather rapidly attacked by halogens. I did not see striations in any one of the three substances, or well defined dark space.

Every metallic vapour, so far as I have yet observed, shews but one spectrum at both electrodes and throughout the tube.

Mercury, when a globule is made the kathode, shews a brilliant green negative glow at the surface of the metal, and a less bright patch of light where the kathode rays strike the surface of the metal when this is the anode. In both the spectrum is the same, with the well known green line, pair of yellow lines and indigo blue line, of mercury. When the tube is warmed to increase the pressure of the mercury vapour the glow which fills the tube shews the same spectrum, and there is no other spectrum at the anode nor is any definite positive column developed, or any striation.

Other metallic vapours examined were those of sodium, cadmium, and thallium. Plain tubes, without capillary, about 15 cm. long, enclosing fragments of the several metals, were pumped out, and, while still connected with the pump, were heated by gas jets, in a small furnace, built up of tiles from a combustion furnace, so that the whole tube was tolerably equably heated. Small openings left between the tiles allowed the spectrum to be observed. After the carbonic acid gas emitted by the glass had been pumped out as completely as possible, the tube and its contents were gradually heated, and though the pressure of the vapour could not be measured, it was varied by raising the temperature more or less.

The sodium vapour at very low pressure shewed its presence on passing the discharge by a long positive column with a well marked dark space about the kathode. The kathode glow was

not strong but fairly well seen. At first the spectrum shewed lines of hydrogen as well as sodium, and the positive column shewed striation, but these disappeared when the hydrogen had been well pumped out. The light about the kathode then shewed only the *D* lines of sodium and a trace of the pair in the citron at about λ 568, while the bright column shewed in addition the pairs in the green and blue at about λ 515 and 498. The head of the bright column was concave to the kathode and was really the negative glow, separated from the kathode by a relatively dark space in which the spectrum was weak. There was no break in the column, and it was about equally bright quite up to the anode and gave the same spectrum throughout. At a somewhat higher pressure of vapour the light about the kathode, and throughout the column, gave brightly, besides *D*, the pairs at about the wave-lengths 616, 568, 515, 498, 475, and 466. They were identified by comparison with the spectrum of the spark in air between two beads of sodium carbonate. The fragments of solid sodium, or rather the skin of oxide they had acquired before the tube was pumped out, fluoresced brilliantly in the kathode rays with an emerald green light, which however gave only a continuous spectrum.

With cadmium vapour, when all carbon dioxide had been pumped out, there was no striation. The spectrum was the same in all parts of the tube, and shewed the characteristic triplet, about λ 5085, 4799, 4677, the orange line about λ 6438, and the green line about λ 5153. At the kathode, when the vapour pressure was not too low, there appeared in addition the two green lines about λ 5378, 5337, and the indigo blue line about λ 4415, usually seen only with a jar discharge in the spark between cadmium electrodes in air. As these three lines were not observed at the anode it may be supposed that they belong exclusively to the kathode glow. I think, however, that as they are seen only at higher pressure when the absorption of the kathode rays is most rapid and the consequent intensity of the glow greatest, their appearance is a question of relative intensity, and they are not part of a different spectrum.

In thallium vapour there was a good negative glow which shewed the green thallium line only. The anode shewed the same. The whole tube was filled with a green glow, unstriated.

In this respect all the metals examined behaved alike.

Turning to compound gases. Hydrochloric acid gives no spectrum which I can ascribe to the undecomposed molecules. It gives, in a tube with platinum electrodes, a good kathode glow with a spectrum which consists of the lines of chlorine and the first spectrum of hydrogen, and nothing else that I can detect. The capillary shews in addition a continuous spectrum in the

orange and green, due, I suppose, to chlorine; and the anode shews this continuous spectrum.

Striæ of a reddish tint, very faint and thin, are formed in succession close to the kathode and move slowly away from it, becoming more developed as they cross what would be the dark space if the head of the positive column were stationary. They were not bright enough to allow of my making sure that they shewed the second spectrum of hydrogen, but from their red tint I conclude that they are due to hydrogen. If so their gradual formation probably arises from decomposition of hydrochloric acid at the kathode. They move up to the anode and then disappear. This continuous movement implies some sort of circulation such as we may suppose to occur if the ions of hydrochloric acid are dissociated in a strong electric field and are reassociated in a weak one. The continuous movement of the striæ, if we except a little flickering now and then, does not, so far as I have observed, occur except when there is reason to suppose that chemical change attends the discharge. For example, striæ are formed and move from kathode to anode in a mixture of cadmium vapour with carbonic acid gas, where probably the latter gas gives up half its oxygen to the metal in one part of the field, and the reverse reaction occurs in another part of the field.

The behaviour of water vapour was analogous to that of hydrochloric acid. When the carbonic acid gas had been thoroughly pumped out, and the pressure of the residual water vapour reduced to about one and a half millimeters by cooling a prolongation of the tube to -15° C., there was a bright crimson kathode glow, a red capillary, and hardly any light on the anode. Platinum electrodes were used in this case. The kathode glow gave the first spectrum of hydrogen very bright, and much less brightly four lines in the blue, which by comparison with an oxygen tube I identified as oxygen lines. They were not easily seen at first, more easily after a little sparking. I looked for the orange and green lines of Schuster's compound line spectrum, but failed to detect them. Nor could I see the two broad green bands. The blue lines were, however, observed and compared with the oxygen lines several times. At the anode there was no bright spot, nor a trace of the second spectrum of hydrogen. There was a faint flickering striation, of the same colour and appearance as that in hydrochloric acid, too faint to shew a spectrum. The capillary gave the first spectrum of hydrogen without the second spectrum, but no oxygen lines.

That some decomposition of the vapour was produced by the discharge was shewn by a little increase of pressure, and the fact that after sparking for a few minutes a little permanent gas could always be pumped out of the tube.

Other gases which I have studied are the oxides of carbon. So far as I can see they each give, with a continuous discharge, but one spectrum in all parts of the tube, in the negative glow, the capillary, the striæ and the anode, it is the same spectrum for both carbonic oxide and carbon dioxide, and was faithfully described and figured by Ångstrom and Thalén, in their *Recherches sur les spectres des métalloids* before referred to, as the spectrum of carbonic oxide. It consists of a number of channelled bands degraded on the more refrangible side, and of a number of weaker bands, almost equally diffuse on their two sides, in the spaces between the degraded bands. These gases are very sensitive to the kathode rays, and give a bright negative glow, by which the course of the kathode rays may be readily traced. The light in all parts of the tube is nearly white, and in the striæ, which are easily produced, there is no perceptible difference of colour between the anode and kathode sides of the striæ, as there is in the striæ of hydrogen and of nitrogen, nor such a well-marked position of the maximum of luminosity. There was no trace of the bands of the candle-flame spectrum in either of the two gases in any part of the tube, with such a discharge as I have used.

Another carbon compound examined was cyanogen. The observations were impeded by the rapid decomposition of the compound, and the blackening of the tube, but I was able to observe some effects of the discharge. When all carbonic acid gas had been completely removed, the tube refilled with cyanogen and the pressure reduced, on first passing the discharge striæ appeared fairly sharply defined and moving from the kathode rather quickly, but as decomposition proceeded the striæ became very diffuse and ceased to move. After pumping out and refilling and reducing the pressure to about 2 mm., the kathode glow was fairly bright and the dark space defined. The spectrum of the glow shewed a set of bright bands in the orange and yellow, which appeared, by their number, character and arrangement, to be the same as the bands in that region of the spectrum which are developed in the flame of cyanogen burning in oxygen, though I could not prove identity, not having such a flame at hand for comparison. Then there were besides two sets of bright bands in the blue and violet, which from their position and appearance I had no doubt were the cyanogen groups beginning at about λ 4600 and 4215 respectively. At the anode no blue or violet bands could be seen, only some continuous light in the orange and on to green, with traces of the bands in that region that were bright and well-defined in the kathode glow. At higher pressure the anode light gave these same bands distinctly, though not so bright as the kathode glow, while the blue and violet sets were

still absent. At still higher pressure the blue and violet bands appeared at the anode as well as at the kathode. By this time the whole tube had become very black with the products of decomposition which could not be removed by pumping. The best observations were obviously those made at first, from which I gather that all the bands above mentioned belong to the kathode glow. In no case, when once the carbonic acid had been thoroughly removed, did I see anything of the spectrum either of carbonic oxide, or of the candle-flame, though the latter is always very brilliant in the flame of cyanogen fed with oxygen.

The series of bands less refrangible than the blue, so bright in the flame of cyanogen, are not connected with the series in the blue and violet. The latter are very bright, in both arc and spark, between carbon electrodes in nitrogen, without a trace of the former. The latter are degraded on the more refrangible side, while the former are degraded on the less refrangible side, which is such an important difference that I feel sure that the vibrating molecules in which they originate are different. This is probable enough because of the rapid decomposition of cyanogen in the discharge, and the ease with which paracyanogen is produced, but at present I have no evidence for attributing the orange bands to any particular product of cyanogen.

Coming to the theoretical considerations which the facts observed suggest, we have to account for (1) the two perfectly distinct spectra, at the kathode and anode respectively, of hydrogen, nitrogen, and the halogens, (2) the absence of any anode light in oxygen and sulphur, (3) the identity of kathode and anode spectra in the metallic vapours, and in carbonic oxide, (4) the absence of visible spectra which can be ascribed to the compound molecules of hydrochloric acid and water, while the compound molecules of carbonic oxide and cyanogen give very characteristic spectra, (5) the identity of the spectra of the two oxides of carbon.

The appearances of the positive column in hydrogen and nitrogen agree well with Professor Thomson's theory[1] that the light of the column is excited by association of the ions, and arises in the positive ions. The head of the column nearly coincides with a region in which the strength of the electric field is a maximum, and this would occur where the association of ions is a maximum. Moreover, the spectrum of the head of the column, especially the brightest part of the head of each stria when striation occurs, is essentially that of the anode, and the head of the column is driven from the kathode as the exhaustion proceeds and the free path of the negative ions lengthened, until, when

[1] *Phil. Mag.* [5], L. 278, 1900, and *Conduction of Electricity through Gases*, Cambridge University Press, 1903.

the dark space reaches to the anode, the second spectrum of hydrogen, and the closely set yellow bands of nitrogen, are seen only on the anode. These spectra are both extremely complicated, whence I infer that the molecules in which they originate are of complicated structure and comparatively massive. The continuous spectra of the halogens at the anode emphasises this point, they are, in a sense, still more complex, and it is noticeable that the brightness of the continuous spectrum is greatest in iodine and least in chlorine, in fact follows the molecular masses. In connexion with this it seems probable that oxygen may emit some continuous spectrum at the anode, feebler than that of chlorine, and not easily detected, because not lying in the part of the spectrum most easily seen.

The kathode glow follows so closely the course of the kathode rays, and the gases behave to them so precisely in the way that a fluorescent liquid behaves to ultra-violet light, that the inference immediately suggests itself that the excitement of luminescence is of the same sort in these two cases, and also in the case of the glass which fluoresces under the rays when the density of the gas is insufficient to absorb them.

The essential character of fluorescence, as I understand it, is that the fluorescing substance absorbs certain kinds of radiant energy and forthwith emits it again in a more or less modified form as light, and continues to do this as long as the exciting cause continues and for no appreciable time longer, and is not itself permanently changed in constitution or material by the process.

Percival Lewis' observations[1] have shewn that various metallic vapours exposed to kathode rays, without being in the course of the electric discharge, or forming part of the conducting gas, emit light, of which he has examined the spectra.

In the case of each of the four metals that I have examined, the rays that he found in its spectrum are all to be seen in the kathode glow of the same metal, though in sodium and cadmium the glow gives a greater number of rays than Lewis observed. But the most convincing argument that the molecules of the gas emitting the kathode glow are unaltered in constitution is that in many cases the spectra are reversible. This could not be unless the molecules which produce the reversed spectra by absorption, were the same as those which under the stimulus from the kathode emit the same rays bright. The line spectrum of hydrogen is well known to be reversed in the sun, and the reversal of C and F has often been observed in looking at a spark in dense hydrogen. Most of the metallic lines also which I have seen in the glow, as well as those Lewis has observed in the kathodo-

[1] *Astro-Phys. J.*, XVI. 31.

luminescence of vapours of potassium, zinc, and magnesium, are reversible. But perhaps the most striking case is that of the cyanogen bands. The two sets of these bands which are conspicuous in the kathode glow in cyanogen are invariably associated with two other sets, ultra-violet, near the solar lines L and N respectively. Of these that near L is reversed when the flash of an arc is taken in an atmosphere of cyanogen, as Professor Dewar and I found[1].

Wood and Moore[2] have recently obtained, as a fluorescence of sodium vapour in sun-light, a bright banded spectrum corresponding to the absorption spectrum of the same vapour described by Roscoe and Schuster[3]. I have little doubt that the kathode rays would produce a similar spectrum in sodium vapour of sufficient density, but could not try it because my glass tubes would give way at the temperature required to produce such dense vapour.

It is true that the oxygen lines and bands of the kathode glow have not been observed to be reversed, whereas some other absorption bands of oxygen are well known. I am not inclined to attach much weight to the former fact, and with regard to the latter it must be borne in mind that it requires a great thickness of compressed oxygen to make those absorptions visible, so that we could not expect to see the corresponding emissions from a small mass of gas at low pressure.

It is noteworthy that the persistent glow of radium salts in atmospheric air is that of the kathode in nitrogen, as Sir W. and Lady Huggins have shewn[4].

The behaviour of the metals suggests that the positive ions consist of molecules of the vapour, which when deionized vibrate to the same tune as do the like molecules under the influence of kathode rays. In this connexion the monatomic character of metallic vapours may have some significance. But be that as it may, the positive ions may very well be molecules of the vapour, and no reason occurs to me why chemically compound molecules, such as those of carbonic oxide, should not become positive ions. In fact these molecules often play the part of elementary atoms in chemical combination.

Of hydrochloric acid no emission spectrum has, so far as I am aware, been observed, and the only absorption spectrum it gives is a continuous one in the upper region of the ultra-violet, so that if it fluoresces at all the light it emits may be in that region. It is equally probable that the ions it furnishes may be derived from it by chemical dissociation. The absorption by water vapour is chiefly in the low red, and is only visible when the light has traversed a great thickness of vapour, while its emission

[1] *Proc. R. S.* XLIV. 247, *note.*
[2] *Astro-Phys. J.* XVIII. 94.
[3] *Proc. R. S.* XXII. 362
[4] *Proc. R. S.* LXXII. 409.

spectrum is chiefly ultra-violet, so that it is *à priori* not improbable that it would give no visible fluorescence. It is also not improbable that the ions are furnished by dissociation of the elements, and the facts that water vapour gives no anode light, hydrochloric acid a continuous spectrum at the anode, lend confirmation to the supposition that oxygen furnishes the positive ions in the former, chlorine in the latter medium.

The identity of the spectra given by the two oxides of carbon suggests that the ions of carbonic acid gas are furnished by dissociation of carbonic oxide and oxygen. It is true that I have not been able to trace the presence of oxygen at the kathode as I was able to do in water vapour, but considering that in the latter case it was only blue lines of oxygen that were seen, and that they could not have been detected but for the fact that hydrogen gives no light in that region, it is not surprising if the very brilliant spectrum of carbonic oxide overpowered the oxygen.

It has been maintained that the common spectrum of the two oxides of carbon is emitted by carbon, but the fact that cyanogen, when all carbonic acid from the glass has been well pumped out, does not shew the carbonic oxide spectrum, is a strong reason for concluding that molecules of carbonic oxide are really the origin of the vibrations producing the spectrum in question.

In conclusion, the examination of the kathode glow, very incomplete as it is, has suggested to me that the explanation of the solar chromosphere and corona will be found in regarding them as a huge kathode glow.

G. D. L.

On the Distribution and Spectra of Metallic Vapours in Electric Sparks. By HUGH RAMAGE, B.A., St John's College.

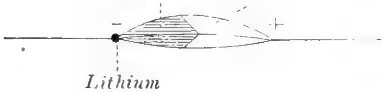
[*Read* 18 January 1904.]

Some peculiarities observed in certain lines in the arc and spark spectra of lithium were described by the writer in a paper communicated to the Royal Society in November 1902[1]. The blue line, for instance, was found to be broadened in the spectra of the regions near the negative element both in the arc and spark. The broadening of this line was easily observed and photographed in the spectrum of the spark when no Leyden jar was used, and a slight broadening was observed near the positive electrode when no air break was inserted in the secondary circuit and no precautions were taken to keep the electrodes cool. Between these two regions the line was thin and sharp. A further study of electric sparks in air at ordinary pressures has since been made and the results will now be described.

Two clean platinum wires were fixed horizontally in a holder with their adjacent ends about a centimetre apart. When they were connected with an induction coil and the coil was started, the flame surrounding the sparks had the characteristic colour due to the gases of the atmosphere. The anode was then moistened with a solution of a compound of lithium and the coil again started. Flashes of red were observed in the sparks, but when the anode was heated the flame round the sparks became red throughout its whole length, and there were seen in its spectrum the red lithium line and the yellow sodium lines. It would appear from this that some of the lithium and sodium, present as an impurity, were vapourised by the heat and reduced to the free state by the spark, otherwise, according to Gouy's observations[2], no colour should be communicated to a flame in presence of so much free oxygen. A quantity of lithium salt was then fused on the end of one of the wires, forming a globule about 2 mm. diameter. When the coil was started with this as the anode the red flashes were more frequent and brighter than before, and the orange line of lithium was also observed in the spectrum, but it was extremely feeble. When an air break was inserted in the secondary circuit no red colour was observed as long as the anode remained cold.

[1] *Proc. Roy. Soc.*, Vol. LXXI. p. 164.
[2] *Comptes Rendus*, Vol. LXXXIII. p. 70.

On reversing the spark, thus making the globule of lithium compound the kathode, a bright red colour developed in the flame near the kathode, but it did not pass the whole way across to the anode unless the electrodes were brought nearer together.



The appearance of the spark and flame is shown in the figure. There were observed in the spectrum of the red portion, viewed through a vertical slit, the red lithium line as a long line, the orange line as a short line of feeble intensity, and the sodium lines as lines of medium length. The orange line is the strongest line in the subordinate series of lithium, and it is only seen near the central axis of the spark. An exceedingly bright spot of white light was always observed on the salt where the spark touched it, and in the spectrum of this spot the red lithium line and the yellow sodium lines were somewhat brighter than they were in the red portion of the flame, but the lines of the two subordinate series of lithium were exceedingly bright. It was this spot, and possibly the vapour very close to it, which gave the broadened lines in the diffuse series which were described in the above-mentioned paper. The salt on the kathode becomes hot and sometimes it fuses; if the current is then reversed the flame becomes coloured throughout its whole length but the colour fades away as the temperature of the bead falls. A single spark passing to the kathode produces both the bright spot and the red flame.

The electrodes were next placed vertically in a line, the bead of salt being first placed on the lower end of the upper electrode. When this was made the kathode, the red colour appeared, but it only extended for about one-half the distance from the electrode that it did when these were horizontal; the upward current of hot air from the spark evidently opposed its motion downwards. The bead and electrodes were allowed to cool and the sparks were then passed in the opposite direction, but no red colour appeared until the bead became heated by the upward current of hot air and then the red colour appeared and extended down to the clean kathode.

The bead was next put on the upper end of the lower electrode. When it formed the kathode the red colour extended up to the anode but it was deepest near the kathode. When the electrodes

were cold and the bead was made the anode no red colour appeared in the spark.

The lithium vapour liberated from a hot anode would appear to carry a positive charge and thus convey part of the current to the kathode, but the lithium vapour liberated from the kathode does not appear to be charged. In no case has it been observed to be repelled by the anode, and it certainly does not appear to be attracted by it; if it is charged negatively at first the charge must be quickly lost. The lithium would appear to be liberated in definite quantity by each spark, in a very short interval of time, but there is no decisive evidence given by these experiments to show whether the vapour passes away in an explosion wave or merely by diffusion in the gases heated to a high temperature by the passage of the spark. Sodium vapour passes about as far from a kathode of sodium salt as lithium does; this is in favour of the former view.

There is evidently an intense chemical action taking place in the bright spot on the kathode, and there seems no doubt but that lithium is liberated from combination by electrical action at this point. It is probable that the production of the subordinate series is associated with the process of liberation of the metal, that the vibrations producing the lines of these series are peculiar to the atom at the moment of its liberation, perhaps to the atom and the electric charge then given to it, whilst the vibrations producing the lines of the principal series are peculiar to some change, combination or otherwise, of the free atom of the metal. The lines of the subordinate series become broader and apparently brighter as the negative pole of the arc is approached, and this pole, according to Violle, has a temperature of about 2700° when the positive pole is about 3500°, and the temperature of the arc itself is still higher. The apparent brightening of the lines is not a temperature effect.

The results obtained with a compound of sodium are similar to those obtained with lithium. Potassium gives a different result when a bead of its carbonate is on the kathode. There is no flame near the kathode, the spark is reduced to a thin line whilst it retains its original size and greenish colour near the anode. The bright kathode spot gives the subordinate series as strong lines. Rubidium and caesium resemble potassium but they give their colours to a portion of the flame near the kathode.

Calcium, strontium, and barium give brilliant colours to the flame near the kathode when beads of their chlorides are used, and their spectra at the kathode resemble their oxyhydrogen flame spectra, whilst the spectra of the coloured flames are more like those of their Bunsen flame spectra.

Experiments have been made with metallic electrodes of

magnesium, aluminium, zinc, tin, and lead. These metals give bright spots on both electrodes and there are differences between the spectra of these. The spectra of the anode spots on magnesium and aluminium contain the "oxide bands" of the metals, but these bands have not been observed in the kathode spots. The writer is continuing the study of this subject in the hope of gaining further knowledge on the constitution of electric sparks and the origin of the spectra.

NOTE. Since the above paper was written it has been found by examination of the sparks in a rotating mirror that the coloured flames actually shoot out from the kathode. There was practically no colour given to the spark by the matter thus expelled in an atmosphere of hydrogen; colours were obtained in some instances in nitrogen but the experiments are not yet concluded. The spectra of the bright spots on the kathodes are always brighter in hydrogen than in air or oxygen and they are usually brighter under pressures somewhat below the ordinary atmospheric pressure. Zinc and cadmium were reduced from electrodes of their oxides when these were in hydrogen. Their vapours were shot off from the electrode and films of the metals condensed in the form of rings on the tube and on pieces of perforated mica placed in the path of the spark.

On the transmission of Earthquake waves through the earth. By Rev. O. FISHER, M.A., F.G.S., Hon. Fellow of Jesus College, Cambridge, and of King's College, London.

[*Received* 23 November 1903.]

In Mr Milne's *Report on Seismological Investigation* at the Southport meeting of the British Association, 1903, the conclusion arrived at is that the high velocity of preliminary tremors of 10·5 to 12 kilometres per second suggests a high rigidity for the earth; while the approximate uniformity of speed within its core indicates approximate uniformity in those properties which determine the rate at which it transmits vibrations.

I have always doubted the doctrine of extreme rigidity of the earth's interior, and have felt that the phenomena of instability of the crust, and the enormous amount of steam emitted by volcanoes point to the substratum of the crust being a liquid magma holding water in solution at a temperature above the critical. I have therefore made the following attempt to account for the peculiarities of the transmission of earthquake waves on this hypothesis, for if that can be done an objection to the theory of liquidity will be removed.

I shall assume that the disturbance originating a world-shaking earthquake is caused by a relaxation of pressure of the earth's crust upon the subjacent magma, which we may conceive to be brought about, either by a fitful spasm of mountain elevation due to their mass being lightened by denudation, or by some other cause. Hence on the subjacent portion of the magma the pressure is slightly relieved, and a disturbance is produced.

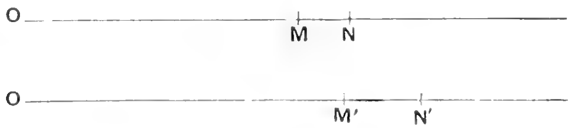
Henry's law of the absorption of gases by liquids asserts, that the ratio of the volume of the gas which can be held in solution by a given volume of the liquid is the same at all pressures[1]. Thus if rV be the volume of gas which can be held in solution by the volume V of the liquid, rV is the same at all pressures.

It apparently follows that if the liquid be in a state of compression, so that when the pressure is relieved the volume V expands to $V+v$, the volume of gas which it can then hold in solution will be $r(V+v)$. Consequently an additional volume of gas rv, proportional to the increase of volume of the liquid, can be held in solution, and no gas will be extruded in consequence

[1] *Phil. Trans.* 1803.

of the relief of pressure until the limit of expansibility of the liquid is reached, after which gas will be extruded as the pressure continues to fall.

We will consider the simplest case, *i.e.* of a column of unit section filled with the liquid in a state just saturated with the gas. Let r be the constant volume of gas held in solution in unit volume, where r will be independent of pressure.



Let $OM = x$ be the distance from the origin of a particle in the magma in the undisturbed and saturated condition, and let the corresponding pressure be P.

Let OM' be its distance in the disturbed condition.

Let the density of $MN = D$.

The density of $M'N'$ is different, on account of the expansion of the liquid and of the vesicles of gas which are liberated owing to the relaxation of pressure.

Let p be the pressure of the liberated gas at M'. Then this pressure will be transmitted throughout the magma, so that the pressure of the mixed gas and liquid at M' will be p, which will act upon the expanded mass $M'N'$, and it will be observed that the mass of $M'N'$ is the same as that of the original mass MN.

Let $$MN = \delta x, \quad OM' = x + \xi.$$

Then $$M'N' = \frac{d}{dx}(x + \xi)\,\delta x + Q\delta x^2 = \delta x + \frac{d\xi}{dx}\,\delta x + Q\delta x^2.$$

Now consider a volume V of the undisturbed and saturated magma, and let S be what the density of the gas dissolved in it would be at pressure P. Let s be the density of the same at the pressure p. Then because at the pressure P this gas is wholly absorbed, its total volume at the pressure P would be rV.

Hence its total volume at the pressure p would be $rV\dfrac{S}{s}$, which by Boyle's law $= rV\dfrac{P}{p}$.

Suppose that at the pressure p some of the gas is set free so as to form vesicles. Then we should have

volume of free gas + volume absorbed = total volume.

And by Henry's law the volume absorbed is constant, and $= rV$.

$$\therefore \textit{ volume of free gas} + rV = \frac{P}{p} rV;$$

$$\therefore \textit{ volume of free gas} = rV\left(\frac{P}{p} - 1\right).$$

We may suppose that the magma as a liquid is subject to voluminal expansion from V to V' owing to the change of P to p; and let e be the coefficient of voluminal compression at the pressure P.

Then $e =$ increment of pressure $\div$ voluminal compression

$$= (P - p) \div \frac{V' - V}{V'}.$$

Whence
$$V' = \frac{V}{1 - \dfrac{P - p}{e}}.$$

Hence, supposing if possible the magma to expand and gas to be simultaneously extruded, we should have for the total expanded volume at the pressure p,

$$\textit{volume of free gas} + \textit{volume of expanded magma}$$
$$= rV\left(\frac{P}{p} - 1\right) + \frac{V}{1 - \dfrac{P - p}{e}}.$$

Therefore putting δx for V, we should have,

$$\frac{M'N'}{MN} = r\left(\frac{P}{p} - 1\right) + \frac{1}{1 - \dfrac{P - p}{e}} = 1 + \frac{d\xi}{dx}.$$

$$\therefore -r\frac{P\dfrac{dp}{dx}}{p^2} - \frac{\dfrac{1}{e}\dfrac{dp}{dx}}{\left(1 - \dfrac{P - p}{e}\right)^2} = \frac{d^2\xi}{dx^2}.$$

Now $P - p$ is the relaxation of pressure which causes the disturbance. It is obvious that this must be small compared with P, for P being the pressure of the crust, if $P - p$ was equal to P it would imply that the crust was entirely removed above the area of disturbance.

Let then $\dfrac{P - p}{P} = \alpha$, where α is small.

$$\therefore r\frac{P\dfrac{dp}{dx}}{p^2} = \frac{r}{P}\frac{\dfrac{dp}{dx}}{\left(\dfrac{p}{P}\right)^2} = \frac{r}{P}\frac{dp}{dx}(1 + 2\alpha).$$

We may assume the change of pressure corresponding to a change in x to be small, otherwise the equation will not be linear. Hence we neglect the product $\frac{dp}{dx}\alpha$.

Also e being a large pressure, we neglect $\frac{P-p}{e^2}\frac{dp}{dx}$, and our equation becomes,

$$\left(-\frac{r}{P}-\frac{1}{e}\right)\frac{dp}{dx}=\frac{d^2\xi}{dx^2}.$$

Since p is the pressure at M', and is a function of ξ, and ξ is a function of x, therefore p is a function of x, and when ξ becomes $\xi+\frac{d\xi}{dx}\delta x$, p becomes $p+\frac{dp}{dx}\delta x$.

Hence $p+\frac{dp}{dx}\delta x$ is the pressure at N', and $\frac{dp}{dx}\delta x$ is the difference of the pressures on the two faces of $M'N'$, and acts towards the origin, and is therefore negative.

The mass of $M'N'$ on which it acts is the same as that of the original mass MN, viz. $D\delta x$.

Hence by the equation of motion

$$-\frac{dp}{dx}\delta x=\frac{d^2\xi}{dt^2}D\delta x.$$

$$\therefore \left(\frac{rD}{P}+\frac{D}{e}\right)\frac{d^2\xi}{dt^2}=\frac{d^2\xi}{dx^2}.$$

If there was no extruded gas, or the disturbance due to the expansion of the liquid only, we should have

$$\frac{e}{D}\frac{d^2\xi}{dx^2}=\frac{d^2\xi}{dt^2}.$$

This is the differential equation to a wave of which the velocity is $\sqrt{\frac{e}{D}}$. It indicates a wave which will be propagated within the substance of the liquid, independent of any gas.

If on the other hand the liquid was inexpansible, or the disturbance due to the extruded gas alone, then we should have

$$\frac{P}{rD}\frac{d^2\xi}{dx^2}=\frac{d^2\xi}{dt^2}.$$

This would be the differential equation to a wave whose velocity would be $\sqrt{\frac{P}{rD}}$, and would probably be less than that of the wave due to the expansion of the liquid alone.

These two effects cannot be simultaneous because, as we have already shown, so long as the liquid expands no gas will be extruded.

Consider then the effect of a diminution of pressure upon the magma at the origin of disturbance. The relaxation of pressure, although impulsive, cannot be instantaneous. The magma being under the compression corresponding to saturation, the relief of compression up to a certain point will in the first place cause voluminal expansion, which, as already shown, will not be accompanied by extrusion of gas. This expansion will be propagated as an elastic wave with the velocity $\sqrt{\frac{e}{D}}$. As the relief of compression continues, the expansion of the liquid magma will reach the limit of voluminal expansion, and gas will begin to be extruded. This stage of the disturbance will be propagated as a second, gaseous, wave with a velocity $\sqrt{\frac{P}{rD}}$, less than that of the elastic wave, with extrusion of gas but without further expansion of the liquid element of the magma. Hence the disturbance due to the elastic wave will reach a distant station first, and the disturbance due to the gaseous wave will after an interval arrive.

Similar changes will occur at every place which the disturbance reaches during its passage. Thus the first effect will be voluminal expansion, so that an elastic wave will be continually started in front of the gaseous wave. The consequence will be that tremors arising from elasticity will continue without intermission to arrive at a distant station up to the time of the arrival of the gaseous wave.

This sequence of events is similar to what is observed in the case of a world-shaking earthquake, such as that of the great Indian Earthquake of 12th June, 1897. Mr Oldham, in his exhaustive memoir on this earthquake, divides the movement propagated to distant observatories into

(1) Waves of elastic compression;

(2) Wave of [supposed] distortion;

(3) Undulatory gravitational waves;

and states that the mean velocity of the second type is 5·29 km. per second[1].

I use the term "supposed" distortion, because, as I understand the question, this supposition has been made to account for the two classes of waves having different velocities, not because the type of disturbance specially indicates distortion. And it is because these two sets of waves travelling with different velocities might

[1] *Memoirs of the Geological Survey of India*, Vol. XXIV. p. 248.

coexist in a rigid mass that the phenomena have been held to prove a high rigidity of the interior[1]. If however the present hypothesis be valid that argument loses its force.

Let us now enquire what conclusions may be drawn from the present hypothesis regarding the constitution of the magma.

The velocity of the wave of expansion of the liquid alone will be $\sqrt{\frac{e}{D}}$, and if this is the wave which produces the preliminary tremors then $\sqrt{\frac{e}{D}} = 11$ km. per second.

The velocity of the gaseous wave will be $\sqrt{\frac{P}{rD}}$. If this is the wave of the second observed type its velocity is about 5·29 km. per second. This will give $\frac{P}{rD} = (5{\cdot}29 \text{ km. per second})^2$.

Reducing this to feet, since 1 km. = 0·621 mile,

$$\frac{P}{rD} = (5{\cdot}29 \times 0{\cdot}621 \times 5280)^2.$$

Here P is the pressure on the magma in its saturated condition. We may take it as fairly certain that the magma is normally saturated at the bottom of the crust, otherwise a layer of gas would be formed in that position, which, though perhaps locally occurring before explosive outbursts like that of Bandaisan[2], is as a rule highly improbable. Hence P may be considered to be the pressure of the crust and D the density of the magma under it in a saturated condition.

Let then w be the mass of a cubic foot of water, ρ the specific gravity of the crust, k its thickness in feet, and g the acceleration of gravity. Then the pressure arising from the weight of the crust will be $\rho kwg = P$.

Assuming the thickness of the crust to be 25 miles, and its specific gravity 2·68, we shall then have,

$$\frac{2{\cdot}68 \times 25 \times 5280 \times 32 \times w}{rD} = (5{\cdot}29 \times 0{\cdot}621 \times 5280)^2,$$

which gives $$rD = 0{\cdot}037627w.$$

Assuming further that the density of the magma is three times that of water, which is the case with a heavy basic rock, we have

$$r\,3w = 0{\cdot}037627w,$$

and $$r = 0{\cdot}01254,$$

[1] Milne's *Seismology*, pp. 115, 116. Also Darwin's *Tides*, p. 237.
[2] *Nature*, 13 Sept. 1888.

which implies that the volume of gas dissolved in a volume of magma at the pressure just under the crust is 0·01254 of that volume.

If RV be the volume to which the gas in the volume V of magma would expand on reaching the surface, where it will be exposed to the atmospheric pressure only, then, by the expression already found for the free gas,

$$RV = rV\left(\frac{P}{p} - 1\right),$$

where P is the weight of the crust and p that of the atmosphere, which is about that of 34 feet of water.

This will give about 130 cubic feet of gas at atmospheric pressure given off by every cubic foot of lava when it reaches the surface of the earth.

It must not be supposed however that this represents the whole expansive force in a volcano, because throughout the entire duct gas is being liberated, so that the expansive force is the aggregate of the pressure of the liberated gas throughout the height of the duct until the supply is temporarily exhausted.

Our data enable us to find the value of e, the volume elasticity of the magma: for, comparing the velocities of the two types of waves, we have

$$\left(\frac{11}{5{\cdot}29}\right)^2 = \frac{er}{P}.$$

Now expressed in C.G.S. units

$$P = 2{\cdot}68 \times 25 \times 160933 \times 981{\cdot}17,$$

which with the value of r already found gives

$$e = 3{\cdot}647 \times 10^{12}.$$

This is of the same order of magnitude as some values I obtained for the compressibility of specimens of granite, calculating from values of Young's modulus, and the coefficient of rigidity as measured by Dr H. Nagaoka and by Dr Knott[1].

We have been considering the magma just beneath the crust. At greater depths P will be increased, and D will be increased, and probably the temperature also, e will also probably be increased. But if the interior is liquid, the temperature may be more equable than if it was rigid. So long however as the properties of the gas and liquid remain the same, r would be constant. According to Laplace's law, the pressure increases more rapidly than the density. The velocity will therefore

[1] "Rival theories of Cosmogony," by the Author, *American Journ. Science*, June, 1901.

probably increase in passing through the deeper parts of the interior.

Besides these two types of wave there is a third, supposed to be of a gravitational character, which does not fall within the limits of the present enquiry.

I cannot but think that many geologists, like myself, would be pleased to be permitted to regard the interior of the earth as liquid, and if what I have now written is a step in that direction I shall be satisfied.

Some High Andine and Antarctic Umbelliferae. By A. W. Hill, M.A., King's College.

[*Read* 1 February 1904.]

The communication dealt with the genera *Crantzia* Nutt. and *Azorella* Lmk., which are widely distributed in the Southern hemisphere.

The genus *Crantzia* which occurs in Chatham Island, New Zealand and Australia, and is found again in the Falkland Islands and all through S. America to Mexico and in North America, has always been considered as a monotypic genus, with a singularly wide range. It is a small swamp- or water-plant without any well-marked vegetative features, but an examination of the fruits of specimens from various localities shews that apparently several species corresponding to definite geographical areas are included under one very similar vegetative form, and that the fruit characters afford a sound basis for their classification.

The genus *Azorella*, which includes the well-known *A. selago* Hook. of Kerguelen's land, etc. and is allied to the Balsam Bog, *Bolax Glebaria* Comm., of the Falkland Islands and S. Chile, is of interest from its peculiar habit of growth. The plants usually form large turf-like masses or mounds some two or three feet high and they are distributed all along the Andes, occurring in Bolivia and Peru, where six species were collected by the author, between about 12,000 and 15,000 feet above the sea.

A peculiar form collected in Bolivia, which approaches *Laretia acaulis* in the structure of its fruit and which forms great mounds in the high Andes, was described, and a collection of seedlings was exhibited shewing the way in which the mound-like masses are developed.

Free-living fresh-water New Zealand Nematodes. By N. A. COBB, with note by K. LUCAS, B.A., Trinity College. (Communicated by Mr A. E. SHIPLEY.)

[*Read* 1 February 1904.]

The Nematodes whose descriptions follow are of some little interest in the following particulars.

In view of the few species at present known from New Zealand it is of interest to see that all the present ones belong to well known genera, and that as species they present no remarkable variations from the normal types of their respective groups. This leads us a step further towards the belief that many of the Nematode genera living free in the soil and in water will be found to have a very wide geographical distribution. This is precisely what was to be expected from what we know of the life histories in a number of these genera. The small size of the individuals, their fecundity, their adaptability to transportation by a great variety of agencies, and their resistance to desiccation, at least in certain stages of their existence, are all in favour of wide distribution.

The depths from which some of the present specimens were dredged throws an interesting light on the fauna of such depths. There have been comparatively few Nematodes taken in fresh water from such depths. That all the species thus taken should belong, in the present instance, to known genera and be closely related to known species indicates that even very considerable changes of environment effect small changes in the form of these worms.

That all the species here presented are for the most part of large size may be an idiosyncrasy of the collector, an effect of the apparatus used, or a mere coincidence, and hence little weight can be attached to the fact.

So far as I can see the species resemble most closely other austral species, though some of them are certainly closely like those of Europe. The *Mononchus rex* finds its closest known relative in Fiji, while the *Dorylaimi* are most closely related to Australian species.

The fact that the two species of *Dorylaimus* dredged from New Zealand lakes should belong one to the North Island and the other to the South Island may be indicative of variation from a common stock, especially as the species appear to be closely related. The localities from which the two species are derived are six to eight hundred miles apart on a north and south line, and hence the differences in climate are considerable.

There is no clue to the host of the species of *Mermis*. The discovery of a species of *Mermis* at such a depth living free in fresh water is rather interesting.

Otherwise than in these few small points these descriptions appear to add nothing of particular value to our knowledge of the Nematode group.

Formula used in the descriptions.

Fig. 1 illustrates the formula used in the text to express the necessary measurements. The formula derived from the

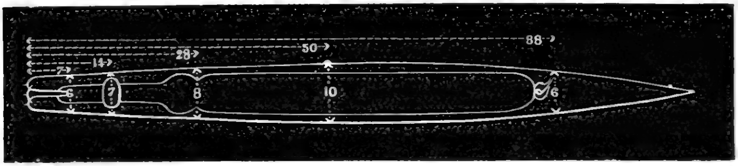


FIG. 1. Diagram in explanation of the descriptive formula used for Nematode worms. 6, 7, 8, 10, 6 are the transverse measurements, while 7, 14, 28, 50, 88 are the corresponding longitudinal measurements. The formula in this case is:—

$$\frac{7\cdot \quad 14\cdot \quad 28\cdot \quad 50\cdot \quad 88\cdot}{6\cdot \quad 7\cdot \quad 8\cdot \quad 10\cdot \quad 6\cdot}.$$

The unit of measurement is the hundredth part of the length of the worm, whatever that may be. The measurements become, therefore, percentages of the length.

The measurements are taken with the animal viewed in profile; the first is taken at the base of the pharynx, the second at the nerve-ring, the third at the cardiac constriction (base of the "neck"), the fourth at the vulva in females and at the middle (M) in males, the fifth at the anus.

diagram is however capable of considerable amplification, for by simple modifications the fourth term above the line can be made to express both the size and form of the internal sexual organs. Thus, letting – (a dash) represent an outstretched, and ' (an apostrophe or quotation mark) a reflexed or contorted, ovary or testicle, we have in the case of a male:—

– M for a simple outstretched apparatus extending forward.

= M for a double outstretched apparatus extending forward.

– M – for outstretched testicles extending in opposite directions.

' – M for two testicles extending forward, one of them reflexed.

" M for two testicles extending forward, both reflexed, or contorted. Similarly in the case of a female, in which for the sake of illustration we will place the vulva at 66 per cent. of the length from the head:—

' 66 ' for two reflexed ovaries extending in opposite directions.

" 66 for two reflexed ovaries extending forward.

– 66 – for two ovaries outstretched in opposite directions.

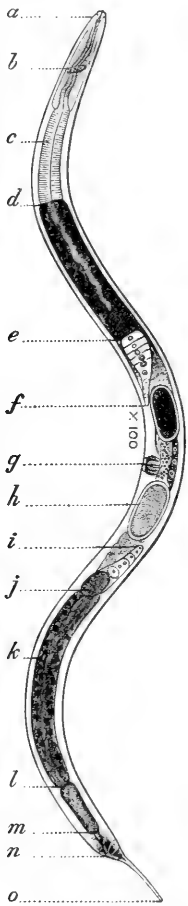
' 66 for a single reflexed ovary extending forward.

– 66 for a single outstretched ovary extending forward.

66 – for a single outstretched ovary extending backward.

66 ' for a single reflexed ovary extending backward.

Furthermore by adding an exponent to the fourth term we may indicate roughly the size of the organs. Thus $-\,66\,-^{33}$ indicates a female having the vulva located 66 per cent. of the length from the head end, and whose uteri and ovaries are outstretched and occupy 33 per cent. of the length of the body. Again $-\,\mathrm{M}\,-^{33}$ indicates a male with straight testicles extending in opposite directions, and whose internal sexual apparatus extends forward from the anus a distance equal to 33 per cent. of the length of the body.

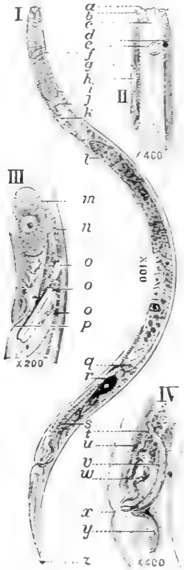


a, spear.
b, nerve-ring.
c, enlarged part of œsophagus.
d, cardiac collum.
e, reflexed anterior ovary.
f, blind end of same.
g, vulva.
h, egg.
i, blind end posterior ovary.
j, flexure in posterior ovary.
k, intestine.
l, *m*, special posterior part of intestine.
n, anus.
o, terminus.

Fig. 2. *Dorylaimus Bastiani.*

$$\frac{\cdot 4 \quad 6\cdot \quad 20\cdot \quad `50\cdot'^{32} \quad 93\cdot}{\cdot 9 \quad 2\cdot 6 \quad 3\cdot 7 \quad 3\cdot 9 \quad 1\cdot 5}\ 1\cdot 3\ \text{mm.}$$

The annexed illustrations of some known species together with their formulae will serve to make the application of this decimal system perfectly clear. (Figs. 2 and 3.)



a, lip.
b, cephalic seta.
c, pharynx.
d, eye.
e, lateral organ.
f, nerve-ring.
g, excretory pore.
h, ampulla.
i, posterior end œsophagus.
j, cardia.
k, ventral gland.
l, blind end of testicle.
m, egg.
n, intestine.
o, *o*, *o*, diatoms in intestine.
p, vulva.
q, junction of vas deferens and ejaculatory duct.
r, one of the caudal glands.
s, ejaculatory duct.
t, proximal end left spiculum.
u, posterior part of ejaculatory duct.
v, shaft, left spiculum.
w, rectum.
x, anus.
y, accessory piece.
z, terminus.

FIG. 3. *Monhystera diplops.*

$$\frac{1\cdot \quad 7\cdot \quad 15\cdot \quad -\mathrm{M}^{50} \quad 85\cdot3}{1\cdot8 \quad 3\cdot \quad 3\cdot3 \quad 4\cdot \quad 3\cdot}\ \cdot8 \text{ mm.}$$

I, male worm. II and IV, head and anal region of the same worm more highly magnified. III, portion of the body of a female, showing the vulva.

I think it would not be too much to expect reviewers to always include the decimal formula in even the briefest of notices if authors would take the trouble to give their measurements in that form. If such a custom could be brought into vogue it would constitute, it seems to me, a considerable improvement in helminthological literature, for we should then have, even in the briefest notices in the *Zoological Record*, *Zoologische Anzeiger* and other similar publications considerable highly condensed information in a form that would not take up too much of their valuable space.

The present paper might be noticed for instance in somewhat the following fashion:—

Cobb, *Proc. of the Cambridge Philos. Soc.*, Vol. XII. 1904, "Free-living Fresh-water New Zealand Nematodes"; describes new species as follows:—

Mermis Novae Zealandiae

$$\frac{\cdot0 \quad 1\cdot7 \quad 4\cdot7 \quad {}^{\backprime}48\cdot^{,26} \quad ?}{\cdot1 \quad \cdot4 \quad \cdot6 \quad \cdot7 \quad ?}\ 28\cdot \text{ to } 38\cdot \text{ mm.}$$

Mononchus rex

$$\frac{2\cdot\quad 6\cdot\quad 20\cdot\quad \text{‘}58\cdot\text{’}^{27}\quad 81\cdot}{1\cdot6\quad 1\cdot7\quad 2\cdot\quad 2\cdot2\quad 1\cdot4}\ 6\cdot5 \text{ to } 7\cdot\text{ mm.,}$$

$$\frac{2\cdot\quad 5\cdot5\quad 20\cdot\quad M^{50}\quad 85\cdot5}{1\cdot5\quad 1\cdot9\quad 2\cdot1\quad 2\cdot3\quad 1\cdot5}\ 6\cdot5 \text{ to } 7\cdot\text{ mm.}$$

Dorylaimus Novae Zealandiae

$$\frac{\cdot0\quad 5\cdot5\quad 20\cdot\quad \text{‘}45\cdot\text{’}\quad 95\cdot}{\cdot5\quad 1\cdot6\quad 2\cdot5\quad 2\cdot8\quad \cdot8}\ 6\cdot7\text{ mm.,}$$

$$\frac{\cdot0\quad 5\cdot3\quad 21\cdot8\quad -M-^{27}\quad 99\cdot2}{\cdot4\quad 1\cdot4\quad 2\cdot1\quad 2\cdot3\quad \cdot8}\ 6\cdot36\text{ mm.}$$

Dorylaimus profundis

$$\frac{\cdot0\quad 5\cdot9\quad 22\cdot\quad \text{‘}50\cdot\text{’}^{34}\quad 98\cdot2}{\cdot5\quad 1\cdot8\quad 2\cdot3\quad 2\cdot8\quad 1\cdot}\ 7\cdot\text{ mm.,}$$

$$\frac{\cdot0\quad 6\cdot4\quad 23\cdot6\quad -M-^{24}\quad 99\cdot}{\cdot5\quad 2\cdot1\quad 2\cdot7\quad 3\cdot\quad 1\cdot1}\ 6\cdot \text{ to } 6\cdot3\text{ mm.}$$

The sketch from which the formula is to be derived may be as complete as one wishes, but all that is necessary is a very rudimentary skeleton of the form of the worm. Draw a line with the aid of the camera lucida representing the axis of the worm. Across this line at the proper places draw cross lines representing the diameters. The result will resemble Fig. 4 from which the formula can be easily computed. The measurements of the skeleton shown in Fig. 4 give the formula

$$\frac{2\cdot\quad 9\cdot3\quad 17\cdot4\quad 45\cdot5\quad 93\cdot}{2\cdot8\quad 4\cdot2\quad 5\cdot9\quad 8\cdot6\quad 3\cdot6}\ \cdot78\text{ mm.}$$

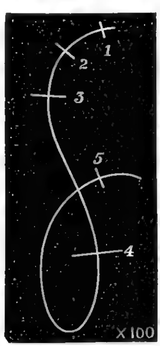


Fig. 4. Skeleton camera lucida drawing from which to compute the nematode formula.

If one possesses and uses a slide rule or other calculating machine the labour of producing the formula from such a skeleton is much reduced. I estimate that in most cases by the above method the formula may be derived from a specimen in about five minutes, provided there is a clear perception beforehand of what is to be done.

Where the worm is of large size, too large for measurement under the microscope, the camera lucida may be mounted on a lens holder without any lens, in which case the worm may be drawn natural size or even be reduced.

The rules etched upon glass so that the scale can be laid in contact with the paper, thus doing away with parallax, are very convenient for use in measuring the skeleton drawings.

Mermis Novae Zealandiae, n. sp.

$$\frac{\cdot 0 \quad 1{\cdot}7 \quad 4{\cdot}7\,? \quad `48{\cdot}{,}^{26} \quad ?}{\cdot 1 \quad \cdot 4 \quad \cdot 6 \quad\quad \cdot 7 \quad ?}\ 28{\cdot} \text{ to } 38{\cdot}\text{ mm.}$$

is the best formula derivable from the half dozen specimens examined, only one of which was beyond question adult. The thin, smooth, whitish, translucent skin is destitute of hairs and striae, and it would appear that the musculature is weak. The body is threadlike and uniform in diameter, and ends anteriorly in a neck which in its anterior half is slightly convex-conoid. The head end is truncate, apparently without papillae, and certainly without setae, lips, lateral organs and eyes. There is no proper pharynx, the pore-like mouth opening being continued in the œsophagus as a tube of the same diameter. The cylindrical œsophagus is two-fifths as wide as the neck and in all the specimens is somewhat indefinite, though the length given in the formula is believed to be at least approximately correct. The intestine was plainly visible in most parts of the body but was manifestly becoming obsolete. No anus was to be seen. The posterior extremity for a distance equal to two to three body widths was more transparent owing to the absence of intestine. No ventral gland was seen.

The nerve ring was not very satisfactorily located but the measurement given in the formula is probably fairly correct.

The tail is rather suddenly convex-conoid at the somewhat blunt terminus, and it is quite destitute of hairs or papillae. There are no caudal glands and no spinnerets. The body continues to have its usual width until within a short distance of the tail end, the tapering taking place in a distance no greater than two to three times as great as the width of the body. The inconspicuous vulva is located very slightly in front of the middle of the body. There are two uteri, extending in opposite directions. In the only gravid female seen they contained towards

one thousand somewhat spherical eggs, measuring 50 to 60 micromillimetres in their greatest diameter, each egg being therefore one-fifth as wide as the body.

No males were found in the collection.

Habitat, Lakes Taupo and Waikaremoana, New Zealand, at a depth of 370 to 840 feet.

Mononchus rex, n. sp.

$$\frac{2\cdot \qquad 6\cdot \qquad 20\cdot \qquad {}^{\prime}58\cdot{}^{\prime 27} \qquad 81\cdot}{1\cdot 6 \qquad 1\cdot 7 \qquad 2\cdot \qquad 2\cdot 2 \qquad 1\cdot 4} \; 6\cdot 5 \text{ to } 7\cdot \text{mm.}$$

is the formula of an adult female. The skin is without hairs and there are no striae to be seen with moderate powers. As usual the neck is cylindroid and ends in a head destitute of setae, though there are papillae. These latter are low, broad and inconspicuous. While the lips do not project, the region is elevated enough to give the front of the head a rounded appearance. The lips are bulky and powerful, being one-third as thick as the pharynx is wide. There are no eyes, and no lateral organs were seen.

The pharynx is armed with very powerful muscles that can be easily seen passing from the lips to the region near the anterior end of the œsophagus. The teeth are less well developed than in some other species of the genus. There is a dorsal tooth, small in comparison with the size of the pharynx, and reaching not more than one-third the way to the lips. There are also ventrally submedian teeth located opposite the dorsal tooth, but they are smaller, and one of them is seen only with difficulty in the specimens before me.

The œsophagus is cylindroid and is surrounded rather squarely by the nerve ring, which, as is usual in this genus, is situated well forward.

The intestine is dark in colour and is about one-half as wide as the body, the cells showing a tessellated arrangement of their contents. The rectum is shorter than the body diameter. No ventral gland or *porus excretorius* was seen.

The lateral fields are one-fifth as wide as the body.

The conoid tail tapers more rapidly at first, being nearly cylindroid in the posterior two-thirds. The terminus is slightly expanded, the usual spinneret being accompanied by two ventrally sub-median papillae after the manner of the species *longicaudatus*, except that here the papillae are slightly larger in proportion. Three caudal glands are present.

Two uteri lead from the inconspicuous vulva, one forward and the other backward. Each uterus is capable of carrying a single egg at a time, or possibly two. The eggs are nearly a fourth of a

millimetre long, and are half as wide as long. The ovaries are reflexed.

$$\frac{2\cdot \quad 5\cdot5 \quad 20\cdot \quad M^{50} \quad 85\cdot5}{1\cdot5 \quad 1\cdot9 \quad 2\cdot1 \quad 2\cdot3 \quad 1\cdot5}\ 6\cdot5 \text{ to } 7\cdot \text{ mm.}$$

The tail of the male is shaped like that of his mate. In front of the anus it is armed with a ventral row of seventeen prominent papillae, extending forward from the anus a distance equal to half the length of the tail. There is no bursa. The spicula were not plainly seen, but are believed to be of the form usual to the genus. They are about twice as long as the anal body diameter.

Habitat, Lakes Manapouri and Wakatipu, New Zealand, at depths of 200 to 1150 feet. The species is a near ally of the writer's *M. gymnolaimus,* of Fiji.

Dorylaimus Novae Zealandiae, n. sp.

$$\frac{\cdot0 \quad 5\cdot5 \quad 20\cdot \quad `45\cdot2' \quad 95\cdot}{\cdot5 \quad 1\cdot6 \quad 2\cdot5 \quad 2\cdot8 \quad \cdot8}\ 6\cdot7 \text{ mm.}$$

is the formula of a somewhat immature female, the largest seen. The smooth yellowish skin is destitute of transverse markings so far as can be seen with moderately high powers. Longitudinal markings (internal), due to the muscles, are plainly to be seen. The skin, especially near the head, is innervated, but the nature of the connected external organs was not made out; there are however no setae anywhere on the body. Very likely the innervations indicate the presence of sensory papillae.

The conoid neck becomes a trifle convex-conoid at the head end, where it is simply truncate, the lip region being neither contracted nor expanded. There appear to be six very inconspicuous lips each bearing a single minute papilla. The width of the head may be gauged by the fact that the lip region is about twice as wide as the spear. It seems that lateral organs are present in the shape of U-shaped markings immediately behind the lips and opening backwards. The spear is of the usual form and slides in a ring located a short distance behind the lips.

The œsophagus is of the form typical for the genus, being however simply conoid in form, enlarging gradually from two-fifths as wide as the corresponding part of the neck in front, to three-fifths as wide as the corresponding part of the neck behind. The lining is rather conspicuous in the posterior part. The œsophagus is set off from the intestine by a shallow but distinct constriction.

The dark coloured intestine is one-half to three-fifths as wide as the body. The cardia is inconspicuous and the cardiac cavity is nearly obsolete.

The rectum and pre-rectum vary in the two sexes, being longer

in the male. In the female the rectum is one and one-half times to two times as long as the anal body-diameter, while the pre-rectum is nearly or quite three times as long as the rectum. In the male the pre-rectum is about three-fourths as long as the neck. Numerous glands are to be seen in the neighbourhood of the anus, especially in the male.

The lateral fields are about one-fifth as wide as the body.

The inconspicuous nerve-ring surrounds the œsophagus rather squarely.

The tail of the female is concave-conoid in the anterior half and setaceous in the posterior half, where it ends in an acute terminus, destitute of opening or spinneret. The tail of the male is obtusely conoid, and bears, near its extremity, at least two slightly lateral papillae. The posterior extremity of the male is invariably incurved. The anal region is not raised or made conspicuous by any special external markings or structures.

From the large depressed vulva the vagina leads half-way across the body where it branches into two symmetrical uteri extending in opposite directions so as to occupy the middle third of the body. Each ovary is reflexed and reaches about two-fifths the way back to the vulva. As only immature females were seen nothing is known about the eggs.

$$\frac{\cdot 0 \qquad 5\cdot 3 \qquad 21\cdot 8 \qquad -\mathrm{M}-^{27} \qquad 99\cdot 2}{\cdot 4 \qquad 1\cdot 4 \qquad 2\cdot 1 \qquad 2\cdot 3 \qquad \cdot 8}\ 6\cdot 36 \text{ mm.}$$

is the formula of an adult male.

External tactile supplementary sexual organs are present on the males in the form of a row of ventral papillae arranged in close juxtaposition. Taking the adjacent body-diameter as a measuring unit, this row of papillae begins three measures from the anus and extends two measures forwards. Each papilla is innervated, and the total number is twenty-nine, the anterior one being somewhat the largest. These papillae are most plainly to be seen when the body is strongly incurved. There is no bursa. The immediate region of the anus was not supplied with any conspicuous papillae.

The two equal, plain, linear, arcuate, somewhat pointed spicula were of the form usual to the genus and measured two and one-half times as long as the anal body-diameter. Accessory pieces were present. Powerful oblique copulatory muscles occupied part of the space opposite the row of papillae. The two testicles were straight and extended in opposite directions from a point a little in front of the middle of the body, where they occupied a space over one-quarter as long as the body.

The species is a typical Dorylaimus of large size.

Habitat, Lake Taupo, New Zealand, at a depth of 350 feet.

Dorylaimus profundis, n. sp.

$$\frac{\cdot 0 \quad 5\cdot 9 \quad 22\cdot \quad {}^{\prime}50\cdot{}^{,34} \quad 98\cdot 2}{\cdot 5 \quad 1\cdot 8 \quad 2\cdot 3 \quad 2\cdot 8 \quad 1\cdot}\ 7\cdot \text{ mm.}$$

is the formula of a female that, though having the appearance of being full grown, had not yet produced eggs. The structure of the skin was entirely typical, and so closely resembled that of *Novae Zealandiae* that the same description would apply to both, and it may be here remarked that in spite of the marked differences pointed out later on, the two species resemble each other outwardly in a marked degree.

The conoid neck becomes convex-conoid at the head end, where there is a distinct lip region, which though presenting the same contour as that of *Novae Zealandiae* is of quite a different character. In speaking of *Dorylaimus latus* in Vol. VI. (1891) of the *Proceedings of the Linnaean Society of N. S. W.* I described the manner in which the lip region of that species can be protruded, and gave a figure of the protruded condition of the lips. It seems that the head of *D. profundis* has a similar structure. In the present case I have not seen the lips protruded, but the chitin at the head presents that infolding characteristic of the head of *latus* and some other species. This structure of the head together with differences in the structure of the posterior extremity enable one to distinguish readily between the two species *Novae Zealandiae* and *profundis.*

As usual there are six obscure lips each bearing a single almost invisible papilla.

There is nothing striking about the structure of the pharynx, the stout spear sliding in a ring located behind the infolded lips at a distance one and one-half times as great as the width of the head.

The œsophagus is simply conoid, being anteriorly one-third and posteriorly one-half as wide as the corresponding part of the neck. The lining is plainly to be seen in the posterior part. The œsophagus is set off from the intestine by a prominent constriction.

The dark coloured intestine is one-half as wide as the body and is composed of cells presenting a tessellated arrangement. In the female the pre-rectum is one-third to one-half as long as the neck, while in the male it is much longer, namely somewhat longer than the neck.

There seems to be no gland emptying ventrally as in many Nematode genera, but certain organs, apparently glandular, are to be seen in the neck region and these probably debouch laterally.

The lateral fields are one-third to one-fourth as wide as the body. The nerve-ring encircles the œsophagus rather squarely.

The tail of the female is concave-conoid to a rather blunt point, while that of the male is conical to a rounded terminus, where a few innervated papillae are to be seen. The anus is neither raised nor depressed, and though easily to be discovered is not rendered conspicuous by any accessory apparatus in its immediate vicinity.

The vulva is depressed, and from it the vagina extends half way across the body, where it divides into two uteri, one extending forward and the other extending backward. These, together with the reflexed ovaries, occupy the middle third of the body. No eggs or young were seen.

$$\frac{\cdot 0 \quad 6\cdot 4 \quad 23\cdot 6 \quad -\mathrm{M}-^{24} \quad 99\cdot}{\cdot 5 \quad 2\cdot 1 \quad 2\cdot 7 \quad 3\cdot \quad 1\cdot 1}\ 6\cdot \text{ to } 6\cdot 33 \text{ mm.}$$

A row of ventral tactile papillae, numbering thirty-five to forty-two, is found some distance in front of the anus on the male. Reckoning from the anus these occupy the second fourth of the pre-rectal region. These papillae are plainly to be seen only when the body is strongly incurved, and then only as doubly innervated, low and inconspicuous contiguous elevations.

The two equal arcuate spicula are two and one-half to three times as long as the anal body-diameter, and are of the nature characteristic of most of the species of the genus. Powerful oblique copulatory muscles exist in the pre-rectal region.

The species is a typical *Dorylaimus* of large size, readily distinguished from *Novae Zealandiae* by the structure of the head, the shortness of the female tail and the number of the male accessory papillae.

Habitat, Lakes Wakatipu, Waikaremoana, and Manapouri, New Zealand, at a depth of 200 to 1100 feet.

Register of the Species, Localities and Depths.

	Depth in feet.	
Lake Taupo,	350	*Dorylaimus Novae Zealandiae.*
„ „	350	„ „ „
„ „	370	*Mermis Novae Zealandiae.*
Lake Waikaremoana,	840	„ „ „
„ „	300—600	*Dorylaimus profundis.*
Lake Manapouri,	200—600	*Dorylaimus profundis* and *Mononchus rex.*
„ „	360	*Dorylaimus profundis.*
Lake Wakatipu	500—1100	„ „
„ „	200—300	*Dorylaimus profundis* and *Mononchus rex.*
„ „	1150	*Dorylaimus profundis* and *Mononchus rex.*

NOTE. At the Author's request I add a note on the dates of collection of the Nematodes described above. They were collected during 1902, as follows:—

Lake Taupo, during May and June.
Lake Waikaremoana, August and September.
Lake Wakatipu, October and November.
Lake Manapouri, November and December.

Lakes Waikare, examined during April, and Rotoiti, during July, yielded no specimens. K. LUCAS.

Relative Amount of Ionization produced in Air and Hydrogen by Röntgen Rays. By R. K. McClung, B.A., Trinity College, 1851 Exhibition Science Scholar.

[*Read* 1 February 1904.]

The question of the relative amount of ionization produced in different gases by means of Röntgen rays has been investigated by different experimenters. Professors Thomson[1], Rutherford[2], and Perrin[3] have each made independent determinations on this question. In the case of most of the gases experimented upon the results obtained by the different observers were in very fair agreement, but hydrogen proved to be a marked exception. In the case of hydrogen there was a very considerable difference in the values obtained by the three experimenters. The conditions under which these different experiments were carried out differed, of course, somewhat in the different instances, and no doubt the results were influenced thereby. The experiments described in the present paper were undertaken therefore to determine, if possible, what are the conditions which cause variations in this particular gas.

The method adopted for comparing the amount of ionization in the two gases was a balance method. This promised to be the most satisfactory, as the ionization in hydrogen is so much less than in air that comparisons could be more easily made in this manner than by separate and direct measurements.

The gases to be ionized were contained in two similar metal cylinders which were made gas-tight. Inside each of these cylinders were placed two parallel plates between which the Röntgen rays passed, in the direction parallel to the length of the plates, and ionized the gas between them. These vessels were placed symmetrically with regard to the source of the rays so that the same amount of ionization was produced in each of them when they were both filled with air at atmospheric pressure. In one vessel one of the plates was connected to the positive pole of a battery, while the corresponding plate in the other vessel was connected to the negative pole, the centre of the battery being connected to earth. The other two plates were connected together and to the electrometer. When the gas was ionized in both cylinders the plate connected to the electrometer

[1] J. J. Thomson, *Proc. Camb. Phil. Soc.* x. p. 10, 1900.

[2] Rutherford, *Phil. Mag.* v. 43, p. 241, 1897.

[3] Perrin, *Annales de Physique et de Chimie*, xi. p. 496, 1897.

in one would receive a charge opposite in sign to that received by the corresponding plate in the other. If these charges were equal in amount they would of course neutralize each other and produce no effect on the electrometer. When a balance was once obtained, by careful adjustment of the position of the different portions of the apparatus, any alteration of the ratio of the ionization in the one cylinder to that in the other could easily be detected by the alteration in the balance. This method of comparing the ionization in the two cases can be made a very sensitive one.

To compare the ionization produced in air with that produced by the same source in an equal volume of hydrogen, the following method was followed. The two cylinders, containing air at atmospheric pressure which had been carefully dried, were carefully adjusted so that as good a balance as possible was obtained. One of the cylinders was then exhausted and filled with hydrogen at atmospheric pressure, the operation being repeated more than once so as to insure that the hydrogen was as free from air as possible. The ionization in hydrogen being quite different from that in air the balance was now completely destroyed. To restore the balance the cylinder containing air was partially exhausted until the ionization in the air at the reduced pressure was sufficient to balance the ionization in the hydrogen. Now since the amount of ionization in air is proportional to the pressure of the air, therefore the amount of ionization in hydrogen at atmospheric pressure can thus be compared with the amount in an equal volume of air at the same pressure.

Several experiments were made, using a Röntgen ray bulb with an automatic vacuum regulator attached as the source of the rays. The rays given out by this bulb remain pretty constant, and exceedingly constant results were obtained with it. If Q_A is the amount of ionization produced in a given volume of air, and Q_H the amount in an equal volume of hydrogen at the same pressure, then for this bulb the value obtained for the ratio $\frac{Q_A}{Q_H}$ was 0·11. Several other focus bulbs were tried, and they gave results which differed considerably from this value, and also differed from one another. These were not automatic regulating bulbs but simply the ordinary type of focus bulb, and they did not give nearly so constant results as the automatic regulating one. One of these bulbs gave values for the ratio $\frac{Q_A}{Q_H}$ varying between 0·060 and 0·084, and another one gave the two values 0·052 and 0·056, while a third one gave the value 0·059. Different bulbs usually give out different types of rays, and even the type of rays given out by the same bulb under different conditions

may vary. It is evident from these experiments that when the rays, which produce the ionization in the two gases, are kept constant, the relative amount of ionization remains constant, but that certain alterations in the rays produce an alteration in the relative amount of ionization. Of course it has been known for some time that the amount of ionization produced in a given volume of any gas is dependent upon the type of rays used, but it is evident from this investigation that an alteration in the type of rays affects the ionization in these two gases in different proportions.

Experiments are at present being made, using a bulb in which the pressure can be altered by means of an exhaust pump, to determine what effect any alteration in the pressure of the gas in the bulb has upon the ratio of the ionization for the two gases. These experiments are as yet incomplete, but the indications are that an alteration of the pressure in the bulb does affect this ratio. They appear to show that the more highly exhausted the bulb is, and consequently the harder the rays, the greater is the ionization in hydrogen as compared with air. This question is being more fully investigated, and it is hoped to have more definite results shortly.

In conclusion I would thank Professor Thomson for his advice in connection with these experiments.

On the Calculation of Capacities in terms of the Coefficients of Electrostatic Induction. By G. F. C. Searle, M.A., Peterhouse, University Lecturer in Experimental Physics, and Demonstrator in Experimental Physics.

[*Read* 15 February 1904.]

Introduction.

§ 1. In elementary theoretical electrostatics we meet with the capacity of an isolated body, such as a sphere in free space, and with the capacity of a condenser. But, in dealing with the problems connected with the distribution of electrical charges upon the various conductors in a cable used for the transmission of polyphase currents, it is convenient to so extend the definition of the capacity of a conductor that we can employ the term even though other conductors are in the neighbourhood. In like manner it is convenient to define the capacity between two conductors in such a way that we can employ the term not only when the two conductors do not form a theoretical condenser, but also when other conductors are in the neighbourhood.

Maxwell* contemplated the extensions of the definitions, which are necessary in attacking the more complex problems, but he did not pursue the matter except in a single instance. As the subject is of some interest in practical electrical engineering, I propose to give some definitions of capacity and to shew how to calculate the capacities, so defined, in terms of Maxwell's coefficients of induction.

Capacity of a Conductor.

§ 2. The simplest case is that of a single conductor at an infinite distance from all other bodies. In this instance, the capacity of the conductor is defined by means of the relation between the charge and the potential of the body. Thus, if C be the capacity of the conductor and e and v be its charge and its potential, then the capacity is defined by the relation

$$C=\frac{e}{v} \quad \text{.............................(1).}$$

In practice, however, we do not have infinite distances to deal

* Maxwell, *Electricity and Magnetism*, Vol. II. § 87, third edition.

with, and thus we are obliged to consider a conductor in relation to its surroundings; when we do this, we find that the ratio of the charge on a conductor to its potential no longer depends only upon the form and magnitude of the conductor itself, but also upon all the other conductors in its neighbourhood. To meet the case, in which the field contains n conductors, we require a new definition of capacity. When $n-1$ conductors are kept at zero potential, Maxwell defines the capacity of the remaining conductor as its charge when its own potential is unity, but this extension of the definition is not sufficient, for problems occur in which some of the $n-1$ conductors are insulated instead of being at zero potential. The definition I propose is as follows:

DEFINITION A. *Let there be the n conductors* [1], [2], ... [n] *in the field and let them all be initially uncharged. Then, if the potential of an* insulated *conductor* [1] *rise from zero to v_1 when a charge e_1 is given to* [1], *the ratio of e_1 to v_1 is called the capacity of* [1] *in the presence of* [2], [3], ... [n].

The capacity of the conductor [1], as thus defined, depends in general not only upon the forms and positions of the remaining $n-1$ conductors, but also upon their electrical conditions, and thus it is necessary, as Maxwell hints, to specify in each case which of the conductors [2], [3], ... [n] are insulated and which are kept at the constant potential zero.

A problem, at first sight more general than the last, is that in which, initially, the insulated conductors have given charges and the other conductors have given potentials, which are maintained constant while the charge on [1] is changed. In this case we may say that if the potential of an insulated conductor [1] rises by V_1 when its charge is increased by E_1, the ratio of E_1 to V_1 is equal to the capacity of [1] in the presence of [2], ... [n]. It will, however, be seen that, since the n equations connecting the potentials of the conductors with their charges are linear, the value of the capacity of [1], yielded by the last definition, is identical with that obtained by the use of Definition A, provided that the same conductors be insulated in the two cases. It will therefore suffice, in the present paper, to use the simpler definition.

Coefficients of Induction.

§ 3. If, when [1] is at unit potential and all the other conductors are at zero potential, the charge on [1] is q_{11}, the quantity q_{11} is called the Coefficient of Self Induction for the conductor [1]. We see at once that, according to Definition A, q_{11} is the capacity of [1] in the presence of [2], ... [n], when the latter conductors are all kept at zero potential.

If, when [1] is raised to unit potential, and the $n-1$ remaining conductors are at zero potential, the charge on [2] is q_{21} and if, when [2] is raised to unit potential and the $n-1$ remaining conductors are at zero potential, the charge on [1] is q_{12}, it can be shewn that

$$q_{12} = q_{21} \quad \text{.............................(2).}$$

On account of this relation, q_{12} or q_{21} may be called the Coefficient of Mutual Induction between [1] and [2]. To facilitate the mathematical operations upon the determinants, which will appear in the course of the work, it will be often convenient to treat q_{12} and q_{21} as separate quantities. If we wish to do so, we can substitute q_{12} for q_{21} in the final results.

The relations between the charges $e_1, e_2, \dots e_n$, and the potentials $v_1, v_2, \dots v_n$ of the n conductors can now be expressed, as Maxwell shews, by means of the n equations,

$$\left.\begin{array}{l} q_{11} v_1 + q_{12} v_2 + \dots + q_{1n} v_n = e_1 \\ q_{21} v_1 + q_{22} v_2 + \dots + q_{2n} v_n = e_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ q_{n1} v_1 + q_{n2} v_2 + \dots + q_{nn} v_n = e_n \end{array}\right\} \quad \text{...............(3).}$$

Maxwell says that "a coefficient in which the two suffixes are the same is called the Electric Capacity of the conductor to which it belongs," but, in view of the extended definitions of capacity which are given in the present paper, it is necessary to abandon Maxwell's term and, in its stead, to speak of the coefficient of self induction of the conductor. It must be remembered that when there are n conductors in the field, every coefficient, whether of self or of mutual induction, is, in general, a function of the form and position of every one of the n conductors.

Capacity of [1] *in the presence of* [2], [3], ... [n], *when* [2], ... [m] *are insulated and* [$m+1$], ... [n] *are kept at zero potential.*

§ 4. If $v_1, v_2, \dots v_n$ be the potentials of the conductors after [1] has received the charge e_1, as in Definition A, then, since,

$$e_2 = e_3 = \dots = e_m = 0,$$

and
$$v_{m+1} = v_{m+2} = \dots = v_n = 0,$$

the first m of Maxwell's equations become

$$\left.\begin{array}{l} q_{11} v_1 + q_{12} v_2 + \dots + q_{1m} v_m = e_1 \\ q_{21} v_1 + q_{22} v_2 + \dots + q_{2m} v_m = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ q_{m1} v_1 + q_{m2} v_2 + \dots + q_{mm} v_m = 0 \end{array}\right\} \quad \text{............(4).}$$

These equations suffice to determine the potentials of the m insulated conductors [1], [2], ... [m] in terms of e_1. When these potentials have been found in terms of e_1, the remaining $n-m$ equations, of which the first is

$$q_{(m+1)1}v_1 + q_{(m+1)2}v_2 + \ldots + q_{(m+1)m}v_m = e_{m+1} \quad \ldots\ldots(5),$$

give the charges upon the uninsulated conductors [$m+1$], ... [n] in terms of e_1. In the present problem, however, we are not concerned with the values of these charges.

Let Δ denote the determinant of m rows formed from the coefficients in equations (4), so that

$$\Delta = \begin{vmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{vmatrix} \quad \ldots\ldots\ldots\ldots(6).$$

In (6) no suffix higher than m appears, but it must be remembered that every coefficient appearing in (6) is a function of the forms and positions not only of the conductors [1], [2], ... [m], but also of the conductors [$m+1$], ... [n].

Let Δ_{11} denote the determinant formed by deleting the row and the column which contain q_{11}, so that

$$\Delta_{11} = \begin{vmatrix} q_{22} & q_{23} & \cdots & q_{2m} \\ q_{32} & q_{33} & \cdots & q_{3m} \\ \cdots & \cdots & \cdots & \cdots \\ q_{m2} & q_{m3} & \cdots & q_{mm} \end{vmatrix} \quad \ldots\ldots\ldots\ldots(7).$$

Thus Δ_{11} is the minor of Δ which is complementary to q_{11}.

Solving equations (4) for v_1, we at once obtain

$$v_1\Delta = e_1\Delta_{11} \quad \ldots\ldots\ldots\ldots(8).$$

Hence, if C_1 be the capacity of [1] under the specified conditions, we have, by Definition A, $C_1v_1 = e_1$ and thus

$$C_1 = \frac{\Delta}{\Delta_{11}} \quad \ldots\ldots\ldots\ldots(9).$$

It must be remembered that Δ is a determinant of m rows and not of n rows. Coefficients with suffixes referring to one or more of the conductors, which are kept at zero potential, do not appear in Δ.

§ 5. As a simple example of the problem of § 4, I will take the case in which all the conductors except [1] and [2] are kept at zero potential. We then have

$$\Delta = \begin{vmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{vmatrix} = q_{11}q_{22} - q_{12}q_{21},$$

and $$\Delta_{11} = q_{22}.$$

Hence $$C_1 = \frac{q_{11}q_{22} - q_{12}q_{21}}{q_{22}} = q_{11} - \frac{q_{12}q_{21}}{q_{22}}.$$

Similarly $$C_2 = \frac{q_{11}q_{22} - q_{12}q_{21}}{q_{11}} = q_{22} - \frac{q_{12}q_{21}}{q_{11}}.$$

Since q_{11} and q_{22} are positive and q_{12} and q_{21} are negative, we see that the capacity of [1] is smaller when [2] is insulated than it is when [2] is kept at zero potential.

Relations between the potentials and the charges of the insulated conductors [1] *and* [2], *when* [3], ... [m] *are insulated and uncharged and* [$m+1$], ... [n] *are kept at zero potential.*

§ 6. So far we have been concerned with the relation of the potential of a *single* conductor to its charge under specified conditions. It will, however, be convenient, for use in later problems, to obtain the relations which exist between the potentials v_1 and v_2 and the charges e_1 and e_2, when the conductors [3], ... [m] are insulated and [$m+1$], ... [n] are kept at zero potential.

In this case, the first m of Maxwell's equations are

$$\left.\begin{aligned} q_{11}\,v_1 + q_{12}\,v_2 + \ldots + q_{1m}\,v_m &= e_1 \\ q_{21}\,v_1 + q_{22}\,v_2 + \ldots + q_{2m}\,v_m &= e_2 \\ q_{31}\,v_1 + q_{32}\,v_2 + \ldots + q_{3m}\,v_m &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ q_{m1}v_1 + q_{m2}v_2 + \ldots + q_{mm}v_m &= 0 \end{aligned}\right\} \ldots\ldots\ldots\ldots(10),$$

the quantities $e_3, e_4 \ldots e_m$ being all zero, since the corresponding conductors are insulated and are initially without charges. As in § 4, the remaining $n-m$ equations will give, in terms of e_1 and e_2, the charges upon the conductors [$m+1$], ... [n], which are kept at zero potential.

If Δ_{11}, Δ_{12}, Δ_{21} and Δ_{22} denote the minors of the determinant (6), which are complementary to q_{11}, q_{12}, q_{21} and q_{22} respectively, we can write down the values of v_1 and v_2 in terms of e_1 and e_2 in the following forms:

$$\left.\begin{aligned} v_1\Delta &= \quad e_1\Delta_{11} - e_2\Delta_{21} \\ v_2\Delta &= -e_1\Delta_{12} + e_2\Delta_{22} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(11).$$

If we solve these equations for e_1 and e_2, we shall find the charges which must be given to the conductors [1] and [2] in order to raise their potentials from zero to v_1 and v_2 under the specified conditions. Thus

$$\left.\begin{aligned} e_1(\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}) &= (v_1\Delta_{22}+v_2\Delta_{21})\,\Delta \\ e_2(\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}) &= (v_1\Delta_{12}+v_2\Delta_{11})\,\Delta \end{aligned}\right\}\text{.........(12).}$$

Since a determinant is unchanged in value when its rows are turned into columns and its columns into rows, and since $q_{rs}=q_{sr}$ for every pair of conductors, it follows that $\Delta_{21}=\Delta_{12}$.

§ 7. If $\Delta_{11,22}$ denote the minor of Δ complementary to

$$\begin{vmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{vmatrix},$$

then it is known* that

$$\Delta\Delta_{11,22}=\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}\text{.................(13),}$$

and thus (12) may be written

$$\left.\begin{aligned} e_1\Delta_{11,22} &= v_1\Delta_{22}+v_2\Delta_{21} \\ e_2\Delta_{11,22} &= v_1\Delta_{12}+v_2\Delta_{11} \end{aligned}\right\}\text{.................(14).}$$

We notice that equation (13) requires us to write $\Delta_{11,22}=1$, when Δ contains only *two* rows.

Capacity of [1] *and* [2] *"in parallel," when* [3], ... [m] *are insulated and* [$m+1$], ... [n] *are kept at zero potential.*

§ 8. The words "in parallel" imply that [1] and [2] are connected by a very fine wire, and the capacity in question is the capacity, according to Definition A, of the single conductor formed by joining [1] and [2]. It is further implied that the relative positions of the n conductors are the same before and after [1] and [2] are joined. Since the wire is very thin, the charge upon it is negligible in comparison with the charges upon the conductors themselves, and the only function of the wire, as far as Maxwell's equations are concerned, is to make the potential of [2] equal to that of [1].

It is easy to calculate, in terms of the n^2 coefficients† belonging to the original n conductors, the $(n-1)^2$ coefficients for the $n-1$ conductors, to which the system is reduced by joining [1] and [2], and, when this has been done, the capacity of [1] and [2] "in parallel" can be at once written down by means of equation (9).

* Burnside and Panton, *Theory of Equations*, third edition, § 136.

† In this estimate, q_{rs} and q_{sr} are treated as distinct quantities. If we do not distinguish q_{rs} from q_{sr} the number of coefficients is only $\frac{1}{2}n(n+1)$.

I shall, however, first solve the problem by the aid of the general equations obtained in § 6.

If C_{1+2} denote the capacity of [1] and [2] "in parallel," and v be the common potential of the two conductors, then, by Definition A,

$$C_{1+2} = \frac{e_1 + e_2}{v},$$

since $e_1 + e_2$ is the charge upon the single conductor formed by joining [1] and [2]. Putting $v_1 = v_2 = v$ in (12), we find at once that, when [3], ... [m] are insulated and [$m+1$], ... [n] are kept at zero potential,

$$C_{1+2} = \Delta \frac{\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} \quad \ldots\ldots\ldots\ldots\ldots(15).$$

§ 9. I now pass on to calculate, in terms of the original n^2 coefficients, the $(n-1)^2$ coefficients which occur when [1] and [2] are joined. If we put $v_1 = v_2 = v$ in Maxwell's equations (3), and then add together the first two equations, we obtain the $n-1$ equations

$$\left.\begin{array}{llll} (q_{11}+q_{12}+q_{21}+q_{22})\,v & +(q_{13}+q_{23})\,v_3 & +(q_{14}+q_{24})\,v_4+\ldots & =e_1+e_2 \\ (q_{31}+q_{32})\,v & +q_{33}\;v_3 & +q_{34}\;v_4+\ldots & =e_3 \\ (q_{41}+q_{42})\,v & +q_{43}\;v_3 & +q_{44}\;v_4+\ldots & =e_4 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots \end{array}\right\} \ldots(16).$$

If, for brevity, the single conductor formed by joining [1] and [2] is denoted by the symbol [0], and if the coefficients for the new system are denoted by accented letters, we see, on comparing the above equations with Maxwell's scheme, that

$$q'_{00} = q_{11} + q_{12} + q_{21} + q_{22} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17),$$

$$q'_{0s} = q_{1s} + q_{2s}, \quad q'_{s0} = q_{s1} + q_{s2} \quad \ldots\ldots\ldots\ldots\ldots(18),$$

$$q'_{rs} = q_{rs}, \quad q'_{sr} = q_{sr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19),$$

where s and r range from 3 to n.

§ 10. If Δ' denote the determinant of $m-1$ rows, formed on the plan of (6) from the coefficients for the $m-1$ conductors [0], [3], ... [m], and if Δ'_{00} denote the minor of Δ' complementary to q'_{00}, it follows from (6) that

$$\Delta'_{00} = \begin{vmatrix} q_{33} & q_{34} & \cdots \\ q_{43} & q_{44} & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix} = \Delta_{11,22}.$$

We can now write down the capacity of [0], i.e. of [1] and [2] "in parallel," by the aid of (9). Thus

$$C_{1+2} = C_0 = \frac{\Delta'}{\Delta'_{00}} = \frac{\Delta'}{\Delta_{11,22}} \quad \text{.................(20).}$$

The determinant Δ' can easily be expressed in terms of the four minors Δ_{11}, Δ_{12}, Δ_{21} and Δ_{22} of the original determinant Δ. Writing Δ' as the sum of two determinants, we have

$$\Delta' = \begin{vmatrix} q_{12}+q_{22} & q_{13}+q_{23} & \cdots \\ q_{32} & q_{33} & \cdots \\ q_{42} & q_{43} & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix} + \begin{vmatrix} q_{11}+q_{21} & q_{13}+q_{23} & \cdots \\ q_{31} & q_{33} & \cdots \\ q_{41} & q_{43} & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix}.$$

The first determinant on the right may be written

$$\begin{vmatrix} q_{22} & q_{23} & q_{24} & \cdots \\ q_{32} & q_{33} & q_{34} & \cdots \\ q_{42} & q_{43} & q_{44} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} + \begin{vmatrix} q_{12} & q_{13} & q_{14} & \cdots \\ q_{32} & q_{33} & q_{34} & \cdots \\ q_{42} & q_{43} & q_{44} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix},$$

or

$$\Delta_{11} + \Delta_{21}.$$

In a similar way the second determinant on the right may be written

$$\Delta_{12} + \Delta_{22}.$$

Hence

$$\Delta' = \Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}.$$

Substituting this value of Δ' in (20), we obtain

$$C_{1+2} = \frac{\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}}{\Delta_{11,22}} \quad \text{...............(21).}$$

The result obtained in § 8 was

$$C_{1+2} = \Delta \frac{\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} \quad \text{..............(22).}$$

Comparing (21) with (22), we obtain the known formula

$$\Delta\Delta_{11,22} = \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21},$$

which was the result employed in § 7.

§ 11. When all the conductors except [1] and [2] are kept at zero potential, we find from (22)

$$C_{1+2} = q_{11} + q_{12} + q_{21} + q_{22},$$

as might have been inferred from (17).

Capacity between two conductors.

§ 12. In many, perhaps in the majority of cases, the relation between *two* insulated conductors in the presence of other conductors is in question. To lead up to the mathematical definition appropriate to this relation, it will be convenient to consider the manner in which this relation is examined in engineering practice, in the case of a cable containing several conductors. Two of the conductors, say [1] and [2], are selected and insulated; the sheath of the cable and some of the other conductors are earthed and the remaining conductors are insulated. The conductors [1] and [2] are then joined to the two poles of an insulated battery, and thus receive equal and opposite charges—say $+e$ and $-e$—provided, as is the case in practice, that the charge upon the battery itself be negligible in comparison with that on either conductor. The charge on either conductor is then measured by disconnecting the conductors from the battery and connecting them through an insulated ballistic galvanometer, whose "throw" measures e. Since e is proportional to the E.M.F. of the battery, it is convenient to have a term to express the ratio of e to the E.M.F. of the battery. We are thus led to speak of the capacity *between* two conductors and to define it as follows:—

DEFINITION B. *Let there be the n conductors* [1], [2], ... [n] *in the field and let them all be initially uncharged. Then, if the potentials of two* insulated *conductors* [1] *and* [2] *become v_1 and v_2 when the equal and opposite charges e and $-e$ are given to* [1] *and* [2], *the ratio of e to $v_1 - v_2$ is called the capacity between* [1] *and* [2] *in the presence of* [3], ... [n].

The capacity between the conductors [1] and [2], as thus defined, depends, in general, not only upon the forms and positions of the remaining $n-2$ conductors, but also upon their electrical conditions, and thus it is necessary to specify in each case which of the conductors [3], [4], ... [n] are insulated and which are kept at the constant potential zero.

If, initially, the insulated conductors have given charges, while the other conductors are maintained at given constant potentials, the definition would run as follows:—

Let [1] and [2] be two insulated conductors with any initial charges e_1 and e_2, and let any number of the other $n-2$ conductors be insulated and have given charges, while the remainder are kept at given constant potentials. Then, if the potentials of [1] and [2] are changed from v_1 and v_2 to $v_1 + V_1$ and $v_1 + V_2$, when the charges are changed from e_1 and e_2 to $e_1 + E$ and $e_1 - E$, the ratio of E to $V_1 - V_2$ is called the capacity between [1] and [2] in the presence of [3], ... [n].

Since, however, Maxwell's equations are linear, the value of the capacity, in terms of $q_{11}, q_{12}, \ldots$, given by this definition is identical with that obtained by the use of Definition B, provided, of course, that the same conductors be insulated in the two cases.

Capacity between [1] *and* [2] *when* [3], ... [m] *are insulated and* [$m+1$], ... [n] *are kept at zero potential.*

§ 13. Putting $e_1 = -e_2 = e$ in equations (11), we obtain

$$v_1\Delta = \quad e(\Delta_{11} + \Delta_{21}),$$

$$v_2\Delta = -e(\Delta_{12} + \Delta_{22}).$$

Thus, if the capacity between [1] and [2] under the specified conditions be denoted by C_{1-2}, we have

$$C_{1-2} = \frac{e}{v_1 - v_2} = \frac{\Delta}{\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}} \quad \ldots\ldots\ldots\ldots(23).$$

§ 14. When all the conductors except [1] and [2] are kept at zero potential, we have, by (23),

$$C_{1-2} = \frac{q_{11}q_{22} - q_{12}q_{21}}{q_{11} + q_{12} + q_{21} + q_{22}}.$$

§ 15. We may notice the relations between the capacity of [1] and [2] "in parallel" and the capacity between [1] and [2]. Thus, from (22) and (23),

$$\frac{C_{1+2}}{C_{1-2}} = \frac{(\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22})^2}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}}$$

$$= \frac{(\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22})^2}{\Delta\,\Delta_{11,22}}$$

$$C_{1+2} \cdot C_{1-2} = \frac{\Delta^2}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} = \frac{\Delta}{\Delta_{11,22}}.$$

§ 16. In conclusion I wish to express my thanks to Mr A. Russell for bringing the subject to my notice and for helping me in framing the definitions of capacity adopted in the paper. My thanks are also due to my former colleague, the Rev. P. E. Bateman, for advice and assistance in connexion with the mathematical methods employed, and to Mr G. T. Bennett for reading the manuscript of the paper.

On a soluble colloidal form of ferric and of other phosphates.

By W. J. Sell, M.A., F.R.S., Christ's College.

[*Read* 18 January 1904.]

It is well known that ferric phosphate is soluble to a small extent in an excess of disodium hydrogen phosphate in presence of ammonia. In the course of some work on the ammonium phosphates it was found that when the corresponding ammonium phosphate was employed in presence of ammonia a larger amount of the ferric phosphate was dissolved and that if the strengths of various solutions were properly adjusted a clear reddish brown solution was obtained. The change may be thus represented:

$$n\,(NH_4)_3\,PO_4 + FeCl_3 = FePO_4 + (n-1)\,(NH_4)_3\,PO_4 + 3NH_4Cl.$$

Now since ferric phosphate as ordinarily produced by precipitation is only known as a jelly-like substance insoluble in water, it seemed possible and desirable to ascertain if from the solution produced as above the compound could be obtained in solution in water by the process of dialysis. It seemed at least probable that ferric phosphate would diffuse with extreme slowness while the other ingredients are known to have high rates of diffusion: accordingly the following experiments were carried out with this object.

A solution of ferric chloride was made by dissolving about a hundred grams of the hydrated salt in 800 c.c. of water, the iron determined in the solution and the strength subsequently adjusted so as to contain 50 grams of the anhydrous salt in one litre. The diammonium hydrogen phosphate contained 100 grams of the crystallised salt in one litre. The ammonia used was the ordinary ·880 diluted with an equal volume of water.

If to 100 c.c. of ferric chloride be now added 400 c.c. of the diammonium hydrogen phosphate and, after shaking, sufficient ammonia to make the whole smell decidedly of that reagent, the large white precipitate turns slightly brown in colour and gradually but completely dissolves to a clear brownish red solution.

The solution was now dialysed through parchment paper in the ordinary way until every trace of chloride had disappeared. The dialysed solution had a somewhat lighter colour than before dialysis and had to some extent increased in volume by osmosis from the surrounding water. Positive tests having shewn the

presence of iron and phosphoric acid in the solution, measured portions were now removed with the object of determining the ratio of Fe_2O_3 to P_2O_5.

The P_2O_5 was determined by Woy's modification of the molybdate process (*Chem. Ztg.* Vol. XXI. pp. 441 and 469, 1897) and his results have been extensively verified by Sherman and Hyde (*Jour. Amer. Chem. Soc.* Vol. XXII. p. 652, 1900).

Expt. (1) 100 c.c. gave 2·228 grms $P_2O_5\,24MoO_3$ = ·0879 P_2O_5.

" (2) 100 c.c. " 2·224 " " " = ·0878 "

" (3) 250 c.c. after reduction of the iron to the ferrous state required 34·5 c.c. $\frac{N}{10}$ potassium permanganate.

" (4) 250 c.c. after reduction required 34·5 $\frac{N}{10}$ potassium dichromate.

In 100 c.c. this number gives ·11 grm Fe_2O_3. We have therefore in 100 c.c. ·0891 P_2O_5 to ·11 Fe_2O_3. Dividing these numbers by their respective molecular weights we have

$$P_2O_5 : Fe_2O_3 :: 61 : 68 \text{ or as } 1 : 1{\cdot}11,$$

thus shewing that the solution contains the compound $FePO_4$ with a minute quantity of $Fe(OH)_3$.

Further experiments were now carried out with a different sample of the dialysed solution and a sufficient supply was prepared and evaporated to dryness on the water bath, the dried residue tubed and separate portions taken for quantitative analysis. For this purpose the P_2O_5 after precipitation by the molybdate process was dissolved in ammonia and weighed as magnesium pyrophosphate.

The sample was found to contain water to the extent of 5·40 per cent. expelled on ignition.

The iron was determined as in the former experiments.

The percentages work out as follows:—

$$\begin{aligned} Fe_2O_3 &= 53{\cdot}69 \\ P_2O_5 &= 40{\cdot}71 \\ H_2O &= 5{\cdot}40 \\ \hline & \, 99{\cdot}80 \end{aligned}$$

which gives the molecular ratio $P_2O_5 : H_2O : Fe_2O_3$ as ·29 : ·3 : ·33 and the formula $2FePO_4, H_2O$ indicates that the substance as dried on the water bath is a hydrated ferric phosphate somewhat basic from the presence of a small quantity of ferric hydrate. This small quantity of ferric hydrate is to be expected both from

the hydrolysis of the aqueous ferric chloride and more especially the action of free ammonia in the dialyser on the ferric phosphate carrying away ammonium phosphate.

The dialysed solution is tasteless, of a reddish brown colour, and without action on litmus. Solutions of alkaline chlorides and of most other salts cause the solution to gelatinise. With silver nitrate a yellow precipitate consisting for the most part of ferric phosphate but containing a minute quantity of silver phosphate is produced. The electric conductivity of the solution was not accurately determined but was very small, approximating to that of the distilled water used. This is in accordance with its behaviour with such reagents as potassium sulphocyanate which gelatinises the solution but produces no change of colour, thus indicating the absence of free iron ions in the solution. Potassium ferrocyanide also gelatinises the solution, the separated material being slightly darkened but no blue colour is produced. On the other hand sulphuretted hydrogen and also ammonium sulphide give black precipitates.

The rays emitted from radium bromide have no visible effect on the solution.

The soluble colloidal forms of aluminium and of chromium phosphate were prepared in a precisely similar way to the ferric phosphate. The solutions however are weaker, especially the chromium phosphate. Their behaviour with reagents and also with salts generally is similar to that of ferric phosphate.

On the presence of Radio-Active Matter in ordinary substances. By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 15 February 1904.]

In a paper read before the Society in May, 1903, I showed that there is in the water supplied by the Cambridge Water Works, and also in the water from many of the wells in the neighbourhood a radio-active gas, and by the kindness of Professor Dewar I was able to show that this gas could be liquefied. Since the date of that paper I have examined specimens of water from wells in various parts of the country; for these specimens I am indebted to the kindness of Professor Bedson, Middlesborough; C. Joyner, Esq., Icknield Square, Birmingham; The Buddow Brewery Co., Chelmsford; Messrs Salt and Co., Burton-on-Trent; Messrs Worthington, Burton-on-Trent; The Preston Colliery Co., North Shields; A. P. Jenkin, Esq., Redruth; A. Bosville, Esq., Bridlington; E. J. Russell, Esq., Wye, Kent; E. Hinks, Esq., Croydon; F. Everett, Esq., Dover. Eighteen specimens were examined and in 15 of these considerable quantities of the radio-active gas were found; the amount of this gas varied considerably in the different specimens, in two cases it greatly exceeded the amount in the Cambridge water. Measurement of the rate of decay of the radio-activity of the gas expelled from these waters, as well as that of the induced radio-activity due to it, showed that the gas was identical with that contained in the Cambridge water, which again is identical with the emanation from radium. The same gas has recently been found by Professor Bumpstead in the water at Newhaven, Connecticut, U.S.A. There is thus reason for believing that this gas is a constituent of most waters which have not been allowed to stand out of contact with the soil for too long. For as the radio-activity of the gas dies away to half its value in about four days and diminishes in geometrical progression with the time, it naturally ceases to be appreciable in water which has stood out of contact with the ground through which it has percolated for more than a fortnight or so. As the gas is identical with the emanation from radium it seemed desirable to see whether traces of radium could be found in the soil through which the water had percolated. Elster and Geitel had already shown that a radio-active gas is given off by the soil. I have therefore made a series of experiments, which have shown that a radio-active substance, apparently radium, is exceedingly widely distributed, occurring in the most unexpected places.

Wide prevalence of radio-active substances. The method used to detect whether or not a substance was the source of a radio-active emanation, was to reduce the substance under examination to as fine a powder as possible and then shake the powder up in distilled water which did not contain any radio-active gas. The presence of a radio-active gas given off by the substance was tested in two ways. In the first filtered air was bubbled through the mixture of the powdered substance and distilled water, and then passed through a vessel within which a gold-leaf electroscope was enclosed, the gold leaves of the electroscope were observed through a microscope with a micrometer eye-piece. The rate of movement of the leaves when charged gave a measure of the ionization of the gas in the closed vessel. Measurements were made of the rate of fall of the leaves before and after the introduction of the air which had bubbled through the mixture, the existence of a radio-active emanation from the substance mixed with the water showed itself by the increase in the rate of leak of the electroscope after the introduction of this gas.

The other method of testing for the emanation was to boil the water with which the powdered substance was mixed, to introduce the gas expelled from the mixture by boiling into the vessel containing the electroscope and to see whether this increased the rate of leak of the electroscope. By either of these methods an emanation whose life is longer than the time required to transfer the gas extracted from the water into the testing vessel could readily be detected; a very short lived emanation such as that given off by actinium might however escape detection by these methods.

A large number of substances were tested, of which the following gave unmistakable evidence of containing a radio-active emanation. Soil from the garden adjoining the laboratory, Cambridge gault, gravel from a pit at Chesterton, powdered bricks, powdered glass, sea sand from the beach at Whitby, Yorks. (this was exceedingly rich in the emanation), blue lias from Whitby; one specimen of powdered silica contained a very large quantity of the emanation, other specimens little or none; one specimen of wheaten flour contained an appreciable amount of emanation, other specimens none; one or two specimens of flowers of sulphur gave off emanation, while other specimens obtained from a different source did not. Practically all the clays, sands and gravels tested gave off the emanation. The emanation is continually being given off by these substances, for though the water containing the powdered substance can be temporarily deprived of the emanation either by boiling or bubbling, yet if the water with the powder in it is allowed to stand for a few days, the emanation gradually accumulates and can be collected by the same process

as before. The water containing the powder thus differs from tap water, for if the emanation is once expelled from the latter, it does not again accumulate to anything like the original extent, though even in this case there is, as Mr Adams has shown, a very slight recovery of the emanation.

Properties of the Emanation. The emanation given out by all these substances seems to be the same as that given out by radium; thus its radio-activity falls to one-half its value in about four days, while the induced radio-activity due to it, falls to about one-half its value in 40 minutes. In fact the phenomena are just those which would occur if radium were a very widely diffused substance, minute traces of it occurring in almost all specimens of clay, sand or shale. The emanation obtained from these substances is due to an impurity mixed with them, and does not come from the main bulk of the substance; this is shown by the great differences in the amount of emanation given off by specimens of the same substance obtained from different sources; thus for example in the case of precipitated silica, one specimen bought in Cambridge was richer in the emanation than any substance (other than radium or thorium) I have ever examined, while two other specimens, one of which had been for a long time in the Cavendish Laboratory, and the other obtained from the Chemical Laboratory, hardly gave a trace of emanation. The same variation was observed to a smaller extent in all the substances examined.

The question as to whether all substances give off emanations to a slight extent is one to which I have given a good deal of attention, but so far I have not obtained any emanations other than those whose capriciousness indicated that they were due to minute traces of a radio-active impurity. Two methods were used; in the first of these, salts of the metal to be tested (for the most part lead, tin, zinc, bismuth, or copper) were dissolved in distilled water, and the air expelled from the solution by boiling; tests were made soon after the solution was formed and again after it had stood for some weeks, but in neither case was any emanation found in the air expelled from the solution. In the second method in order to get a large surface of the metal exposed to the water I deposited the metal in a colloidal state by making an electric arc pass beneath distilled water between terminals of the metal, the metals tested in this way were platinum, bismuth, and lead; enough metal was deposited to make a litre flask of the solution opaque, the solution was boiled and the air from it tested, but I could not obtain any evidence of an emanation.

I also tried whether a metal could be stimulated to give off emanation by exposure to Röntgen rays; for this purpose I used

a closed lead vessel with a gold-leaf electroscope inside; the vessel was provided with a large aluminium window through which the rays passed before falling on lead, the lead was exposed on several occasions to strong Röntgen rays for four hours consecutively, but after stopping the rays the rate of collapse of the gold leaves was the same as before the exposure, showing that either no emanation had been formed or if formed had not lasted over the interval between the stoppage of the rays and the observation of the electroscope. As the observation of the electroscope occupied several minutes a very evanescent emanation might escape detection by this method. I therefore examined whether any induced radioactivity was produced. For this purpose a long wire was electrically connected with the gold leaves and maintained at a negative potential of 1000 volts during the exposure to the rays, but again after the exposure had ceased the rate of collapse of the gold leaves was the same as before, thus if any emanation is given off by the lead the life of both the emanation and the induced radioactivity due to it must be exceedingly short.

I also tried whether a salt of uranium could be made to give off an emanation by exposure to the radiation from radium. To test this 15 mg. of pure radium bromide were sealed in a thin glass tube and placed in a solution of the uranium salt, but again after an exposure of several days no emanation could be detected in the solution.

Though no evidence has been obtained that the property of giving off an *emanation* is at all general, there is I think a considerable amount of evidence that most if not all bodies are continually emitting radiation which like the Röntgen rays can ionize a gas through which it passed. This evidence is derived from the study of the ionization of a gas shut up in a closed vessel. McClennan and Cooke have observed that the ionization inside a closed vessel is diminished when the outside of the vessel is surrounded by lead, the maximum amount of reduction being about 30 per cent. A layer of lead about 2 cm. thick was sufficient to produce the maximum reduction, no further reduction taking place even when as in Cooke's experiments nearly a ton of lead was placed outside the vessel. This result shows that part of the ionization is due to radiation coming from outside; having regard to the wide distribution of radium and its emanation the most obvious view is that the radiation arises from radium and its emanation diffused through the air and through the soil. A closer study of the effect of screens on the ionization inside a closed vessel will I think show that this explanation is insufficient. Let us consider what would be the intensity of the radiation coming from a large mass of substance containing a radio-active impurity. In consequence of the absorption of the radiation by

the matrix the radiation from an infinitely large mass of material will reach a finite limit, and will equal the radiation given out by the mass of radio-active substance within a distance from the observer proportional to the thickness of a layer of the material which would by its absorption reduce the intensity of the radiation to one-half its initial value. Let us call this thickness the coefficient of absorption of the substance. Thus for example if radium or its emanation were uniformly diffused through the air, the radiation coming to the earth from the sky would be proportional to that emitted by the radio-active substance within a hemisphere whose centre is at the place of observation and whose radius is the coefficient of absorption of air, while the amount coming from the ground would be proportional to the radiation emitted by the radium within a hemisphere whose radius is the coefficient of absorption of the soil. Now if the radium is present as an impurity there is no reason why the quantity per unit volume in air should be to the quantity per unit volume in the ground in the proportion of the coefficient of absorption of air to that of the soil; so that the effect of screening off the radiation from the sky might be expected to be very different from that produced by screening off the radiation from the ground and also the effect of screening off this radiation might be expected to be very different in different countries. Now Cooke found in his experiments in Montreal that the effect of screening off the radiation from the sky was the same as the effect produced by screening off the radiation from the soil, and experiments made in Cambridge have shown that the reduction in the ionization made by screening in Cambridge is practically the same as in Montreal.

Again, the effect of screens made of different materials in cutting off the radiation from outside is not what we should expect if this radiation were one of the forms of radium radiation. The absorption of these has been shown by Strutt to be roughly proportional to the density, so that screens of different materials but of the same mass, should produce approximately the same effect. I have found that this is by no means the case. The experiments were as follows: the vessel in which the ionization was observed was a closed brass cylinder 40 cm. long, 9 cm. in radius and ·75 mm. thick, a gold-leaf electroscope observed by a telescope with a micrometer eye-piece fixed inside the cylinder gave the ionization. This cylinder was placed in the middle of a galvanized iron tank which could be filled with water and sand, gravel, sawdust, or bricks. The dimensions of the tank were 80 cm. × 60 cm. × 60 cm., so that when the tank was full the vessel containing the air to be tested was, except at the top, surrounded by a screen about 30 cm. thick. Lining the tank with lead, about 2 cm. thick, produced a diminution of about 17 per cent. in the

ionization, but if instead of the lead screen the tank was filled with water, I could detect no diminution in the ionization; the sand and sawdust likewise produced no effect, while the bricks produced a slight increase in the ionization. To compare the effects of these screens on radium radiation, I brought near the tank 15 mg. of radium bromide enclosed in a layer of lead about 4 cm. thick and placed it at such a distance from the testing vessel that the ionization was about doubled; in this case the introduction of water in the tank produced a most marked diminution, amounting to about 40 per cent. of the ionization due to the radium.

Patterson found that the ionization inside a closed vessel was largely due to easily absorbed radiation proceeding from the walls of the vessel. This result has recently been confirmed by an entirely different method by the experiments made by Campbell at the Cavendish Laboratory. Wood working at the Cavendish Laboratory has shown that the diminution in the ionization produced in a vessel of given volume by a given screen depends on the material of which the walls of the vessel are made, the absolute diminution in a tin vessel being twice as great as for a zinc one of the same size. These results are both explained if we suppose that the radiation which ionizes the gas in the vessel is partly due to *secondary* radiation proceeding from the walls of the vessel, this secondary radiation being excited by radiation which comes from outside and penetrates the walls of the vessel.

Source of the Radiation causing the ionization of gas in a closed vessel. We have seen that the laws governing the ionization inside a closed vessel are such as to make it improbable that the radiation causing this ionization comes from a radio-active impurity present in the neighbourhood of the vessel. If, however, the radiation is not due to an impurity it must come from the non-radio-active substances around the vessel, for the radiation can not be derived entirely from ultra-mundane sources, for as we have seen, radiation of this kind comes upwards from the earth—it cannot however have penetrated the earth, for it is stopped by a few inches of lead.

The study of the ionization of gases in closed vessels seems to me to point most strongly to the conclusion that all substances emit a kind of radiation which, like Röntgen rays, is able to ionize a gas through which it passes; they also absorb such radiation so that the stream of radiation inside an unlimited extent of radiating substance is not infinite, but is such that the amount of energy crossing in one direction unit area in the substance in unit time is $q/6\lambda$, where q is the energy radiated from unit volume of the substance in unit time, and λ is the coefficient of absorption, *i.e.* when energy of radiation I_0 travels through a distance dx, an amount of energy $I_0\lambda dx$ is absorbed. We see from this that the

intensity of the radiation depends on the ratio of the emission to the absorption of the substance, and as long as this ratio remains the same, the radiation will be unaltered; thus for example, since the radiation and absorption of a gas are each proportional to the number of molecules, the ratio of the two will be independent of the pressure, and thus the stream of radiation through a gas due to its own radiation will be independent of the pressure of the gas. If the absorption for all substances bore the same proportion to the radiation, the flow of ionization would be the same for all substances; in this case there would be no screening, for it is easy to see that the screen would emit as much radiation as it would absorb. We see that for a substance to act as a screen I_0/λ must be less for the screen than for the substance it displaces; if I_0/λ is greater for the screen than for the substance the screen will increase the ionization. For a substance to be efficient as a screen the absorption must be abnormal in comparison with the radiation. Now we have no direct information as to the amount of radiation in different substances, but we know that whereas in general the absorption of substances for Becquerel radiation is proportional to the density, the heavier metals absorb considerably more than this law suggests; thus if radiation is proportional to density the substances which obey Lenard's law would not act as screens, it is only substances such as lead, iron, platinum, &c. which give abnormal absorption that would be effective. This view is confirmed by the experiments described above, which show that lead as a screen is enormously more effective in comparison with such substances as sand or water than its density warrants.

Radiation of the kind we are contemplating involves a continual transformation of the internal energy of the atom into heat; for taking the case of air as an example, the radiation absorbed by the air is not wholly used in producing radiation of the same kind, part of it is spent in ionizing the gas, and is, on the recombination of the ions, converted into heat; the mechanical equivalent of the heat developed by the recombination of the ions in a cubic centimetre of air at atmospheric pressure and temperature is between 1 and 10 ergs per century, and this will be a measure of the amount of internal atomic energy lost by a cubic centimetre of air in the same time.

Just as Röntgen radiation in its passage through matter gives rise to more absorbable secondary radiation, and this again to still more absorbable tertiary, so the radiation streaming through bodies will not be all of one type, the primary will give rise to secondary, the secondary to tertiary, and so on, and it is I think probable that in small vessels part of the ionization in the vessel is due to the secondary and tertiary radiation proceeding from its walls.

The Temperature Effect in the combination of Hydrogen and Chlorine under the influence of Light. By P. V. BEVAN, M.A., Trinity College.

[*Read* 15 February 1904.]

It has been recognized for some time that the accelerating influence of rise of temperature on chemical actions, which are produced by the action of light on the reacting substances, is very considerably less than the corresponding effect in actions where the influence of light is not required for the action to take place. With these facts in view, it occurred to the author that an investigation into the increase in rate of combination of Hydrogen and Chlorine caused by an increase of the temperature at which the action takes place might throw some light on the mechanism of this action. The combination being conditioned by the presence of water vapour in the mixture of the two gases, it was thought that, as the specific influence of temperature on the velocity constants of the action is probably small for small ranges of temperature, the comparatively rapid change of water vapour pressure with rise of temperature might lead to changes in the rate of combination which would enable some insight to be obtained into the part played by the water vapour in the action. The results to be described do not throw any very clear light on this general question, but they are of interest as indicating that the action is not of a simple character, and that the relations of van 't Hoff and others concerning the change of velocity constants with temperature do not hold in this particular action. It will appear later that the assumption of the formation of molecular aggregates, or of some special kind of molecules, as a preliminary phase of the action will enable us to understand the phenomena at any rate qualitatively.

The method adopted for the investigation was that of Bunsen and Roscoe, which has proved the best possible arrangement for studying this particular action. The combination was made to occur in a glass bulb containing a little water in its lower part. As hydrochloric acid is formed it is absorbed by the water in the bulb, and the consequent diminution of volume of the gas is observed by the motion of an index of water in a capillary tube*. In the experiments now described the bulb in which the combination takes place was arranged so as to hang in a water bath which itself stood in another bath which was kept at the required

* For a complete description of the apparatus used and the method of generating the mixture of Hydrogen and Chlorine see a paper by the author in *Phil. Trans.* A. 202, p. 71.

temperature by a thermostat. The inner bath was stirred and kept at a temperature varying not more than 2 or 3 degrees. The inner bath was of glass, and was covered almost entirely with a frame of wood which served to keep all extraneous light from the bulb and also to stop loss of heat. This wooden frame had a hole cut in it through which light could be admitted to the bulb. For a constant source of light an ordinary gas burner was used; in front of this was an iron screen with five holes cut in it at the corners and the centre of a square. The centre hole alone served for transmitting light to the bulb, a lens sending light transmitted through this hole in parallel rays on to the bulb. The other four holes with the lens in position gave images on the side of the frame over the water bath, and by adjusting the size of the flame these four images could be made of equal brightness, thus insuring that the light through the centre hole was from the brightest part of the flame, and that the flame was of practically constant size. The constancy of the light thus obtained was quite satisfactory. The constancy was tested at various times, and rates of action consistent to within four or five per cent. were obtained, individual experiments ranging over

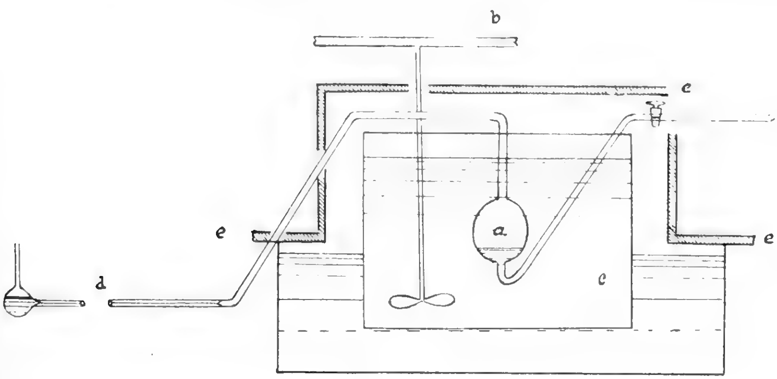


FIG. 1.

a bulb containing gas mixture.
b stirrer.
c inner water bath.
d index tube.
e wooden case.

a period of three months. Fig. 1 shews the arrangement of the bulb and baths.

When the temperature of the bath is changed, changes in the composition of the gas mixture take place, and several days are

required for further saturation of the liquids involved and for obtaining a mixture of gases of the maximum sensitiveness. It was found best, in the long run, to keep a constant small current of gas evolved by electrolysis from concentrated hydrochloric acid running steadily through the apparatus, and not to increase the rate of evolution so as to quickly drive out gas of less than the maximum sensitiveness. Altering the rate of evolution of gas was found to diminish the sensitiveness to light, probably owing to changes of concentration round the electrodes giving rise to small changes of composition of the evolved gas. The individual experiments each required about ten days' preparation, so that a long time was necessary for a series of observations. At a particular temperature the rate of combination was observed daily until a maximum rate was reached, after which no further increase in the rate was observed on further saturation. This rate, therefore, is the rate of combination for the pure mixture at that temperature.

The following table shews the way in which the maximum action is reached. Each observation represents the rate of combination after the period of induction is passed, that is, the steady rate of combination.

Date	Rate of action at temp. 12° C.
Nov. 5	2
,, 7	5
,, 10	22
,, 11	23
,, 12	24

Date	Rate of action at temp. 50° C.
Nov. 21	13
,, 22	21
,, 23	26
,, 25	48

The following table gives the results obtained with the dates on which the final readings were taken for each temperature.

Date	Temperature	Final rate of action
January, 1904	11°	22
Nov. 12, 1903	12·5	24
	30·5	31
Dec. 17 ,,	33·5	32
,, 14 ,,	35·5	33
,, 2 ,,	46	38
Nov. 25 ,,	50	48
January, 1904	60	56

The date of the first and last observations I have not recorded, having omitted them in my rough notes. But the value for the rate of action at 11° taken in January 1904 agreeing nearly with the value at 12·5° in November 1903, gives a sufficient guarantee that the quality of the gas mixture evolved during that time had not altered. The same acid was used all through the experiments for generating the mixture experimented on. The observation at 46° is not a good one, as appears from the curve Fig. 2. It will

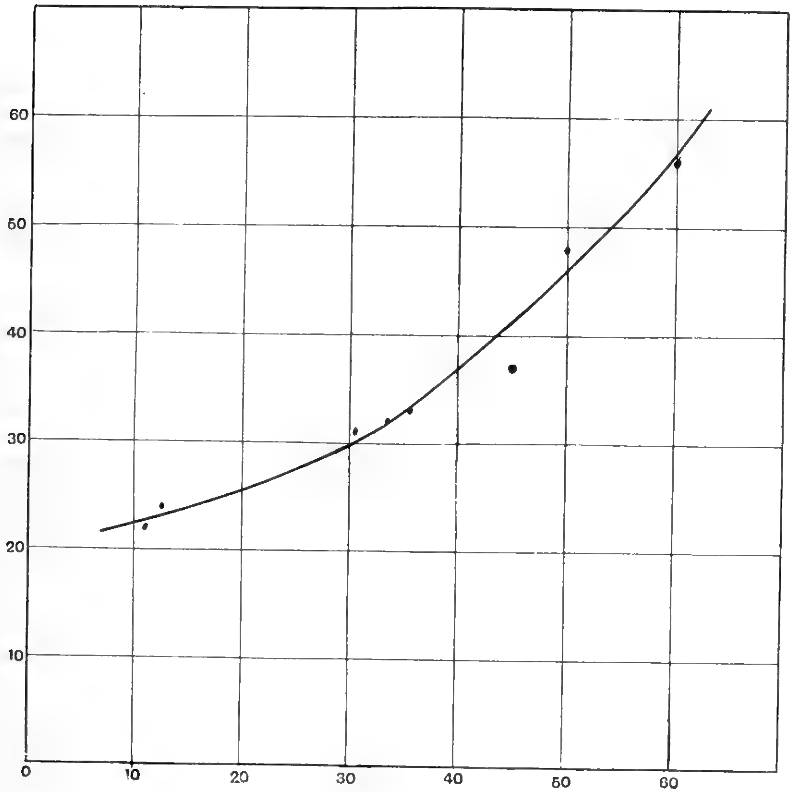


Fig. 2.

be noticed that for this observation the gas mixture had cooled from the former observation at 50° C. It was found afterwards that after cooling it takes a longer time for the state of maximum sensitiveness to be reached than after heating. This is probably owing to the fact noticed by Bunsen and Roscoe, that an excess of Hydrogen in the mixture produces a much greater effect in retarding the action than an equal excess of Chlorine. After

heating, if the mixture is kept at a high temperature, there is for some time a slight excess of Chlorine evolved from the water in the bulb. If the whole apparatus be cooled we get a slight excess of Hydrogen, and apparently for the observation of December 2, sufficient fresh gas had not been passed through the apparatus.

On the ordinary theory of this action, if we attribute the Induction Period to some interfering action at present not understood, the relation between the velocity and temperature should be according to van 't Hoff's theory,

$$\frac{d \log K}{dT} = \frac{A}{T^2} + B,$$

where K is the velocity constant and T the absolute temperature. In this case K can be taken to be the velocity itself as the concentration of the Hydrogen and Chlorine remains constant during the action*. This expression is derived on the assumption that the action is reversible, which is probably not valid in this case. However, the generally accepted theory of actions of this type is that they are reversible and so we should examine the application of the relation to the experimental results.

We obtain on calculation from the smoothed curve the following numbers for B, using the equation in the form

$$B = \frac{T_1^2 \left(\frac{d \log K}{dT}\right)_1 - T_2^2 \left(\frac{d \log K}{dT}\right)_2}{T_1^2 - T_2^2}$$

and taking for T_2 10° C.

T_1	B
20°	·0168
30	·0143
40	·0123
50	·0094
60	·0078

Another assumption on which the relation of van 't Hoff depends is that the heat developed in the action is independent of the temperature within the range considered. This is probably true for this action. But the equation of Kooy

$$\frac{d \log K}{dT} = \frac{A + BT}{T^2},$$

which takes into account variation of the heat developed in the action, gives results which are very little better.

* We here neglect the influence of the water vapour in diminishing the concentration of the Hydrogen and Chlorine, but this makes only a small difference until high temperatures are reached.

We obtain

T	B
20°	·0097
30	·0084
40	·0073
50	·0057
60	·0048

The temperature effect shews, therefore, little agreement with the theory applicable to reversible actions.

The author (*loc. cit.*) has developed to some extent a theory of the action, regarding it as an irreversible action taking place by means of intermediate compounds, or at any rate aggregates of molecules, the first step being the formation of groups involving chlorine and water molecules. On this theory we should expect that the rate of action when this reaches its steady value should be proportional to the product of the concentrations of the chlorine and water molecules. Or if the combination takes place through aggregates of water molecules acting as regions in which the Hydrogen and Chlorine can combine, then the final rate should be proportional to the concentration of the water aggregates.

Consider the consequences of assuming that an action takes place first between the chlorine and water molecules, the rate of action should then at a given temperature be proportional to

$$x(a-x)$$

where x is the concentration of water molecules and $\frac{a}{2}$ the concentration of chlorine molecules, which under the conditions of the experiment remains constant. Now x is proportional to p, the vapour pressure of the water at that temperature, and so the rate of action may be put equal to

$$kp(c-p).$$

c in this expression is the pressure at which the experiment is conducted, and may be put $= 76·0$, measuring pressure in centimetres of mercury. If we calculate the factors $p(c-p)$ we obtain the following table:

Temperature	$p(c-p)$	Temperature	$p(c-p)$
0°	34	60°	912
10	68	70	1225
20	129	80	1440
30	229	90	1235
40	386	95	800
50	615	100	0

And dividing the rates of combination obtained from the smoothed curve we obtain for k the following values:

Temperature	k
10°	·331
20	·194
30	·131
40	·097
50	·076
60	·063

The value of k, therefore, diminishes rapidly with rise of temperature. If the action take place first between chlorine molecules and aggregates of water molecules or with individual water molecules in a special state, as seems probable from the character of the induction period and from the fact that damp chlorine which has been illuminated combines more readily with Hydrogen than unilluminated Chlorine, then k will depend on the number of such aggregates or special molecules. And further, the number of these special molecules or aggregates will depend on the temperature of the water vapour. We have then to consider the equilibrium condition of water vapour with regard to the number of these special systems present. And as such a state must evidently be a state of dynamical equilibrium, we can apply thermodynamical results to find the variation in concentration of such molecules. Considering the case of molecules in a special condition, their concentration, as far as pressure is concerned, is proportional to the concentration of the ordinary water molecules present. This is expressed in our values of $kp(c-p)$ by the factor p.

k, therefore, must be assumed to be of the form KK' where K' is the constant for the water molecules, and K is the velocity constant for the action between the Cl molecules and these special molecules. The constant K' being an equilibrium constant is subject to the thermodynamical relation

$$\frac{d \log K'}{dT} = \frac{A'}{T^2}.$$

And for K the velocity constant we have the equation of van 't Hoff,

$$\frac{d \log K}{dT} = \frac{A''}{T^2} + B,$$

so that

$$\frac{d \log KK'}{dT} = \frac{A}{T^2} + B,$$

or

$$\frac{d \log k}{dT} = \frac{A}{T^2} + B;$$

from the values of k we find for B using the equation in the form

$$B = \left(T_1^2 \frac{d \log k_1}{dT_1} - T_2^2 \frac{d \log k_2}{dT_2}\right) \frac{1}{T_1^2 - T_2^2},$$

T_2	B	$T_1 = 10°$
20°	·410	
30	·392	
40	·372	
50	·391	
60	·403	

And using the mean value of B, ·398, we obtain for A

10°	– 488
20	– 483
30	– 480
40	– 483
50	– 485
60	– 484

The constancy of these values is more accurate than we could have expected. It seems, however, to shew that the assumption of special molecules taking part in the first stages of the action or some equivalent supposition will lead us to an understanding of the mechanism of the action. As we should expect the constant A is negative, which indicates that the higher the temperature the smaller the number of special molecules in the water vapour. B is positive, which to some extent indicates that the action between the chlorine molecules and the special water molecules is one which increases with temperature. Aggregates of water molecules as a step in the action would require higher powers of the vapour pressure in the expression for the velocity, and this would lead to results inconsistent with the experiments, unless in some way aggregates occurred whose number was proportional to the first power of the pressure, that is to the first power of the concentration of the water vapour. At present it is difficult to see how this could occur, so we assume the existence of the special kind of molecules. We have as yet no evidence to throw light on the question as to how these differ from ordinary molecules.

On Convection of Heat. By HAROLD A. WILSON, M.A., D.Sc., Fellow of Trinity College, Cambridge.

[*Read* 15 February 1904.]

The object of the following paper is to determine the distribution of temperature, in certain cases, in media moving between fixed boundaries, the conditions at the boundaries being supposed known. The theory of conduction of heat in bodies at rest relatively to their boundaries has of course been worked out very completely by Fourier, Lord Kelvin and other investigators, but so far as the writer is aware, very little* has been done towards the solution of the corresponding problems in moving media.

The methods employed in the present paper are two, corresponding with the two methods invented by Fourier and Lord Kelvin respectively for the solution of problems on conduction of heat in bodies at rest. The first method is to find a solution of the differential equation which satisfies the given conditions, while the second method is to regard the sources of heat as made up of point sources and to obtain the temperature at any point as the sum of all the temperatures due respectively to each point source.

It will be convenient first to obtain the general differential equation, in Cartesian coordinates, which the distribution of temperature must satisfy in a medium moving in any manner. Let the thermal conductivity of the medium be denoted by k, its density by ρ and its specific heat by s. We shall suppose that k is the same in all directions and independent of the temperature and density of the medium. The specific heat s will be taken to be proportional to ρ so as to include the case where the medium is a gas at constant pressure, but in the problems which will be considered in this paper the variations of temperature will be supposed so small that ρ may be regarded as a constant. If the medium is a viscous fluid, then, when it flows past fixed boundaries, heat will be developed in it, but in what follows we shall suppose that the amount of heat developed in this way is negligible.

Let x, y, z be the coordinates of a point P in the medium and let the velocity components of the medium at P be u, v, w respectively. Consider a small rectangular parallelipiped whose sides are Δx, Δy, Δz situated with P at its centre. The heat

* A case of the flow of a liquid through a tube has been solved by L. Graetz, *Wied. Ann.* XVIII. p. 78.

going into the parallelipiped in unit time through the face $\Delta y\Delta z$ in the direction of the axis x is

$$\Delta y\Delta z\left[s\left(\rho\theta u-\frac{1}{2}\frac{\partial(\rho\theta u)}{\partial x}\Delta x\right)-k\left(\frac{\partial\theta}{\partial x}-\frac{1}{2}\frac{\partial^2\theta}{\partial x^2}\Delta x\right)\right],$$

where θ denotes the temperature at P. The heat going out through the opposite face is

$$\Delta y\Delta z\left[s\left(\rho\theta u+\frac{1}{2}\frac{\partial(\rho\theta u)}{\partial x}\Delta x\right)-k\left(\frac{\partial\theta}{\partial x}+\frac{1}{2}\frac{\partial^2\theta}{\partial x^2}\Delta x\right)\right].$$

The first term in the brackets represents the convection of heat and the second the conduction. We suppose that θ is measured in degrees centigrade, and that the amount of heat in the medium per unit volume is $s\rho\theta$. This amounts to taking the amount of heat in unit volume as zero at 0° C. Since we are only concerned with variations in the amount of heat present in unit volume this latter assumption is legitimate.

The difference between the above two expressions gives the heat gained by the parallelipiped and is

$$\Delta x\Delta y\Delta z\left[k\frac{\partial^2\theta}{\partial x^2}-s\frac{\partial(\rho\theta u)}{\partial x}\right].$$

Similar expressions for the other two pairs of faces may be obtained and the sum of all three is

$$\Delta x\Delta y\Delta z\left[k\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-s\left(\frac{\partial(\rho\theta u)}{\partial x}+\frac{\partial(\rho\theta v)}{\partial y}+\frac{\partial(\rho\theta w)}{\partial z}\right)\right].$$

But the rate at which heat is going into the parallelipiped must be equal to

$$\Delta x\Delta y\Delta z s\frac{\partial(\rho\theta)}{\partial t},$$

so that we obtain the equation

$$k\nabla^2\theta-s\left(\frac{\partial(\rho\theta u)}{\partial x}+\frac{\partial(\rho\theta v)}{\partial y}+\frac{\partial(\rho\theta w)}{\partial z}\right)=s\frac{\partial(\rho\theta)}{\partial t}.$$

We have also the 'equation of continuity'

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0,$$

which reduces the above equation to

$$k\nabla^2\theta-s\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)=\rho s\frac{\partial\theta}{\partial t}.$$

This equation is applicable to solid or liquid media and to gaseous media at constant pressure.

When a steady state has been attained $\frac{\partial\theta}{\partial t}=0$, and the equation becomes

$$k\nabla^2\theta - s\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)=0.$$

All the cases considered in this paper are cases in which a steady state has been attained.

The first problem that will be considered is that of an infinite uniform plane source of heat in a medium moving perpendicularly to it with constant velocity u. Let the initial temperature of the medium be zero, and take the plane yz as the source of heat so that $v=w=0$. Then the differential equation which has to be satisfied is $k\frac{\partial^2\theta}{\partial x^2}-s\rho u\frac{\partial\theta}{\partial x}=0$. This equation on integrating gives $k\frac{\partial\theta}{\partial x}-s\rho u\theta=C$ where C is a constant. Let Q be the amount of heat given out by unit area of the plane source in unit time. Then it is easy to see that when x is negative $C=0$, provided u is positive, and that when x is positive $C=-Q$. When x is negative therefore $\log\theta=\frac{s\rho u}{k}x+A$, where A is a constant, and when x is positive $s\rho u\theta=Q$, so that $A=\log\left(\frac{Q}{s\rho u}\right)$. Hence when x is negative

$$\log\theta=\frac{s\rho ux}{k}+\log\left(\frac{Q}{s\rho u}\right),\ \text{ or }\ \theta=\frac{Q}{s\rho u}e^{\frac{s\rho u}{k}x},$$

and when x is positive $\theta=\frac{Q}{s\rho u}$. If the initial temperature of the medium is θ_0 then these equations become $\theta-\theta_0=\frac{Q}{s\rho u}e^{\frac{s\rho u}{k}x}$ and $\theta-\theta_0=\frac{Q}{s\rho u}$ respectively.

It is easy to see that the time t required to reach the steady state near the plane must be such that $\frac{s\rho u^2}{k}t$ is a large quantity compared with $\frac{Q}{s\rho u}$, or t must be large compared with $\frac{Qk}{s^2\rho^2u^3}$. Thus the time necessary varies inversely as u^3.

The next problem that will be considered is that of a point

source of heat situated at the origin with $v = w = 0$ everywhere, and u constant and positive. In this case the differential equation is $k\nabla^2\theta - s\rho u \frac{\partial\theta}{\partial x} = 0$. Assume as a solution of this $\theta = Ae^{-\alpha x}\phi(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$ and A is a constant. Substituting this in the equation we get, writing ϕ for $\phi(r)$,

$$\nabla^2\phi - 2\alpha\frac{\partial\phi}{\partial x} - \frac{s\rho u}{k}\frac{\partial\phi}{\partial x} + \left(\alpha^2 + \frac{s\rho u}{k}\alpha\right)\phi = 0.$$

Let $\alpha = -\frac{s\rho u}{2k}$, when this equation becomes

$$\nabla^2\phi(r) - \alpha^2\phi(r) = 0.$$

Substituting now the values of $\frac{\partial\phi(r)}{\partial x}$ etc. we get

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} - \alpha^2\phi = 0.$$

The complete solution of this equation is

$$\phi = \frac{Ce^{\alpha r} + Be^{-\alpha r}}{r},$$

where C and B are constants.

When r is infinite $\phi(r)$ must evidently be zero so that $C = 0$, and the solution of the original differential equation becomes

$$\theta = \frac{Ae^{\frac{s\rho u}{2k}(x-r)}}{r}.$$

Hence, when r is very small, $\theta = \frac{A}{r}$ and $\frac{d\theta}{dr} = -\frac{A}{r^2}$.

If Q is the strength of the source we must have, when r is very small,

$$-4\pi r^2 k\frac{d\theta}{dr} = Q,$$

so that $A = \frac{Q}{4\pi k}$, and the solution of the problem is

$$\theta = \frac{Q}{4\pi kr}e^{\frac{s\rho u}{2k}(x-r)},$$

for this expression satisfies the differential equation and all the conditions.

Fig. 1 shows some of the isothermal curves round a point source calculated by means of this equation, taking $Q = 4\pi k$ and $\frac{s\rho u}{2k} = 1$, so that $\theta = \frac{e^{x-r}}{r}$.

Along the axis of x in the positive direction $x = r$ and $\theta = \frac{Q}{4\pi kr}$, consequently the distribution of temperature along the axis of x in the positive direction is the same as if the medium were at rest, however great the velocity of the medium is.

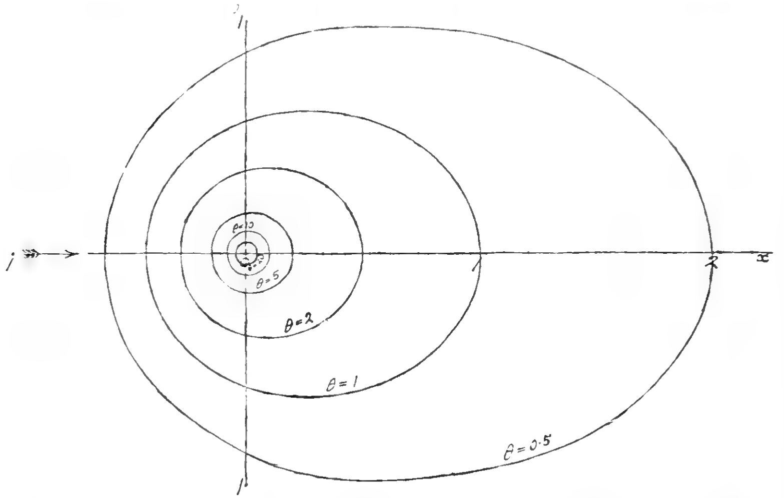


Fig. 1.

The distribution of temperature due to a point source at the origin when $v = w = 0$ can also be obtained by Lord Kelvin's method of point sources. The distribution of temperature due to the instantaneous generation of an amount of heat q at the origin in a medium at rest is given by the equation

$$s\rho\theta = \frac{qe^{-\frac{r^2}{4k't}}}{8(\pi k't)^{\frac{3}{2}}},$$

where $k' = \frac{k}{s\rho}$ and t is the time elapsed since the heat was generated. If the medium is not at rest, but $v = w = 0$ and u is constant and positive, then it is easy to see that the temperature will be given by

$$s\rho\theta = \frac{qe^{-\frac{(x-ut)^2+y^2+z^2}{4k't}}}{8(\pi k't)^{\frac{3}{2}}}.$$

Consequently the temperature due to a continuous point source at the origin must be given by

$$s\rho\theta = \frac{qe^{\frac{ux}{2k'}}}{8(\pi k')^{\frac{3}{2}}} \int_{+\infty}^{0} \frac{e^{-\frac{r^2+u^2t^2}{4k't}}}{(t)^{\frac{3}{2}}} dt.$$

Consequently

$$s\rho\theta = \frac{q}{8(\pi k')^{\frac{3}{2}}} e^{\frac{ux}{2k'}} \frac{2\sqrt{\pi k'}}{r} e^{-\frac{ur}{2k'}}$$

$$= \frac{q}{4\pi k' r} e^{\frac{u}{2k'}(x-r)}$$

as before.

I shall now consider the distribution of temperature due to a straight line source of heat of strength Q per unit length, coinciding with the axis of z when $v = w = 0$ and u has a positive value.

This problem can easily be solved by the first method employed for the point source. This method has however the disadvantage that it is necessary to know the form of solution of the differential equation to assume in order to solve the problem. It is interesting therefore to solve the problem by means of Lord Kelvin's point source method which enables the solution to be obtained by a direct process of integration.

In a medium at rest if a quantity of heat q is instantaneously generated at a point P, then at any subsequent time the rise of temperature due to this is given by

$$s\rho\theta = \frac{qe^{-\frac{r^2}{4k't}}}{8(\pi k't)^{\frac{3}{2}}},$$

where $k' = \dfrac{k}{s\rho}$, r is the distance from P, and t is the time since the heat was generated at P.

The temperature due to an instantaneous line source of heat coinciding with the axis of z of strength q per unit length will therefore be given by

$$s\rho\theta = \int_{-\infty}^{+\infty} \frac{qe^{-\frac{r^2}{4k't}}dz}{8(\pi k't)^{\frac{3}{2}}} = \frac{q}{4\pi k't} e^{-\frac{(x^2+y^2)}{4k't}}.$$

When the medium is in motion so that $v = w = 0$ and u is positive then the distribution of temperature due to an instantaneous line source coinciding with the axis of z will be obtained by

putting $x-ut$ for x in the expression last obtained. So that we have

$$s\rho\theta = \frac{q}{4\pi k't} e^{-\frac{(x-ut)^2+y^2}{4k't}}.$$

If then a continuous line source of strength Q per unit length per second starts at $t=-\infty$ and lasts till $t=0$ the rise of temperature due to it when $t=0$ will be given by

$$s\rho\theta = \int_{+\infty}^{0} \frac{Q}{4\pi k't} e^{-\frac{(x-ut)^2+y^2}{4k't}} dt.$$

Consequently putting $x^2+y^2=r^2$,

$$s\rho\theta = \frac{Q}{4\pi k'} e^{\frac{ux}{2k'}} \int_{+\infty}^{0} \frac{e^{-\left(\frac{v^2t}{4k'}+\frac{r^2}{4k't}\right)}}{t} dt.$$

To solve the problem then it is merely necessary to evaluate the integral

$$\int_{\infty}^{0} \frac{e^{-\left(\frac{v^2t}{4k'}+\frac{r^2}{4k't}\right)}}{t} dt,$$

which we shall denote by y.

Let $$x = \frac{r^2}{4k'} \text{ and } a = \frac{v^2}{4k'},$$

so that $$y = \int_{\infty}^{0} \frac{e^{-\left(at+\frac{x}{t}\right)}}{t} dt, \text{ hence } \frac{dy}{dx} = -\int_{\infty}^{0} \frac{e^{-\left(at+\frac{x}{t}\right)}}{t^2} dt.$$

Now let $$at = \frac{x}{z}, \text{ so that } dt = -\frac{at^2}{x} dz,$$

and $$\frac{dy}{dx} = \frac{a}{x}\int_{0}^{\infty} e^{-\left(\frac{x}{z}+az\right)} dz.$$

Hence $$\frac{d^2y}{dx^2} = -\frac{a}{x}\int_{0}^{\infty} \frac{e^{-\left(\frac{x}{z}+az\right)}}{z} dz - \frac{a}{x^2}\int_{0}^{\infty} e^{-\left(\frac{x}{z}+az\right)} dz.$$

This equation may be written

$$\frac{d^2y}{dx^2} = \frac{a}{x} y - \frac{1}{x}\frac{dy}{dx},$$

or $$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - ay = 0.$$

Let $z = 2\sqrt{ax}$, when this equation becomes

$$\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} - y = 0.$$

The complete solution of this may be written in the form*

$$y = AI_0(z) + BK_0(z),$$

where A and B are constants, $I_0(z) = J_0(iz)$ and

$$K_0(z) = -I_0(z)\left(\log\frac{x}{2} + \gamma\right) + \frac{x^2}{2^2} + \left(1 + \frac{1}{2}\right)\frac{x^4}{2^2 \cdot 4^2} + \left(1 + \frac{1}{2} + \frac{1}{3}\right)\frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots,$$

where $$\gamma = 0{\cdot}57721566\dots.$$

Since when $r = \infty$ the temperature must be zero, we have $A = 0$, so that the solution of the problem takes the form

$$s\rho\theta = \frac{+BQ}{4\pi k'} e^{\frac{ux}{2k'}} K_0\left(\frac{ur}{2k'}\right).$$

It remains therefore to determine B.

When x is very small $K_0(x) = -\log\frac{x}{2} - \gamma$,

so that $$s\rho\theta = \frac{-BQ}{4\pi k}\left(\log\frac{ur}{4k'} + \gamma\right),$$

hence $$s\rho\frac{d\theta}{dr} = \frac{-BQ}{4\pi k' r}.$$

Thus when r is very small the flow of heat from the line source is radial so that we must have

$$-2\pi r k\frac{d\theta}{dr} = Q,$$

from which we see that $B = 2$, so that finally

$$s\rho\theta = \frac{Q}{2\pi k'} e^{\frac{ux}{2k'}} K_0\left(\frac{ur}{2k'}\right),$$

or $$\theta = \frac{Q}{2\pi k} e^{\frac{us\rho}{2k}x} K_0\left(\frac{us\rho r}{2k}\right).$$

* See Whittaker's *Modern Analysis*, p. 307.

When x is large

$$K_0(x) = \sqrt{\frac{\pi}{2x}}\, e^{-x}\left[1 - \frac{1}{8x} + \frac{3^2}{2\,!\,(8x)^2} - \frac{3^2 . 5^2}{3\,!\,(8x)^3} + \dots\right],$$

so that for very large values of r

$$\theta = \frac{Qe^{\frac{us\rho}{2k}(x-r)}}{2\sqrt{us\rho\pi kr}}.$$

For very small values of r

$$\theta = \frac{-Q}{2\pi k}\left(\log \frac{us\rho}{4k} r + 0{\cdot}577 \dots\right).$$

If instead of a line source of heat we have a very fine wire heated by an electric current, then, provided the wire is so thin that it produces no appreciable disturbance of the moving medium outside the region where

$$\theta = \frac{-Q}{2\pi k}\left(\log \frac{us\rho}{4k} r + 0{\cdot}577\right),$$

it is easy to see that the solution obtained for the line source will be very approximately true for the distribution of temperature due to the wire except very near the wire.

The heat given out by such a wire can be calculated from its electrical resistance and the current flowing through it so that Q can be determined. If then the rise of temperature at some point in a known position with respect to the wire were determined, the equation

$$\theta = \frac{Q}{2\pi k}\, e^{\frac{us\rho}{2k}x}\, K_0\left(\frac{urs\rho}{2k}\right)$$

would enable k for the moving medium to be estimated. The quantity $\frac{us\rho}{2k}$ can easily be made very large, so that the approximate formula

$$\theta = \frac{Q}{2\sqrt{\pi s\rho kur}}\, e^{\frac{us\rho}{2k}(x-r)}$$

could be used to determine k in this way. Thus, if two fine platinum wires were stretched parallel to each other, one along the axis of z and the other at $x=a$, $y=0$, and the rise of temperature of the second, due to passing a current through the first, were determined, k could be got by means of the equation

$$\theta = \frac{Q}{2\sqrt{\pi s\rho kua}},$$

since $x-r=0$, when $y=0$ and x is positive.

An expression for the temperature due to a line source can also be got from that for a point source. For a point source at the origin we have

$$\theta = \frac{Q}{4\pi k r}\, e^{\frac{spu}{2k}(x-r)} .$$

Integrating this along the z-axis, we get, for the temperature due to a line source,

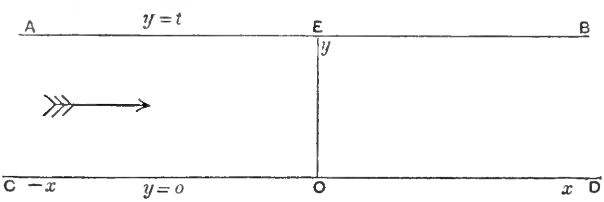
$$\theta = \frac{Q}{4\pi k}\int_{-\infty}^{+\infty} \frac{e^{\frac{spu}{2k}(x-r)}}{r}\, dz = \frac{Q}{4\pi k}\, e^{\frac{spu}{2k}x} \int_{-\infty}^{+\infty} \frac{e^{-\frac{spu}{2k}r}}{r}\, dz.$$

Comparing this with the result previously obtained, we conclude that

$$K_0\left(\frac{ursp}{2k}\right) = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{e^{-\frac{spu}{2k}\sqrt{r^2+z^2}}\, dz}{\sqrt{r^2+z^2}} ,$$

where r is now $\sqrt{x^2+y^2}$ instead of $\sqrt{x^2+y^2+z^2}$.

We shall now consider the distribution of temperature in a slab bounded by two infinite parallel planes between which it slides with a uniform velocity.



Let AB and CD be the two planes, and take the origin of coordinates at O, so that the plane CD is given by $y=0$ and the plane AB by $y=t$, where t is the thickness of the slab. We suppose then that $v=w=0$ and u is positive. The differential equation is therefore

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} - \frac{spu}{k}\frac{\partial\theta}{\partial x} = 0.$$

Assume a solution of the form

$$\theta = \sum_{n=1}^{n=\infty} C_n \sin\left(\frac{n\pi y}{t}\right),$$

where C_n is a function of x only.

Substituting this in the differential equation, we get

$$\frac{d^2C_n}{dx^2} - \frac{s\rho u}{k}\frac{dC_n}{dx} - \left(\frac{n\pi}{t}\right)^2 C_n = 0,$$

a solution of which is $C_n = A_n e^{a_n x}$, where

$$a_n = \frac{s\rho u}{2k} \pm \sqrt{\left(\frac{\pi n}{t}\right)^2 + \left(\frac{s\rho u}{2k}\right)^2}.$$

Consequently, putting $p = \dfrac{s\rho u}{2k}$,

$$\theta = \sum_{n=1}^{n=\infty} A_n e^{\left(p \pm \sqrt{\left(\frac{\pi n}{t}\right)^2 + p^2}\right)x} \sin\left(\frac{n\pi y}{t}\right)$$

is a solution of the differential equation. When $x = 0$ this becomes

$$\theta = \sum_{n=1}^{n=\infty} A_n \sin\left(\frac{n\pi y}{t}\right),$$

which can be made to represent any distribution of temperature or sources of heat, on the part of the plane yz between AB and CD, which is a function of y only.

If $y = 0$ or $y = t$, then $\theta = 0$, so that the solution

$$\theta = \sum_{n=1}^{n=\infty} A_n e^{\left(p \pm \sqrt{\left(\frac{\pi n}{t}\right)^2 + p^2}\right)} \sin\left(\frac{n\pi y}{t}\right)$$

enables the temperature distribution due to any distribution of temperature or of heat supply along Oy, when the planes AB and CD are kept at temperature zero everywhere, to be determined.

If a constant term is added to the above expression for θ it is still a solution of the differential equation. Now of the two values of α it is clear that $p + \sqrt{\left(\frac{\pi n}{t}\right)^2 + p^2}$ will be the appropriate value when x is negative and $p - \sqrt{\left(\frac{\pi n}{t}\right)^2 + p^2}$ when x is positive. The solution with a constant term added will therefore represent the temperature due to any distribution of temperature or heat supply along Oy, when the planes AB and CD are maintained at constant temperatures, θ_1 where x is positive and θ_2 where x is negative.

If $u = 0$, this solution becomes

$$\theta = \sum_{n=1}^{n=\infty} A_n e^{\pm\frac{\pi n x}{t}} \sin\left(\frac{n\pi y}{t}\right),$$

which is Fourier's solution for the corresponding problem in a slab at rest.

As an example, suppose that where x is negative the planes AB and CD are maintained at the temperature unity and where x is positive at the temperature zero, and let there be no sources of heat along Oy. For simplicity also let $t = \pi$.

Then, when x is negative, the solution is

$$\theta = 1 + \sum_{n=1}^{n=\infty} A_n e^{(p+\sqrt{n^2+p^2})x} \sin ny,$$

and, when x is positive, it is

$$\theta = \sum_{n=1}^{n=\infty} B_n e^{(p-\sqrt{n^2+p^2})x} \sin ny.$$

It remains therefore to determine the A's and the B's. When $x = 0$, we have

$$1 + \Sigma A_n \sin ny = \Sigma B_n \sin ny.$$

Hence
$$\Sigma (B_n - A_n) \sin ny = 1.$$

Consequently $B_n - A_n = \dfrac{4}{n\pi}$ when n is odd and zero when n is even.

Also, when $x = 0$,

$$\frac{d\theta}{dx} = \Sigma A_n (p + \sqrt{n^2 + p^2}) \sin ny = \Sigma B_n (p - \sqrt{n^2 + p^2}) \sin ny.$$

This equation is satisfied if

$$A_n = \frac{p - \sqrt{n^2 + p^2}}{p + \sqrt{n^2 + p^2}} B_n.$$

Therefore, when n is odd,

$$B_n \left(1 - \frac{p - \sqrt{n^2 + p^2}}{p + \sqrt{n^2 + p^2}}\right) = \frac{4}{n\pi}.$$

Hence
$$B_n = \frac{2}{n\pi}\left(1 + \frac{p}{\sqrt{n^2 + p^2}}\right),$$

and, when n is even, $B_n = 0$. Also, when n is odd,

$$A_n = B_n - \frac{4}{n\pi} = \frac{2}{n\pi}\left(\frac{p}{\sqrt{n^2 + p^2}} - 1\right),$$

and, when n is even, $A_n = 0$.

The expression for θ is therefore, when x is negative,

$$\theta = 1 + \frac{2}{\pi}\left(\frac{p}{\sqrt{1+p^2}} - 1\right) e^{(p+\sqrt{1+p^2})x} \sin y$$
$$+ \frac{2}{3\pi}\left(\frac{p}{\sqrt{9+p^2}} - 1\right) e^{(p+\sqrt{9+p^2})x} \sin 3y + \ldots,$$

and, when x is positive,

$$\theta = \frac{2}{\pi}\left(1 + \frac{p}{\sqrt{1+p^2}}\right) e^{(p-\sqrt{1+p^2})x} \sin y$$
$$+ \frac{2}{3\pi}\left(1 + \frac{p}{\sqrt{9+p^2}}\right) e^{(p-\sqrt{9+p^2})x} \sin 3y + \ldots.$$

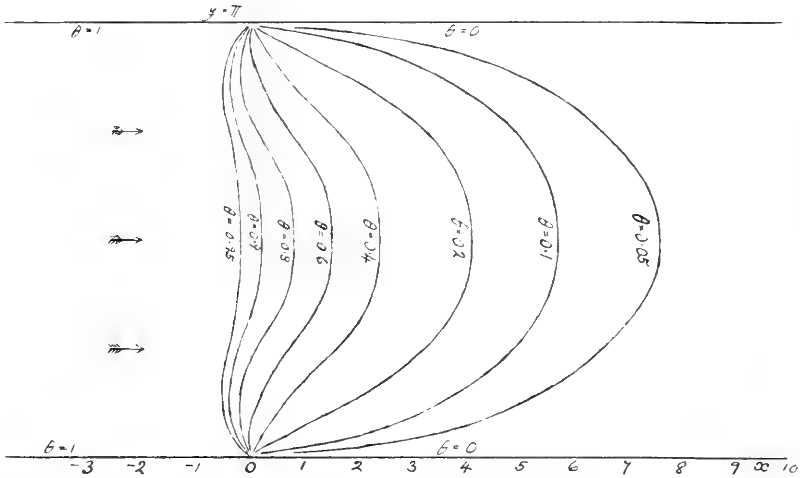


Fig. 2.

Putting $x = 0$, and adding the two series, we get

$$\theta = \frac{1}{2} + \frac{2}{\pi}\frac{p}{\sqrt{1+p^2}} \sin y + \frac{2}{3\pi}\frac{p}{\sqrt{9+p^2}} \sin 3y + \ldots,$$

which, when $p = 0$, reduces to $\theta = \frac{1}{2}$.

Fig. 2 shows some of the isothermal lines for the case $p = 1$.

Next, suppose the planes AB and CD are everywhere kept at temperature zero, and the plane Oy contains an uniform source of heat of strength Q per unit area from $y = \frac{\pi}{4}$ to $y = \frac{3\pi}{4}$, the distance between the planes being π as before.

Then, when x is negative, we have

$$\theta = \sum_{n=1}^{n=\infty} A_n e^{(p+\sqrt{n^2+p^2})x} \sin ny,$$

and, when x is positive,

$$\theta = \sum_{n=1}^{n=\infty} B_n e^{(p-\sqrt{n^2+p^2})x} \sin ny.$$

Putting $x=0$, we see that $A_n = B_n$.

Also when $x=0$,

$$-k\frac{d\theta}{dx_{(x=+)}} + k\frac{d\theta}{dx_{(x=-)}} = Q,$$

when y is between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$, and $=0$ when y is between 0 and $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ and π.

But $$k\left(\frac{d\theta}{dx_{(x=-)}} - \frac{d\theta}{dx_{(x=+)}}\right)$$

$$= k\left\{\Sigma A_n(p+\sqrt{n^2+p^2})\sin ny - \Sigma A_n(p-\sqrt{n^2+p^2})\sin ny\right\}$$

$$= k\Sigma A_n 2\sqrt{n^2+p^2}\sin ny.$$

To determine the A's therefore we require the coefficients in the series

$$\phi(x) = a_1 \sin x + a_2 \sin 2x + \dots,$$

where
$$\begin{aligned} \phi(x) &= 0 \text{ from } 0 \text{ to } \frac{\pi}{4}, \\ &= Q \quad ,, \quad \frac{\pi}{4} \text{ to } \frac{3\pi}{4}, \\ &= 0 \quad ,, \quad \frac{3\pi}{4} \text{ to } \pi. \end{aligned}$$

We have $a_n = \frac{2}{\pi}\int_0^\pi \phi(x)\sin nx\,dx = \frac{2Q}{\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin nx\,dx$

$$= 0, \text{ when } n \text{ is even},$$

$$= \frac{2\sqrt{2}}{\pi n}, \text{ when } n \text{ is } 1, 7, 9, 15, 17, \text{ etc.},$$

$$= -\frac{2\sqrt{2}}{\pi n}, \text{ when } n \text{ is } 3, 5, 11, 13, \text{ etc.}$$

Consequently,

$$\phi(x) = \frac{2\sqrt{2}Q}{\pi}\left[\sin x - \frac{\sin 3x}{3} - \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \frac{\sin 9x}{9} - \frac{\sin 11x}{11} - \frac{\sin 13x}{13} + \dots\right].$$

Hence

$$A_1 = \frac{\sqrt{2}Q}{\pi k\sqrt{1+p^2}}, \qquad A_3 = \frac{-\sqrt{2}Q}{3\pi k\sqrt{9+p^2}},$$

$$A_5 = \frac{-\sqrt{2}Q}{5\pi k\sqrt{25+p^2}}, \qquad A_7 = \frac{\sqrt{2}Q}{7\pi k\sqrt{49+p^2}},$$

etc.

The expression for θ is therefore, when x is positive,

$$\theta = \frac{\sqrt{2}Q}{\pi k}\left(\frac{e^{(p-\sqrt{1+p^2})x}}{\sqrt{1+p^2}}\sin y - \frac{e^{(p+\sqrt{9+p^2})x}}{\sqrt{9+p^2}}\sin 3y - \dots\right),$$

and, when x is negative,

$$\theta = \frac{\sqrt{2}Q}{\pi k}\left(\frac{e^{(p+\sqrt{1+p^2})x}}{\sqrt{1+p^2}}\sin y - \frac{e^{(p+\sqrt{9+p^2})x}}{\sqrt{9+p^2}}\sin 3y - \dots\right).$$

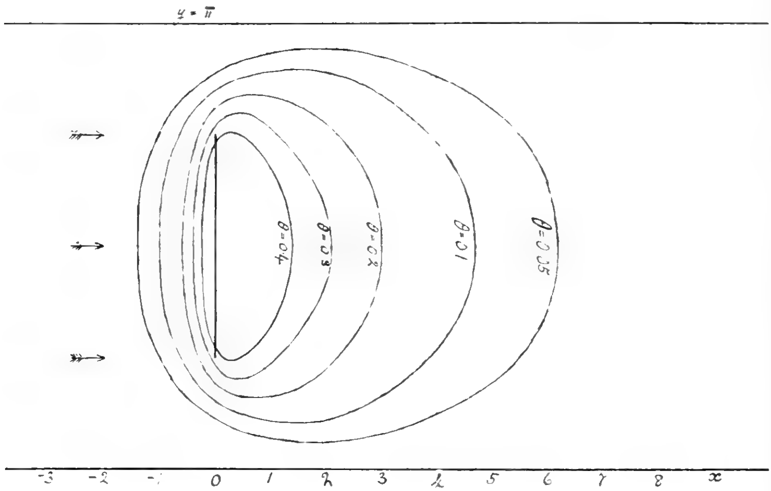


Fig. 3.

Fig. 3 shows some of the isothermal lines given by these formulae, taking $\dfrac{\sqrt{2}Q}{\pi k} = 1$ and $p = 1$.

If the condition given is that no heat shall pass through the planes AB and CD, then we must have $\frac{d\theta}{dy} = 0$ when $y = 0$ and when $y = t$. In this case we take a solution of the form

$$\theta = \sum_{n=0}^{n=\infty} A_n e^{a_n x} \cos\left(\frac{n\pi y}{t}\right),$$

so that

$$\frac{d\theta}{dy} = \sum_{n=0}^{n=\infty} - A_n \frac{n\pi}{t} e^{a_n x} \sin\left(\frac{n\pi y}{t}\right),$$

which is zero when $y = 0$ or t, and so satisfies the given condition.

We shall now consider a problem to which the solution just obtained is not applicable. Let the temperature of the planes AB and CD be given by $\theta = ax + b$, where a and b are constants.

In this case it is clear that $\frac{d^2\theta}{dx^2} = 0$ and $\frac{d\theta}{dx} = a$ everywhere in the slab. The differential equation therefore becomes

$$\frac{d^2\theta}{dy^2} - 2pa = 0,$$

where $p = \frac{s\rho u}{2k}$ as before. The solution of this is

$$\theta = pay^2 + Cay + D,$$

where C and D are constants. When $y = 0$,

$$\theta = D = ax + b,$$

and when $y = t$, $\theta = pat^2 + Cat + ax + b = ax + b$,

hence $C = -pt$. The solution of the problem is therefore

$$\theta = pay^2 - paty + ax + b.$$

In this case therefore the isothermal lines are parabolas with their axes at $y = \frac{t}{2}$. The temperature at the centre of the slab is given by

$$\theta = \frac{pat^2}{4} - \frac{pat^2}{2} + ax + b,$$

or

$$\theta = ax + b - \frac{pat^2}{4},$$

so that the centre of the slab is cooler than the sides by $\frac{pat^2}{4}$.

If instead of a slab we take a circular cylinder sliding in a jacket the differential equation is in cylindrical coordinates, supposing everything symmetrical about the axis of the cylinder,

$$\frac{d^2\theta}{dx^2} + \frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - 2p\frac{d\theta}{dx} = 0.$$

A solution of this of the form $\theta = \Sigma A_n e^{\alpha_n x} J_0(\mu_n r)$ can be obtained where $\alpha_n = p \pm \sqrt{\mu_n^2 + p^2}$ and the μ's are constants. If the radius of the cylinder is a, then, when for example the surface of the cylinder is maintained at temperature zero, the μ's have to be the roots of the equation $J_0(\mu a) = 0$. This solution therefore enables problems for the cylinder to be solved similar to those considered in the case of the slab. It is not proposed to consider any problems on this case in detail.

A problem of some interest from the practical point of view is the distribution of temperature in a liquid flowing through a pipe; the temperature of the pipe being supposed known.

We shall take the axis of the tube as coinciding with the axis of x, and denote the radius of the tube by a. Let the mean velocity of the liquid through the tube be V, then the velocity at any point in the tube is given by the equation

$$u = \frac{2V}{a^2}(a^2 - r^2).$$

Taking everything to be symmetrical about the axis of the tube, the appropriate form of the differential equation is therefore

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} + \frac{d^2\theta}{dx^2} - \frac{2s\rho V}{ka^2}(a^2 - r^2)\frac{d\theta}{dx} = 0.$$

A solution of this equation for the case when the temperature of the pipe is represented by the equation $\theta = cx + d$ can be easily obtained. In this case it is evident that $\frac{d\theta}{dx} = c$ everywhere inside the pipe, so that the differential equation becomes

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \frac{2s\rho V}{ka^2}(a^2 - r^2)c = 0.$$

A solution of this is

$$\theta = A + cpr^2 - \frac{cpr^4}{4a^2},$$

where $p = \frac{s\rho V}{2k}$ and A is a constant.

When $r = a$, we have

$$\theta = cx + d = A + cpa^2 - \frac{cpa^2}{4},$$

so that $$A = cx + d - \tfrac{3}{4}cpa^2,$$

hence the solution of the problem is

$$\theta = cx + d - \tfrac{3}{4}cpa^2 + cpr^2 - \frac{cpr^4}{4a^2}.$$

When $r = 0$, $$\theta = cx + d - \tfrac{3}{4}cpa^2,$$

so that the temperature at the axis of the pipe is lower than that at the circumference by $\frac{3}{4}cpa^2$.

The mean temperature $\bar{\theta}$ of the liquid flowing past the plane $x = q$, is given by the equation

$$\begin{aligned}\bar{\theta} &= \frac{2}{Va^2}\int_0^a u\theta r\,dr \\ &= \frac{4}{a^4}\int_0^a r(a^2 - r^2)\left(cq + d - \frac{3cpa^2}{4} + cpr^2 - \frac{cpr^4}{4a^2}\right)dr \\ &= cq + d - \tfrac{11}{24}cpa^2.\end{aligned}$$

Thus the mean temperature of the liquid flowing through the pipe is less than the temperature of the pipe by $\frac{11}{24}cpa^2$.

As an example, suppose a pipe 2 cms. in diameter with water flowing through it with a mean velocity of 100 cms. per second. Suppose also that the temperature of the pipe falls off in the direction of the flow of the water one degree in 1000 cms. Then we have $c = 10^{-3}$, $a = 1$, and

$$p = \frac{s\rho V}{2k} = \frac{100}{2 \times 0{\cdot}00136} = 3{\cdot}7 \times 10^4.$$

Hence the mean temperature of the water in the pipe at any plane is $\frac{11}{24} \times 10^{-3} \times 3{\cdot}7 \times 10^4 = 17^\circ$ above the temperature of the pipe at that plane. In practice if the velocity of the liquid exceeds a certain value the flow becomes turbulent, in which case the mean temperature of the liquid would be more nearly equal to that of the pipe.

In conclusion I wish to say that my best thanks are due to Prof. J. J. Thomson for several valuable suggestions with regard to this paper.

The decomposition of Hydrogen Dioxide under the influence of Radium Bromide. By H. J. H. FENTON, M.A., F.R.S., Christ's College.

[*Read* 29 February 1904.]

These experiments were originally undertaken with the object of investigating the conditions which affect the stability of solutions of hydrogen dioxide at the ordinary temperature in absence of light.

Pure hydrogen dioxide, prepared by distillation, is now obtainable from Merck of approximately '100 volume' strength—corresponding to about 30 per cent. H_2O_2, or nearly 10 times 'molar' strength. This solution, which is sent out in bottles coated internally with paraffin in order to avoid contact with glass, keeps remarkably well in the dark; a specimen which was obtained about a year previously gave on analysis 28·9 per cent. H_2O_2, or 8·5 times molar strength. But solutions prepared by diluting this specimen with ordinary distilled water exhibited very varying degrees of stability, and in some cases appeared to be practically exhausted after keeping for a few weeks in the dark.

This instability cannot be attributed entirely to the direct action of glass, since it appears also to a considerable degree in paraffin-coated vessels, nor can it be ascribed to dilution which, on theoretical grounds, would be expected to have an opposite effect. Solutions prepared of very varying degrees of dilution both with ordinary and ammonia-free distilled water, were contained either in glass or paraffin-coated vessels and kept in the dark under the same conditions. Analysis of corresponding volumes of these solutions at definite periods shewed only very slight differences compared with the total amount of change, and the acceleration of the change must consequently be largely due to the catalytic influence of some impurity, other than ammonia, already present in the distilled water employed.

During the progress of this enquiry Mr Hardy and Dr Anderson most kindly supplied the loan of 50 milligrams of radium bromide, and it was decided to extend the observations in order to ascertain whether the rate of decomposition is accelerated or otherwise influenced by the rays from this substance.

Bredig and his colleagues in the course of an elaborate investigation have shewn that the rate of decomposition of hydrogen dioxide under the catalytic influence of colloidal platinum and

in presence of various electrolytes, follows the law for a reaction of the first order

$$K = \frac{1}{t} \log \frac{a}{a - x}.$$

Solutions of the pure dioxide, however, under the influence of the platinum solution alone do not decompose strictly in accordance with this law, the value of the constant increasing as the concentration of the dioxide diminishes.

It will be seen from the following results that solutions of the pure dioxide, contained in paraffin-coated vessels, shew a similar increase in the value of the constant as the decomposition proceeds, whether under the influence of radium rays or not. In glass vessels the rate, which is at first irregular, appears to become approximately constant after a few days.

In both series, whether the solution is contained in glass or in paraffin-coated vessels, the value of the constant for the decomposition under the influence of radium appears, in each experiment, to be approximately double of that in the parallel blank experiment.

The observations were carried out in the following manner:—

The pure hydrogen dioxide was diluted to a convenient strength with ordinary distilled water, and the solution accurately titrated with potassium permanganate. This solution was then divided into two similar flasks and kept in separate dark cupboards in the same room, one of the flasks being placed directly over, and almost in contact with, the specimen of radium bromide. Equal portions (5 c.c.) were then withdrawn at stated intervals and the strength determined as before by permanganate (0·0030 $KMnO_4$ per c.c.).

In the following table the concentrations $(a - x)$ are expressed in terms of the permanganate solution used and the time intervals (t) in hours. Ordinary logarithms are used, so that

$$K' = K \times 0{\cdot}4343.$$

I. Paraffin-coated flasks. 0·225 molar H_2O_2.

	Radium		Blank	
t	$(a-x)$	K'	$(a-x)$	K'
0	23·7			
24	22·0	·0013	22·7	·0008
48	19·6	·0017	21·5	·0009
72	17·3	·0019	20·1	·0010
96	14·9	·0021	18·6	·0011
120	12·0	·0024	16·9	·0012
144	9·5	·0027	15·0	·0013
168	7·5	·0030	12·7	·0016

II. Glass flasks. 0·179 molar H_2O_2.

	Radium		Blank	
t	$(a-x)$	K'	$(a-x)$	K'
0	18·9			
24	17·2	·0017	17·8	·0011
48	16·7	·0011	17·7	·0006
72	15·1	·0013	16·8	·0007
96	12·9	·0017	15·4	·0009
120	10·7	·0020	14·1	·0010
144	8·8	·0023	13·1	·0011
168	7·8	·0023	11·8	·0012
192	7·0	·0022	10·8	·0012

Further experiments are being made with the object of ascertaining whether the oxidation of organic substances, such as tartaric acid, either alone or in conjunction with iron, is influenced by radium bromide.

Exhibition of Oribatid Mites taken in the neighbourhood of Cambridge. By C. WARBURTON, M.A., Christ's College, and N. D. F. PEARCE, M.A., Trinity College.

[*Read* 29 February 1904.]

The Oribatidae or Beetle-mites form a compact group of the Order ACARI. They may be recognised by their strongly chitinised integument, which gives to many of them a remarkably beetle-like appearance, and more especially by the possession of a pair of organs on the cephalothorax. These consist of crater-like pits, from the centre of which proceed modified hairs of very varying shape, and of unknown function. The pits are known as *pseudostigmata*, and the hairs as pseudostigmatic organs.

These mites are small creatures, averaging perhaps one-fiftieth of an inch in length. They are vegetable feeders, living on such materials as moss, lichen and rotten wood, and they are most readily taken by shaking out over white paper moss or other substance which has been collected in bags from likely localities. Notwithstanding their small size their movements, as they slowly crawl about, cause them to be easily detected by a practised eye.

The Acari have received extremely little attention in this country, and this is the first attempt to investigate the local fauna of any acarine group. It is, perhaps, worth noting that in four winter months specimens of forty-seven out of the hundred known British species have been taken in the neighbourhood, and that every one of the fifteen British genera is locally represented.

ARACHNIDA.

Order ACARI.

Fam. **Oribatidae.**

Sub-fam. **Pterogasterinae.**

Gen. *Pelops*, C. L. Koch.
P. acromios, Hermann.
P. fuliginosus, C. L. Koch.
P. phaeonotus, C. L. Koch.

Gen. *Oribata*, Latreille.
O. globula, Nicolet.
O. lapidaria, Lucas.
O. orbicularis, C. L. Koch.
O. piriformis, Nicolet.
O. setosa, C. L. Koch.
O. ovalis, C. L. Koch.
O. avenifera, Michael.
O. quadricornuta, Michael.
O. lucasi, Nicolet.
O. fusigera, Michael.
O. parmeliae, Michael.

Sub-fam. **Apterogasterinae.**

Gen. *Scutovertex*, Michael.
S. maculatus, Michael.
S. sculptus, Michael.

Gen. *Cepheus*, C. L. Koch.
C. tegeocranus, Hermann.
C. latus, Nicolet.

Gen. *Tegeocranus*, Nicolet.
T. velatus, Michael.

Gen. *Carabodes*, C. L. Koch.
C. coriaceus, C. L. Koch.

Gen. *Serrarius*, Michael.
S. microcephalus, Nicolet.

Gen. *Liacarus*, Michael.
L. ovatus, C. L. Koch.

Gen. *Notaspis*, Hermann.
N. similis, Michael.
N. exilis, Nicolet.
N. tibialis, Nicolet.
N. oblonga, C. L. Koch.
N. lucorum, C. L. Koch.
N. splendens, C. L. Koch.

Gen. *Damaeus*, C. L. Koch.
D. verticillipes, Nicolet.
D. clavipes, Hermann.
D. geniculatus, Linnaeus.
D. auritus, C. L. Koch.

Gen. *Hermannia*, Nicolet.
- *H. bistriata*, Nicolet.
- *H. scabra*, C. L. Koch.
- *H. arrecta*, Nicolet.

Gen. *Neliodes*, Berlese.
- *N. theleproctus.*

Gen. *Cymbaeremaeus*, Berlese.
- *C. cymba*, Nicolet.

Gen. *Nothrus*, C. L. Koch.
- *N. targionii*, Berlese.
- *N. sylvestris*, Nicolet.
- *N. palustris*, C. L. Koch.
- *N. horridus*, Hermann.
- *N. biverrucatus*, C. L. Koch.
- *N. bicarinatus*, C. L. Koch.

Gen. *Hypocthonius*, C. L. Koch.
- *H. rufulus*, C. L. Koch.

Gen. *Hoploderma*, Michael.
- *H. magnum*, Nicolet.
- *H. dasypus*, Dugès.

Some observations upon the Determination of Sex in Plants.

By R. P. Gregory, B.A., St John's College. (Communicated by Mr W. Bateson.)

[*Read* 29 February 1904.]

The work which forms the subject of this paper has been carried out to investigate the suggestion of Castle ('00), that sex may be an inherited character exhibiting the Mendelian phenomena of segregation and dominance.

The way was prepared for Castle's suggestion by the work of Cuénot ('99), and Strasburger ('00, *b*), who brought together evidence which showed conclusively that sex cannot be determined through the environment during the early life of the embryo.

Strasburger, recognizing that the soma contains both sexes, one in an "active," the other in a "latent" state[1], discussed the possibility of the segregation of the sex characters in the formation of the gametes; but rejected the hypothesis, mainly on cytological grounds, but also on account of the difficulty of so modifying it as to meet the case of those races in which one sex is normally in numerical preponderance over the other[2].

Bateson ('94) and Bateson and Saunders ('02) draw a comparison between the phenomena of sex and those of variation. This comparison is carried further by Castle ('03), who suggests that "Sex is an attribute of every gamete, whether egg or spermatozoon, and is not subject to control through environment. It is inherited in accordance either with Mendel's law of heredity, or with the principle of mosaic inheritance."

(1) *The gametophyte in Plants.*

Accepting for the present the evidence which tends to show that the chromosomes, or groups of chromosomes, contained in the germ cells are the factors concerned in the transmission of inherited characters[3], and further the hypothesis suggested by Cannon ('02), and independently by Sutton ('03), that the segregation of characters in accordance with the Mendelian hypothesis

[1] See Darwin, *Animals and Plants under Domestication*, London; John Murray. 2nd Edit., Vol. II. pp. 25—31.

[2] Certain factors affecting the numerical proportions of the sexes in man are discussed by Punnett ("On nutrition and sex-determination in man," *Proc. Camb. Phil. Soc.* Vol. XII. Pt IV.).

[3] See Wilson ('02) for an account of work previous to that date, and the more recent work of McClung ('00, '02 *a*, '02 *b*), Boveri ('02, '04), and Sutton ('02).

is provided for in the reduction division[1], we may consider certain corollaries which might be expected to follow from Castle's hypothesis.

In plants the problem of sex-determination is rendered more complex by the existence of two separate generations, the gametophyte and sporophyte, which alternate with one another in the life history of the plant.

In the higher Cryptogams and in Phanerogams the gametophyte is dioecious, whatever state prevails as regards the distribution of megasporangia and microsporangia upon the same or different individuals of the asexual generation; but in many of the Archegoniatae the gametophyte is hermaphrodite, bearing both antheridia and archegonia upon the same thallus.

Campbell ('95; p. 335) was not certain whether a reduction in the number of chromosomes takes place at the formation of the spores in Ferns; and while Strasburger ('94) previously gave the number of chromosomes in *Osmunda* as 24 and 12, in the sporophyte and gametophyte respectively, Guignard ('99) has observed that sometimes at least 22 chromosomes were to be counted in the spores. Strasburger ('00, *a*; p. 77) confirms this, at the same time stating that the number, 12, which he had given as that present in the gametophyte, was correct for those examined. Further, while Calkins ('97) described some Leptosporangiate Ferns as possessing about 128 and 64 chromosomes, Stevens ('98) gave the numbers as 64 and 32.

A possible extension of Castle's hypothesis in this connexion was suggested to me by Mr Bateson. It might be supposed that no chromatin reduction takes place at the formation of spores which are destined to give rise to hermaphrodite prothallia; that it only occurs at some later mitosis previous to the formation of the gametes. That is, the nuclei of unisexual prothallia would possess the reduced number of chromosomes; those of prothallia actually or potentially hermaphrodite would contain the same number as occur in the nuclei of the sporophyte[2]. It seemed possible that in this hypothesis lay the explanation of the con-

[1] With, however, the modification that the chromosomes corresponding to the dominant and recessive of each pair of allelomorphic characters will be separated in such a way that any gamete may contain any combination of characters, so long as allelomorphs are separated, and not, as Cannon allows it to be supposed, so that *all* the chromosomes derived from the father pass to one daughter nucleus, and *all* those from the mother to the other daughter nucleus. As Sutton has pointed out the latter hypothesis does not provide for the observed combinations which occur in the offspring of hybrids between two races differing from one another in respect of more than one character.

[2] Farmer ('94) had found reduction of the chromosomes at spore formation in certain *Hepaticae*. The want of uniformity in the nomenclature of this group left room for doubt as to whether the gametophyte was hermaphrodite in any of the species described by that author.

flicting opinions expressed as to the number of chromosomes in Ferns.

The work was carried out by means of a study of spore-formation, and by the culture and examination of large numbers of prothallia.

The results of the cytological examination show

(1) The number of chromosomes is halved at the division of the spore-mother-cells to form the spores.

(2) A true reduction (qualitative) division takes place in preparation for the formation of spores[1].

Large numbers of prothallia were examined to see whether any might be found to be definitely of one sex (♂) only; material grown from spores taken from several plants of *Osmunda regalis* was used chiefly, as being the most likely to yield positive results. The prothallia in two pots were mapped for purposes of identification, and were examined from time to time for a period of nearly two months (May to July; the spores were sown in February, March and April). Of 142 prothallia so examined 128 ultimately bore archegonia as well as antheridia. The remaining 14 were removed, as supposed males, for cytological examination; but on cutting five or six no case was found of complete absence of archegonia, which in sections can be recognized in a very early stage. Several hundreds of prothallia grown in pots were examined in the hope of obtaining purely male material for cytological work; but this was so rare that, among those preserved, no undoubted male was obtained.

A statement has appeared in a text book[2] to the effect that in *Osmunda* the spores formed in any one sporangium tend to be all alike, those of certain sporangia giving rise to an excess of males, those of other sporangia to hermaphrodites exclusively. To test this 24 pure sowings were made, great care being taken to ensure that each one contained spores from one sporangium only. The sporangia used were taken from two plants and from different positions (which were recorded) on the fertile spike. No evidence in support of the view was obtained. Some of the cultures became invaded by a fungus, which in four cases obtained a firm hold; these prothallia were very small, appeared unhealthy and, with one exception, bore no archegonia. In four other cultures fungus appeared to a smaller extent; 51 prothallia were obtained of which 15 bore archegonia as well as antheridia, the remainder bearing antheridia only.

In the cultures which were clean nearly all the prothallia bore archegonia; these too did not entirely escape the attacks of the fungus, so that the experiments were abandoned, while a few

[1] *Roy. Soc. Proc.* Vol. LXXIII., p. 88.

[2] Bennet and Murray, *Cryptogamic Botany*, Longmans, 1889, p. 66.

prothallia still remained without archegonia[1]. In spite of this, there can be no doubt, I think, that *all* the prothallia are potentially hermaphrodite, though, if nutrition is bad, the production of archegonia remains in abeyance. No case of a purely female prothallium was observed.

Experiments were also made to test the possibility of self-fertilization of a hermaphrodite prothallium. For if segregation of the sex-characters takes place at the reduction division in hermaphrodite races, all the gametes produced by any one prothallium must carry the same sexual tendency[2], and two possibilities remain:

(1) The homozygous union is impossible; hence self-fertilization will not take place.

(2) If such a union is fertile, the resulting sporophyte will be pure in respect of one sex, the other being completely absent. The spores produced by such a plant will then give rise to prothallia all of one and the same sex only.

The second alternative is extremely improbable, and may, I think, be at once dismissed[3].

The prothallia used were examined microscopically, and only those were chosen which had at that time no trace of sexual organs. These were washed in distilled water, were planted separately in small pots containing sterilized earth. Of 34 thus treated, 32 died without offspring; but in two cases a healthy sporophyte was produced. The large mortality is not remarkable in view of the necessity for transplanting the prothallia, many of which never obtained a firm hold in the new surroundings. Mr Lynch is able to confirm this observation, for he has found that self-fertilization takes place in *Ceratopteris*.

From the cytological examination of the prothallia it seems probable that the number of chromosomes may vary in different nuclei of the same prothallium, in some mitoses only 18 or 12 can be made out. At the division which leads to the formation of the spermatozoids 22 chromosomes are present, corresponding with the number of about 44 present in the nuclei of the sporophyte, and with the number 22 which is characteristic of the spore.

The above results justify the conclusion that the occasional

[1] Although this experiment has yielded only a negative result, should a tendency be observed for the spores of certain sporangia to produce an excess of *males*, it would hardly be surprising; and might indeed be expected if, by reason of the position of the sporangia or through other causes, the supply of nutriment were reduced. (See p. 435.)

[2] Compare the results obtained by Correns ('00) and de Vries ('00) upon the double fertilization in the embryo sac of *Zea Mais*. The two gametes of each pollen grain, as also the ovum and polar nucleus of each embryo sac, are shown to carry the same somatic character.

[3] Experiments are however being made to test this possibility.

occurrence of a male prothallium can only be ascribed to chance conditions of nutriment and age (see also Heim ('96)). There can be no doubt that in this Fern all the prothallia possess the power to form female organs[1]; but, since the production of a sporophyte must be some tax upon the gametophyte, only those of vigorous and full growth bear the female organs. With this we may compare the results which Klebs obtained in experiments upon the conditions of reproduction. Although able to delay the production of the female organs, he was not able, by artificial means, completely to eliminate them.

(2) *The Sex of the Gametophyte.*

From the experiments upon Ferns recorded above may be drawn two conclusions:

(1) that, although spore-formation is preceded by a qualitative reduction division, no segregation of the sex-characters has taken place at that division;

(2) that the form of gamete (antherozoid or ovum) produced by any prothallium is subject within limits to influence through the conditions of nutrition; that is, that *all* the prothallia have the power of producing either form of gamete, although under some conditions the production of one form may remain in abeyance.

I would suggest that these conclusions are equally true of all those higher plants in which the individual sporophyte bears both megasporangia and microsporangia; and that the second conclusion is true also of those plants in which the sporophyte is dioecious.

The heterosporous hermaphrodite sporophyte is to be looked upon as derived directly from the isosporous form, remaining like the latter undifferentiated as regards the separation of the sexes; as a consequence no segregation of the sex-characters which appear in the sporophyte takes place; and all the gametes which are produced carry the hermaphrodite character, which reappears in the next sporophyte generation. The production of one form of gamete to the exclusion of the other, by the gametophyte of such plants, is then a phenomenon distinct from the separation of the sexes in the sporophyte generation.

The spores derived from one spore-mother-cell, in heterosporous

[1] The prothallia of *Onoclea* are said to be strictly dioecious (Campbell ('95); p. 310). A cytological examination of spore-formation in that Fern was therefore made to see whether any features might present themselves which might indicate a cytological basis for the separation of the sexes. Only a negative result was forthcoming, the divisions leading to spore-formation having an exact similarity with those of the other Leptosporangiate Ferns examined.

plants, are alike in being either all microspores or all megaspores. The production of a particular kind of gamete by the gametophyte of heterosporous plants is therefore not determined by the qualitative reduction division which precedes the formation of the spores. On the contrary it seems probable that the conditions of nutrition determine the fate of the four cells formed at the reduction division—whether they become microspores or megaspores.

In the Ferns and Equisetaceae the tendency of the gametophyte toward dioecism is not fixed, and, as has been shown, may be influenced by environment. (See also Strasburger, '00, *b.*) The hermaphrodite prothallia invariably form a cushion, consisting of a mass of cells which, compared with the cells of the lateral wings, are rich in protoplasmic contents; the result being that only the most highly nourished prothallia assume the function of rearing the next generation. Goebel ('02) has shown that the antheridia and archegonia are exactly homologous structures, the difference lying in (1) the cessation of cell-division in the female organ as compared with the male; (2) the unequal division of the primary cell into dissimilar daughter cells, of which the larger alone is fertile, and, as a consequence, (3) the arrest of certain cells in the female organ whose homologues in the male proceed to the formation of antherozoids. This change in the course of the cell divisions is then brought about by a state of high nutrition.

In the heterosporous plants which still retain the primitive independent condition of the gametophyte (*e.g. Selaginella*) the megasporangia in the early stages are absolutely indistinguishable from the microsporangia. The difference, which only appears at a later stage, is that *all* the spore-mother-cells of the microsporangium divide to form microspores, while in the megasporangium only one of the mother-cells undergoes division, the others remaining abortive, with the result that the four megaspores obtain a vast quantity of nutriment as compared with the microspores, and the protoplasm is abundant and rich in reserve food matter.

In a gametophyte of this type it is to be expected therefore that the development of the sexual organs will be characterized by unequal divisions and by the arrest of one (the smaller) daughter cell; that is, that the gametophyte will bear only female organs.

To put it in another way, it seems as though the spores of certain sporangia receive *from the parent* this tendency towards the formation of antherozoids; those of other sporangia receive the corresponding tendency towards the formation of ova; while the sex-character carried by the gametes so formed, which is manifested in the next sporophyte generation, is independent of this tendency and perhaps may be determined at the reduction division.

In the Phanerogams definite sporophylls, arranged in definite positions, are specialized for the production either of megaspores or of microspores; and in the megasporangium not only does one mother cell alone divide, but only one of the resulting megaspores is permitted normally to come to maturity.

In the dioecious Phanerogams under normal conditions the tendency prevails for each individual to form one kind of spore only; that is, one sex only is present in the active state. The opposite sex is present only in a latent condition, but may be brought into active manifestation by a suitable change in the metabolism of the organism (such as is brought about by the destruction of the anthers of *Melandrium* (= *Lychnis*) *dioica* by *Ustilago violacea*)[1].

Thus sex as it appears in the sporophyte of dioecious plants is so far like a somatic character that the separation of the sexes depends upon a discontinuous variation; such that each of the varieties is characterized by the production normally of one form of sporophyll only.

The analogy of the heterotype division in plants with that in animals leads us to homologize the entire gametophyte of the former with the gamete of the latter[2]. There is evidence that in animals the ova in some cases bear the male character only, and that the spermatozoa carry the female character[3], and it therefore appears probable that the same may be true also of the gametophyte generation in those plants whose sporophytes are dioecious. A segregation of the sex-characters takes place, the sex-character transmitted through any gametophyte to the next sporophyte generation being entirely independent of the particular form of gamete which that gametophyte produces.

A parallel case of the manifestation in the gametophyte of a character opposite to that which is transmitted through it to the sporophyte of the next generation is afforded by the hybrid *Epilobium angustifolium.* The common form of this plant, which has grey-green pollen, crossed by its white variety—whose pollen grains are white—yields a hybrid whose pollen grains are all uniformly grey-green (Correns ('00), p. 232, footnote). If that

[1] Giard ('89); Strasburger ('00, *b*).

[2] In the Thallophyta the reduction division takes place at the formation of the gametes (Farmer and Williams (96)) as in animals.

[3] This occurs in parthenogenetic races; *e.g. Rotifers, Daphnidae, Polyphemidae, Artemia, Cypris, Hymenoptera, Aphides*, and in facultative parthenogenesis in *Liparis* and some Insects. In those cases where parthenogenesis is constant it is known that eggs which give rise to males have undergone a reduction division, those which give rise parthenogenetically to females have formed only one polar body. See Weismann and Ischikawa ('88), Blochmann ('88) and Petrunkewitsch ('03). (For discussion of these phenomena, see especially Castle ('03).) The parthenogenetic *Chara crinata* is possibly a case where the ovum bore the female character, and the male character is now eliminated.

character which is transmitted through the pollen grain were also manifest in it, both white and grey-green pollen grains should have been produced in equal numbers. In fact, as Strasburger remarks, Correns' result proves that the sporophyte determines the colour-character exhibited by the dependent gametophyte.

Sex in the gametophyte, being then a character derived solely from the parent sporophyte and independent of the sex-character carried by the germ cells, is probably not comparable to those somatic characters of the gametophyte which are predetermined and not influenced by the environment, that is, to the somatic characters of, for example, the gametophyte of the Bryophyta,—characters which are sufficiently definite to admit of their use in classification[1].

The differentiation between the megasporangia and microsporangia of heterosporous plants is exactly analogous with that between archegonia and antheridia (p. 435). Such analogy suggests that the determining factor is the same in each case—an effect of nutrition.

In dioecious plants the development either of megasporangia or of microsporangia would be the expression of a difference between the nutrition of the two forms. But this difference is not merely quantitative. Male and female may be looked upon as physiological varieties characterized by distinct physiological processes, in the male tending toward subdivision and the formation of antherozoids, in the female toward arrest of division, the storage of food and the nutrition of the embryo. These processes retain their distinctive features in spite of changes in the environment.

Castle regards hermaphrodite organisms as "sex-mosaics," using the analogy of spotted mice as a mosaic form combining two colour-characters. The use of this term, particularly when an analogy with mosaic mice is drawn, rather leads one to suppose that hermaphrodite races are to be regarded as having arisen through the formation of mosaic gametes by individuals who owed their origin to dioecious parents.

But among the Pteridophyta no instance is known of a plant which, while retaining the primitive independent condition of the gametophyte, also possesses the dioecious character in the sporophyte. So that it seems more logical to regard the primitive hermaphrodite as undifferentiated in respect of the separation of the sexes, which only appears at a higher stage of evolution, and is of the nature of a discontinuous variation (Bateson ('94)).

[1] The characters of the gametophyte in Pteridophyta are so entirely under the influence of environment that any attempt at classification based upon them at once breaks down (Bauke ('78), Heim ('96)).

Summary.

The production of one or other of the two forms of gamete by the gametophyte of the Vascular Cryptogams and higher plants is to be distinguished from the separation of the sexes observed in animals, in the sporophyte of higher plants, and probably from that in the *Thallophyta*.

The form of gamete produced by the gametophyte may be influenced, within limits, during the independent existence of the gametophyte by the environment (Homosporous Plants); or it may be determined under the influence of the parent sporophyte (Heterosporous Plants).

The form of gamete produced by any gametophyte is independent of the sex-character transmitted through that gametophyte to the sporophyte of the next generation.

All plants which have a primitive hermaphrodite sporophyte produce gametes undifferentiated in respect of the segregation of the sex-characters.

I should like to take this opportunity of acknowledging my indebtedness to Mr Bateson for the many suggestions and advice he has given me throughout this work.

BIBLIOGRAPHY.

Bateson, 1894. "Materials for the Study of Variation." London, Macmillan, 1894.

Bateson & Saunders, 1902. "Experimental Studies in the Physiology of Heredity." *Reports to the Evolution Committee of Royal Society*, I.

Bauke, 1878. "Entwickelungsgeschichte des Prothalliums bei den Cyatheaceae verglichen mit derselben bei anderen Farnkräutern." *Pringsheim's Jahrb.* X., p. 29.

— 1878. "Beiträge zur Keimungsgeschichte der Schizeaceae." *Pringsheim's Jahrb.* XI., p. 603.

Blochmann, 1888. "Ueber die Richtungskörper bei unbefruchtet sich entwickelnden Insekteneiern." *Verh. naturh. med. Ver. Heidelberg*, N. F., IV. 1892, p. 239.

— 1889. "Über die Zahl der Richtungskörper bei befruchteten und unbefruchteten Bieneneiern." *Morph. Jahrb.*, Bd. XV. H. 1, p. 85.

Boveri, 1902. "Mehrpolige Mitose als Mittel zur Analyse des Zellkerns." *Verh. d. Phys. Med. Ges. zu Würzburg*, XXXV.

— 1904. "Ergebnisse über die Konstitution der chromatischen Substanz des Zellkerns." Jena, Gustav Fischer, 1904.

CALKINS, 1897. "Chromatin-reduction and Tetrad-formation in Pteridophytes." *Bull. Torrey Bot. Club*, XXIV., p. 101.

CAMPBELL, 1895. "Mosses and Ferns." London, Macmillan, 1895.

CASTLE, 1903. "The Heredity of Sex." *Bull. Museum Comp. Zool. Harvard*, Vol. XL., No. 4, p. 189.

CORRENS, 1900. "Gregor Mendel's 'Versuche über Pflanzen Hybriden' und die Bestätigung ihrer Ergebnisse durch die neuesten Untersuchungen." *Bot. Zeit.* Jahrg. 58, Abt. II., p. 230.

CUÉNOT, 1899. "Sur la détermination du sexe chez les animaux." *Bull. Sci. France et Belg.* T. XXXII. p. 462.

FARMER, 1894. "On *Pallavicinia decipiens*." *Annals of Bot.* VIII., p. 35.

— 1895. "On spore-formation and nuclear division in the Hepaticae." *Annals of Bot.* IX., p. 469.

FARMER & WILLIAMS, 1896. "On Fertilization in *Fucus*." *Annals of Bot.* X., p. 479.

GIARD, 1889. "Sur la castration parasitaire du *Lychnis dioica* L. par l'*Ustilago antherarum* Fr." *Bull. Sci. France et Belg.* T. XX., p. 153.

GUIGNARD, 1898. "La développement du pollen et la réduction chromatique dans le *Najas Major*." *Arch. d'Anat. Microscop.* T. II., p. 455.

HEIM, 1896. "Untersuchungen über Farnprothallien." *Flora*, 1896, p. 329.

KLEBS, 1896. "Die Bedingungen der Fortpflanzung." Jena, Gustav Fischer, 1896.

— 1899. "Zur Physiologie der Fortpflanzung einiger Pilze." *Pringsheim's Jahrb.*, Bd. XXXIII. 1899, pp. 562—567, 582—588.

PETRUNKEWITSCH, 1901. "Die Richtungskörper und ihr Schicksal im befruchteten und unbefruchteten Bienenei." *Zool. Jahrb., Abt. f. Anat. u. Ontog.*, Bd. XIV., H. 4, p. 573.

— 1903. "Das Schicksal der Richtungskörper im Drohnenei." *Zool. Jahrb.*, Bd. XVII., H. 3, p. 481.

PUNNETT, 1903. "On Nutrition and Sex-determination in Man." *Proc. Camb. Phil. Soc.*, Vol. XII., Pt. IV.

STEVENS, 1898. "Ueber Chromosomentheilung bei der Sporenbildung der Farne." *Ber. d. Deutsch. bot. Gesellsch.*, Bd. XVI., p. 261.

STRASBURGER, 1894. "Ueber periodische Reduction der Chromosomenzahl." *Biol. Central.*, Bd. XIV., pp. 817, 849.

— 1900 *a*. "Ueber Reduktionstheilung, Centrosomen und Cilienbildner." *Histologische Beiträge*, Heft VI. Jena, Gustav Fischer, 1900.

— 1900 *b*. "Versuche mit diöcischen Pflanzen mit Rücksicht auf Geschlechtsverteilung." *Biol. Centralbl.*, Bd. XX., pp. 657—665, 689—698, 721—731, 753—785.

SUTTON, 1903. "The chromosomes in Heredity." *Biol. Bull.* 1903, Vol. IV., No. 5.

De Vries, 1900. "Sur la fécondation hybride de l'endosperme chez le Maïs." *Rev. Gén. de Bot.* T. xii., p. 129.

Weismann & Ischikawa, 1888. "Weitere Untersuchungen zum Zahlengesetz der Richtungskörper." *Zool. Jahrb., Abth. f. Anat. u. Ontog.* Bd. iii., H. 3, p. 575.

Wilson, 1902. "The Cell in Development and Inheritance." 2nd Edit., London & New York, Macmillan, 1902.

On Variation in the Number and Arrangement of the Male Genital Apertures and on the Proportion of the Sexes in the Norway Lobster (Nephrops norvegicus). By D. C. McIntosh, M.A., Aberdeen. (Communicated by Francis H. A. Marshall, M.A., Christ's College.)

[*Received* 29 February 1904.]

While engaged last autumn in an investigation on the fauna of the Clyde area, I had my attention drawn to the variation in the number and arrangement of the genital apertures, and to the relative proportion of the sexes, in the Norway Lobster.

Mr F. H. A. Marshall has already considered these two points and has published[1] the results of an examination of 1068 specimens. All of these he procured from the Firth of Forth, where they are much more numerous than on the West coast. The main purpose of the present communication is to consider the bearing of the facts ascertained by me upon Mr Marshall's results.

In my investigation a total of 656 Norway Lobsters was examined, the specimens being dredged from the Firth of Clyde during the first fortnight of September, 1903.

1. *As regards the male genital apertures.*

Mr Marshall found ten forms of variation in the arrangement of the male genital apertures (in addition to the normal arrangement of one opening at the base of each of the last pair of walking legs). The percentage of abnormal specimens was as high as 12·2. On the other hand the percentage of abnormality in the Clyde specimens only amounted to 2·49. Table I, which should be compared with that given by Mr Marshall, indicates the different arrangements of spermatic apertures which I found among the Clyde lobsters. The letters *r* and *l* denote the right and left sides.

TABLE I.

Pair of Walking Legs	A		B		C		D		E		F	
	r.	*l.*	*r.*	*l.*	*r.*	*l.*	*r.*	*l.*	*r.*	*l.*	*r.*	*l.*
Second...										●	●	
Third.....			●	●		●	●			●		
Fourth...	●	●	●	●	●	●	●	●	●	●	●	

[1] *P. Z. S.* London, 1902.

Table II shows the proportion of normal and abnormal males among the specimens obtained from both Forth and Clyde. From this it is seen that in each place the B, C, D, and E arrangements occur. Although the F arrangement was not found in the Forth lobsters, the corresponding arrangement of an extra aperture on the left (instead of the right) second walking leg was noted. Further, there were found in the Forth one specimen with a total of seven apertures, two with six, and five with five apertures, while none of the Clyde specimens had more than four.

TABLE II.

Arrangement	Number for Firth of Forth	Number for Firth of Clyde	Percentage for Firth of Forth	Percentage for Firth of Clyde
A	878	311	87·8	97·49
B	40	2	4·0	·63
C	40	2	4·0	·63
D	31	2	3·1	·63
E	1	1	0·1	·31
Other arrangements	10	1	1·0	·31
Total......	1000	319	100	100

Mr Marshall, however, examined, although he did not include in the figures given above, 80 specimens which were caught off the Isle of Man in the month of August, 1901. He was specially struck with the fact that he found among these only one abnormal male. "The number," as he remarks, "is, of course, not large enough for any definite conclusions to be based upon it; but in view of the fact that I never obtained even smaller batches from the Forth area without finding a much higher percentage of abnormality, the presence of only a single abnormal specimen among the Isle of Man lobsters may point to the percentage being related to the locality." The same remark may be held to apply, though with less force, to the case now under consideration, but the results so far obtained point distinctly to the conclusion that the very marked difference is a local one. Mr Marshall informs me that in the specimens (procured from the same source) which he has examined since the publication of his paper, the high percentage of abnormality therein recorded is almost fully maintained. It ought to be added that the normal apertures were present in every specimen examined, and that while the variation in the male apertures was as stated, no female was found with additional apertures.

2. *As to the proportion of the sexes.*

The difference in the relative proportion of male and female lobsters in the Forth and Clyde areas is even more noteworthy. For example, Mr Marshall found only 68 females (that is, 6·4 per cent.) in a total of 1068 specimens, and he naturally comments on the relative scarcity of these. Among the Clyde lobsters, on the other hand, no fewer than 51·4 per cent. of the specimens examined were females. The difficulty of explaining the difference is rather increased than otherwise by Mr Marshall's statement that of the 80 specimens caught off the Isle of Man only two were females. It would thus appear that the Isle of Man specimens resembled those from the Firth of Clyde in regard to the number with abnormal genital apertures, while markedly differing in the proportion of the sexes.

The Clyde specimens were absolutely unselected, for account was taken of every specimen which the dredge brought up. It is difficult to discover to what extent, if any, Mr Marshall's specimens were selected, since they were obtained from the fish-market at Newhaven (at various times during the spring, summer and autumn of 1901). In view of the fact that females are on the average considerably smaller than males, it is not improbable that a proportion of the females, being smaller, was not landed. For example, I found that the average length of the 656 specimens examined was 110 mm. The average length of the females was 102·4 mm. and the average length of the males was 118·2 mm. This same fact is illustrated in another way in Table III, where are summarized the results of measurement of the length of body of males and females. (The measurement was made from tip of rostrum to end of telson, and it was noted that the length of carapace measured along the dorsal line is almost exactly half that of the abdomen.) An examination of this Table seems to justify the conclusion that on the whole the male is larger than the female.

TABLE III.

Maximum length in mm.	Number of Females	Number of Males
Up to 82	115	70
83—122	129	92
123—162	92	116
163 upwards	1	41
Total.........	337	319

It is well known that Norway lobsters are gregarious, but my observations do not indicate that they are found in shoals according to sex.

In all investigations bearing on this point it is absolutely essential for each investigator to superintend the operation of dredging. It seems useless to draw conclusions as to relative frequency of sex from specimens provided by fishermen in the ordinary course of their calling, for it may very well be that they allow the smaller individuals to escape. It is well known that they do not expose for sale (since to do so is contrary to law) the female crab (*Cancer pagurus*) carrying ova, and it may be that they also allow the female Norway lobster, if carrying ova, to escape. In this connection I found that 10·2 per cent. of the Clyde female lobsters were laden with ova, and one could not examine many of these without remarking on the comparatively small size of mature females.

From the facts recorded in Table III it may probably be concluded that for the Clyde area (1) the proportions of the sexes in *Nephrops norvegicus* are approximately equal, and (2) the size of fully grown males is considerably greater than that of fully grown females. This would be in agreement with Punnett's conclusions[1] as to the relative frequency and relative size of the sexes in *Carcinus maenas.* Table III also illustrates the possibility of error in drawing any conclusion as to the relative frequency of males and females from an examination of specimens at any particular stage of growth.

It may be added as an interesting fact that the Norway lobsters of the Clyde were invariably found associated with those of their own size. One haul, for example, would contain only large specimens, another only medium sized, while a third contained only specimens of small size. I hope to refer more fully to this and other points in a future paper.

In conclusion, my hearty thanks are due to the officials of the West of Scotland Marine Biological Association (and especially to Mr Gray, lately Curator of the Marine Station at Millport) for their kindness in allowing me the use of the S. Y. "Mermaid" to procure the specimens used in this investigation, and to Professor J. Arthur Thomson for permission to occupy the "Aberdeen University Table."

[1] *Proc. Camb. Phil. Soc.* Vol. XII. 1903.

The Boiling Points of Homologous Compounds. By HUGH RAMAGE, B.A., St John's College.

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The relations between certain properties of elements and their atomic masses have been described by the writer in *Roy. Soc. Proc.*, Vol. LXX. pp. 1 and 303, 1902. These relations were discovered by a study of diagrams drawn with the physical constants as abscissae and the atomic masses, or the squares of these, as ordinates. This graphical method does not appear to have been employed by other workers except in the simplest form for showing that the properties are *periodic* functions of the atomic masses.

The method has been extended to other properties of the elements and also to those of compounds. Amongst the latter the boiling points offer the most complete data and in addition furnish constants which are amongst the least affected by other factors such, for instance, as association of the molecules, and moreover they are, especially amongst organic compounds, the most accurately determined of the physical constants. A study of the boiling points of many substances has been made and some facts relating to the boiling points of homologous compounds will be dealt with in this paper.

Professor James Walker has given a general formula, $T = aM^b$, for the boiling points of homologous compounds[1] in which T is the boiling point in absolute degrees, M is the molecular weight and a and b are constants for each series. He applied it to several series and amongst them to the normal paraffins from C_7H_{16} to $C_{16}H_{34}$. He found that it did not apply to the alkyl bromides and iodides, the normal alkylamines, and the normal ketones and aldehydes. He also stated that no formula of this type could possibly be applied to the alcohols.

It was observed when drawing the curve through the boiling points of the paraffins that it would if produced below methane pass very near to the position of hydrogen: it seemed at first when using the older figure for methane (113°) that hydrogen might be regarded as a member of the series. It was however found by drawing a diagram with the logarithms instead of the numbers that hydrogen did not fall on the curve but it fell near it. The change from H_2 to CH_4 is not very different from what it would be were hydrogen the first member of the series.

In referring to the ten hydrocarbons to which his formula applies Walker remarked:—"In this particular case the formula is unusually simple, for the constant b becomes equal to 0·5, the curve of boiling points being therefore a true parabola." He does

[1] *Trans. Chem. Soc.* LXV. pp. 193, 725 (1894).

not discuss the point further. The curve to Walker's formula was drawn on the diagram and in studying the two curves it appeared probable that the simple formula only applied exactly to the CH_2 chain linkage and that the influence of the terminal hydrogen atoms was either a constant in the higher members of the series or it was so small that it might be disregarded. The influence of these terminal atoms increases as the chain shortens and the atoms approach each other until in methane the diagram indicates we practically have the effect of two CH_2 groups. This work has suggested a modification of the formula which applies to the series from methane to hexadecane.

The new formula is $T = a\{M(1-2^{-n})\}^{\frac{1}{2}}$, where n is the number of carbon atoms in the molecule. The constant a is the same as Walker used. It may however be expressed as a function of the pressure and it is given approximately for pressures of 15, 30, 50, 100, and 760 mm. by the equation $a = 23{\cdot}5\, P^{\cdot 07}$, where P is the pressure. Calculated from the first sixteen members of the series for a pressure of 760 mm. its mean value is 37·3775, a figure which is practically identical with that calculated by the simpler formula (37·38) from the higher members of the series. When $n = 11$ and upwards the two formulae give the same results to the first decimal place. The following table gives the observed and calculated numbers and the differences between them.

Paraffin	Boiling points		Differences
	Observed	Calculated	
CH_4	108·3	105·7	− 2·6
C_2H_6	180·0	177·3	− 2·7
C_3H_8	228·0	231·9	+ 3·9
C_4H_{10}	274·0	275·6	+ 1·6
C_5H_{12}	309·3	312·2	+ 2·9
C_6H_{14}	342·0	343·9	+ 1·9
C_7H_{16}	371·4	372·3	+ 0·9
C_8H_{18}	398·5	398·3	− 0·5
C_9H_{20}	422·5	422·5	0
$C_{10}H_{22}$	446·0	445·2	− 0·8
$C_{11}H_{24}$	467·5	466·8	− 0·7
$C_{12}H_{26}$	487·5	487·3	− 0·2
$C_{13}H_{28}$	507·0	507·0	0
$C_{14}H_{30}$	525·5	526·0	+ 0·5
$C_{15}H_{32}$	543·5	544·2	+ 0·7
$C_{16}H_{34}$	560·5	561·9	+ 1·4

The simple formula gives a difference of $+41 \cdot 2$ for methane and of $+24 \cdot 7$, $19 \cdot 9$, $10 \cdot 6$, $7 \cdot 9$, $4 \cdot 6$ and $2 \cdot 4$ respectively for the succeeding members of the series.

Normal Alcohols.

The line which passes through the points given by the molecular weights and boiling points of water and the alcohols is sharply curved at the beginning and then it becomes almost straight. The following linear equation gives the boiling points of the alcohols $T = 286 \cdot 2 + 1 \cdot 41\,M$ and the following table gives the observed and calculated numbers in absolute degrees.

Alcohol	Boiling points		Differences
	Observed	Calculated	
Methyl......	337·7	331·1	−6·6
Ethyl........	351·3	351·1	−0·2
Propyl......	370·4	370·8	+0·4
Butyl........	390·0	390·5	+0·5
Amyl........	411·0	410·3	−0·7
Hexyl.......	430·0	430·0	0
Heptyl......	449·0	449·8	+0·8
Octyl........	469·0	469·4	+0·4

Aldehydes and Ketones.

The boiling points of these are also given by linear equations

(1) Aldehydes $T = 209 \cdot 14 + 1 \cdot 9286\,M.$

(2) Ketones $T = 250 \cdot 07 + 1 \cdot 4643\,M.$

The observed and calculated numbers in absolute degrees are given in the following table.

Aldehyde	Boiling points Observed	Boiling points Calculated	Differences
Formic............	252·0	267·0	+ 15
Acetic	293·8	294·0	+ 0·2
Propylic	321·8	321·0	– 0·8
Butylic............	348·0	348·0	0
Amylic............	376·4	375·0	– 1·4
Capric	401·0	402·0	+ 1·0
Ketone			
Acetone	330·0	335·0	+ 5
Di-ethyl	376·0	376·0	0
Di-propyl.........	417·0	417·0	0
Di-butyl		458·0	
Di-amyl	499·0	499·0	0
Di-hexyl..........	536·0	540·0	+ 4
Methyl-ethyl	354·0	355·5	+ 1·5
Methyl-propyl...	375·0	376·0	+ 1

It is worthy of remark that the difference in boiling points of two ketones differing by CH_2 is about 41·0° and this is the difference between the first constants in the above equations.

The influence of the oxygen atoms in the compounds given in the two last tables is very marked as it also is in the acids. The curve representing the acids is irregular and it indicates that the boiling points of the lower members are abnormal, due doubtless to the association of their molecules, whilst the boiling points assume a normal value in the higher members of the series.

PROCEEDINGS

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On Partial Fractions. By A. C. DIXON.

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Let $u_r = a_r x + b_r y + c_r$ $(r = 1, 2, \ldots n)$ and let U be a rational integral algebraic expression in x, y of degree $n-2$; then, if no three of the lines $u_1 = 0$, $u_2 = 0$, ... meet in a point, the fraction $U/u_1 u_2 \ldots u_n$ can be expressed as the sum of partial fractions of the form $A_{rs}/u_r u_s$ with constant numerators.

For let the expression $\Sigma A_{rs}/u_r u_s$ be brought to a common denominator; the numerator is an expression of the degree $n-2$ and contains $\frac{1}{2}n(n-1)$ arbitrary coefficients, such as A_{rs}, that is, as many as there are coefficients in U. Also none of these arbitrary coefficients are illusory since each may be uniquely determined. To find A_{rs}, put $u_r = 0 = u_s$ in the proposed identity

$$U = u_1 u_2 \ldots u_n \Sigma A_{rs}/u_r u_s.$$

The right-hand side is reduced to one term, containing A_{rs}, and this does not disappear, since no three of the lines $u_1 = 0$, $u_2 = 0$, ... are concurrent. Thus A_{rs} is found.

The identity proposed is thus established, but it may be verified as follows. If the numerators A have the values found for them, the expression

$$U - u_1 u_2 \ldots u_n \Sigma A_{rs}/u_r u_s$$

vanishes at $n-1$ points on the line $u_1 = 0$, namely, those in which this line meets $u_2 = 0$, $u_3 = 0$, ... $u_n = 0$. Since the degree of the expression is only $n-2$, it must contain u_1 as a factor, and similarly $u_2, u_3, \ldots u_n$. It can therefore only vanish identically.

The method fails when three of the lines $u=0$ are concurrent, and a simple case such as the fraction $1/xy(x+y)$ shews that the theorem is not generally true then.

Suppose, as an example, that $u_1=0$, $u_2=0$, $u_3=0$, $u_4=0$ all meet in a point C, through which none of the other lines pass. Then we may determine B_{34}, so that

$$U - B_{34}u_5u_6u_7 \dots \text{ vanishes at } C,$$

and then B_{23}, B_{24}, so that the curve

$$U - B_{34}u_5u_6 \dots - B_{23}u_4u_5u_6 \dots - B_{24}u_3u_5u_6 = 0$$

has a node at C, and then B_{12}, B_{13}, B_{14}, so that

$$U - (B_{34} + B_{23}u_4 + B_{24}u_3 + B_{12}u_3u_4 + B_{13}u_2u_4 + B_{14}u_2u_3)\, u_5u_6 \dots = 0$$

has a triple point at C. Then the partial fractions will be

$$\frac{B_{12}}{u_1u_2} + \frac{B_{13}}{u_1u_3} + \frac{B_{14}}{u_1u_4} + \frac{B_{23}}{u_1u_2u_3} + \frac{B_{24}}{u_1u_2u_4} + \frac{B_{34}}{u_1u_2u_3u_4} +$$

others found as before, with one or two of the expressions $u_5, u_6, u_7, \dots$ in each denominator.

Similarly, if the first m of the lines are concurrent, we have, instead of $\frac{1}{2}m(m-1)$ fractions with denominators

$$u_ru_s\ (r,\ s = 1,\ 2,\ \dots\ m),$$

$m-1$ with denominators, say,

$$u_1u_2,\ u_1u_3,\ \dots\ u_1u_m,$$

$m-2$ with denominators

$$u_1u_2u_3,\ u_1u_2u_4,\ \dots\ u_1u_2u_m,$$

$m-3$ with denominators

$$u_1u_2u_3u_4,\ u_1u_2u_3u_5,\ \dots\ u_1u_2u_3u_m,$$

and so on. The numerators are constants, and there are in all $\frac{1}{2}m(m-1)$ of them. The form of the fractions in which one factor of the denominator is taken from $u_1, u_2, \dots u_m$ and the other from $u_{m+1}, u_{m+2}, \dots u_n$, is not affected.

The denominator may contain repeated factors. In such a case some of the numerators will generally not be constants; they may be quantics in one of the variables only or in an arbitrary linear combination of them. Suppose the variable x to be chosen, and let the denominator contain u_1 in the m^{th} power only, $u_{m+1}, u_{m+2}, \dots u_n$ being different and no three of the lines concurrent. Then the $m-1$ coefficients in an $(m-2)^{\text{ic}}$, $\phi(x)$, and constants B, may be so chosen that

$$U - u_{m+1}u_{m+2} \dots u_n\, \phi(x) - u_{m+1}u_{m+2} \dots u_n \left[\frac{B_{m+1}}{u_{m+1}} + \frac{B_{m+2}}{u_{m+2}} \dots + \frac{B_n}{u_n}\right]$$

contains the factor u_1, provided of course that u_1 is not independent of y. Then

$$\frac{U}{u_1^m u_{m+1} \dots u_n} - \frac{\phi(x)}{u_1^m} - \Sigma \frac{B_{m+s}}{u_1^m u_{m+s}}$$

is equal to a fraction in whose denominator only the $(m-1)^{\text{th}}$ power of u_1 occurs. This fraction can be treated in like manner and thus the original fraction is reduced to a series of fractions: the denominators are

$$u_1^r \; (r = 2, 3, \dots m),$$

$$u_1^r u_{m+s} \; (r = 1, 2, \dots m; \; s = 1, 2, \dots n - m),$$

$$u_{m+r} u_{m+s} \; (r, s = 1, 2, \dots n - m).$$

The numerators in the second and third series are constant: in the first series the numerators are quantics in x only, of degrees lower by two than the respective denominators.

If the lines $u_1 = 0$, $u_2 = 0$, ... are concurrent and the denominator contains the factors u_1, u_2, ... in the degrees m_1, m_2, ..., then the partial fractions corresponding will have denominators of the form

$$u_1^{n_1} u_2^{n_2} \dots,$$

where $n_1 \not> m_1$, $n_2 \not> m_2$, ... and $n_1 + n_2 + \dots \not< 2$.

If x does not vanish at the point of concurrence the numerators may all be made functions of x only, each of degree lower by 2 than the corresponding denominator. Some of the coefficients in these numerators may be arbitrarily chosen: a general rule for finding the others would be to express the numerator U and the other factors of the denominator as homogeneous functions of x, u_1, u_2, and then divide U by the product of these other factors in the denominator, in descending powers of x: each power of x in the quotient could then be treated by itself.

Thus a series of partial fractions would arise from each point of concurrence of two or more of the denominator lines: the sum of all the partial fractions is the original fraction. The case with which we started is that in which each series contains one fraction only.

If u_1, u_2, ... u_n are linear expressions in p variables and U of the degree $n - p$, then $U / u_1 u_2 \dots u_n$ can be similarly resolved into partial fractions. The denominator of each will contain p factors and the numerators will be constant: their number is $\dfrac{n!}{p!\,(n-p)!}$, the same as that of the coefficients in U. The exceptional cases are more complicated and seemingly of no great interest.

The process is of use in discussing the reduction of a ternary n^{ic} V to the form of a symmetrical determinant with linear constituents.

Project to infinity a line meeting the curve $V=0$ in n separate points, no three of the tangents at which meet at a point. This is always possible with a proper n^{ic} curve, for if not, let $A_1, A_2, \dots A_n$ be the points where it meets an arbitrary line, and suppose the tangents at A_1, A_2, A_3 to be concurrent in all positions. Let the line turn about A_1 and come back in such a way that A_2 comes to the position A_4. Then the tangents at A_1, A_4 and another of the n points must be concurrent. Similarly, if any two suffixes r, s are taken there is a third, t, such that the tangents at A_r, A_s, A_t meet in a point, and this holds in all positions of the straight line. Let A_2 move up to A_1 so that the line becomes a tangent. Then the tangent at A_3 must pass through A_1, since this is the point of intersection of the tangents at A_1, A_2, and the line is therefore a bitangent. Every tangent is therefore a bitangent and meets the consecutive in two points: it must therefore coincide with the consecutive and the curve must break up into straight lines.

We may then write $V \equiv u_1 u_2 \dots u_n - U$, where $u_1=0$, $u_2=0\dots$ are the asymptotes of $V=0$, no three of which meet in a point, and U is of degree $n-2$ at most. In the exceptional case U vanishes identically, and V is equal to a determinant having $u_1, u_2 \dots u_n$ in the diagonal and 0 in other places.

When U does not vanish identically, assume

$$u_1 u_2 \dots u_n - \lambda U = \begin{vmatrix} u_1 & a_{12} & a_{13} \dots \\ a_{12} & u_2 & a_{23} \dots \\ a_{13} & a_{23} & u_3 \dots \\ \dots & \dots & \dots \end{vmatrix},$$

a symmetrical determinant of order n in which a_{12}, $a_{13} \dots$ are independent of x, y: these quantities are functions of λ, and when $\lambda=0$ the conditions are satisfied by taking them all to vanish. Suppose λ to be small, and a_{12}, $a_{13} \dots$ also small. Then we have for a first approximation, neglecting powers and products of a_{12}, $a_{13} \dots$ above the second,

$$\lambda U = u_1 u_2 \dots u_n \,.\, \Sigma\, a_{rs}^2 / u_r u_s,$$

or with our former notation

$$a_{rs}^2 = \lambda A_{rs}.$$

The quantities a_{rs} are thus all found to a first approximation. The approximation can be carried to any degree, and as the equations to be solved are algebraic the value of a_{rs} can be found as a series of ascending powers of $\sqrt{\lambda}$ for values of λ within

a certain range, not infinitesimal. Beyond this range the values can be analytically continued if necessary, and the solution thus found when $\lambda = 1$. Hence any ternary quantic can be expressed as a symmetrical determinant with linear constituents: the problem is not poristic. (Compare *Proceedings*, Vol. XI. p. 351.)

Since when λ is small each of the quantities a_{rs} has an ambiguous sign, the number of solutions indicated is $2^{\frac{1}{2}n(n-1)}$; since however in any symmetrical determinant the sign of any constituent in the first row, after the first, may be changed at will by changing the sign of its row and the corresponding column, only one in 2^{n-1} of these solutions is to be counted: there are therefore $2^{\frac{1}{2}(n-1)(n-2)}$ distinct solutions at least. That there may be more is indicated by the fact that $2u_1u_2u_3$ can be expressed in either of the forms

$$\begin{vmatrix} 2u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{vmatrix}, \qquad \begin{vmatrix} 0 & u_3 & u_2 \\ u_3 & 0 & u_1 \\ u_2 & u_1 & 0 \end{vmatrix}.$$

That all the $2^{\frac{1}{2}(n-1)(n-2)}$ solutions given by the process are really distinct follows from a consideration of the terms of the third degree in $a_{12}, a_{13} \ldots$. These will be altered by any change in the signs of $a_{12}, a_{13} \ldots$ other than those we have allowed for. There are, for instance, two terms $a_{12}a_{13}a_{23}u_4u_5u_6 \ldots$ in the expanded determinant, and if these are to be unaffected then two or none of a_{12}, a_{13}, a_{23} must be changed. Similarly for a_{rs}, a_{st}, a_{rt}. Let the suffixes > 1 be divided into two classes, r being of the first class when a_{1r} is changed and of the second when a_{1r} is unchanged. Then a_{rs} is changed if r, s are of different classes, and unchanged if they are of the same. Thus the rows and columns with suffixes of the first class are changed and the others unchanged, which was to be proved.

Note on Plane Unicursal Curves. By A. C. Dixon.

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When a plane curve U of degree n has its full number $\frac{1}{2}(n-1)(n-2)(=p)$, of double points, any curve f of degree m, passing through all the double points, meets U in $mn-2p$ other points, all of which may be arbitrarily chosen. But if f does not pass through all the double points the rest of the common points are restricted by certain conditions.

Since U is unicursal, we have at every point of it

$$x = X\theta/Z\theta, \quad y = Y\theta/Z\theta,$$

where $X\theta$, $Y\theta$, $Z\theta$ denote quantics of the nth degree in the parameter θ which varies from point to point. A node arises when two values of θ, say α and β, give the same values of x and y, that is, when

$$\frac{X\alpha}{X\beta} = \frac{Y\alpha}{Y\beta} = \frac{Z\alpha}{Z\beta} \quad \text{.......................(1)}.$$

To find the intersections of U, f we substitute the above values in terms of θ for x, y in the equation $f=0$, and after multiplying by $(Z\theta)^m$ get an equation

$$F(\theta) \equiv f(X\theta, Y\theta, Z\theta) = 0.$$

This is of the degree mn, and its roots are the values of θ which belong to the points on U that also lie on f. Let these be $\theta_1, \theta_2 \ldots \theta_{mn}$. Then $F(\theta) = C\Pi(\theta - \theta_r)$, where C is a constant coefficient.

Now $f(X\theta, Y\theta, Z\theta)$ is a homogeneous m-ic in $X\theta$, $Y\theta$, $Z\theta$, and therefore

$$f(X\alpha, Y\alpha, Z\alpha)/f(X\beta, Y\beta, Z\beta),$$

in virtue of (1), is independent of the coefficients in f and equal to $(Z\alpha/Z\beta)^m$.

We have then

$$\frac{\Pi(\alpha-\theta_r)}{\Pi(\beta-\theta_r)} = \frac{F(\alpha)}{F(\beta)} = \left(\frac{Z\alpha}{Z\beta}\right)^m.$$

This condition must be satisfied by $\theta_1, \theta_2 \ldots \theta_{mn}$ if α, β are the parameters belonging to any node. Since there are $\frac{1}{2}(n-1)(n-2)$ nodes we have the same number of conditions to be satisfied by $\theta_1, \theta_2 \ldots \theta_{mn}$.

If f passes through r of the nodes each of these counts as two of the intersections, and the number of conditions to be satisfied by the rest is $\frac{1}{2}(n-1)(n-2)-r$, since of those above written r are satisfied already.

In particular, take f to be the $(n-3)$-ic through all the nodes but one, say (α_1, β_1). Its $n(n-3)$ intersections with U correspond to the parameters $\alpha_2, \beta_2, \alpha_3, \beta_3 \ldots \alpha_p, \beta_p$.

Hence $$\prod_{r=2}^{p} \frac{(\alpha_1-\alpha_r)(\alpha_1-\beta_r)}{(\beta_1-\alpha_r)(\beta_1-\beta_r)} = \left(\frac{Z\alpha_1}{Z\beta_1}\right)^{n-3}.$$

There are p relations of this form.

Conversely, if $\theta_1, \theta_2 \ldots \theta_{mn}$ satisfy the p conditions

$$\prod_{r=1}^{mn} \frac{\alpha_i-\theta_r}{\beta_i-\theta_r} = \left(\frac{Z\alpha_i}{Z\beta_i}\right)^m \quad (i=1, 2 \ldots p),$$

the points $\theta_1, \theta_2 \ldots \theta_{mn}$ on U are its intersections with a curve of degree m. Write $F(\theta)$ for $\prod_{r=1}^{mn} (\theta-\theta_r)$, and let $\phi_r = 0\ (r=1, 2 \ldots p)$ be the curve of degree $n-3$ that passes through all the nodes but (α_r, β_r). Thus we have

$$(Z\theta)^{n-3}\phi_r\left(\frac{X\theta}{Z\theta}, \frac{Y\theta}{Z\theta}\right) = \prod_{i=1}^{p} (\theta-\alpha_i)(\theta-\beta_i)/(\theta-\alpha_r)(\theta-\beta_r).$$

Now by proper choice of the coefficients $\lambda_1, \lambda_2 \ldots$ the expression

$$F_1(\theta) \equiv F(\theta) - (Z\theta)^{m-n+3} \prod_{r=1}^{p} (\theta-\alpha_r)(\theta-\beta_r) \sum_{r=1}^{p} \frac{\lambda_r}{(\theta-\alpha_r)(\theta-\beta_r)}$$

can be made to vanish when $\theta = \alpha_1, \alpha_2 \ldots \alpha_p$. The value of λ_1 is

$$\frac{F(\alpha_1)}{(Z\alpha_1)^m} \cdot \frac{(Z\alpha_1)^{n-3}}{\prod_{r=2}^{p} (\alpha_1-\alpha_r)(\alpha_1-\beta_r)},$$

and this is unchanged in value by hypothesis when β_1 is put in the place of α_1. Hence $F_1(\theta)$ also vanishes when $\theta = \beta_1$ and similarly when $\theta = \beta_2, \beta_3 \ldots \beta_p$. If then $m \not> n-3$, $F_1(\theta)$ must vanish identically, since it vanishes for more than $n(n-3)$ values of θ. Thus the curve

$$\lambda_1\phi_1 + \lambda_2\phi_2 + \ldots + \lambda_p\phi_p = 0$$

consists of the line at infinity $n-3-m$ times, with a curve of degree m that passes through the mn given points.

If $m \geqslant n-2$ take a curve of degree m, having, unless $m = n-2$, a multiple point of order $m-n+1$ at an arbitrary point, say the origin, not lying on the curve U: its equation will contain

$$\tfrac{1}{2}m(m+3) - \tfrac{1}{2}(m-n+1)(m-n+2),$$

that is $mn - \frac{1}{2}(n-1)(n-2)$ arbitrary constants, and it can therefore be made to pass through this number of arbitrary points: choose the points on U whose parameters satisfy the equation $F_1(\theta) = 0$, of which points there are $mn - \frac{1}{2}(n-1)(n-2)$, since each node has two parameters. Let $W = 0$ be the equation to

the m-ic curve: then W cannot be a multiple of U, for it only contains terms of n different degrees, whereas in U there are $n+1$, namely $0, 1, 2 \ldots n$.

We have, then, after proper choice of a multiplier,

$$(Z\theta)^m W\left(\frac{X\theta}{Z\theta}, \frac{Y\theta}{Z\theta}\right) = F_1(\theta).$$

Hence

$$F(\theta) = (Z\theta)^m \left[W\left(\frac{X\theta}{Z\theta}, \frac{Y\theta}{Z\theta}\right) + \sum_{r=1}^{p} \lambda_r \phi_r \left(\frac{X\theta}{Z\theta}, \frac{Y\theta}{Z\theta}\right)\right].$$

The curve

$$W(x, y) + \lambda_1\phi_1 + \lambda_2\phi_2 + \ldots + \lambda_p\phi_p = 0$$

therefore meets the given curve in the points $\theta_1, \theta_2 \ldots \theta_{mn}$, which proves the proposition.

In the case of a cusp the form of the condition is modified. Suppose α to be the value of θ at the cusp, and (h, k) its coordinates, then

$$X\theta - h\,.\,Z\theta, \quad Y\theta - k\,.\,Z\theta$$

have the factor $(\theta-\alpha)^2$, and thus the equations (1) are replaced by

$$\frac{X'\alpha}{X\alpha} = \frac{Y'\alpha}{Y\alpha} = \frac{Z'\alpha}{Z\alpha}.$$

Hence, if $\quad F(\theta) \equiv f(X\theta, Y\theta, Z\theta),$

$$\frac{F'\alpha}{F\alpha} = m\,.\,\frac{Z'\alpha}{Z\alpha};$$

or

$$\Sigma \frac{1}{\alpha - \theta_r} = m\,.\,\frac{Z'\alpha}{Z\alpha}$$

is the condition to be satisfied by $\theta_1, \theta_2 \ldots \theta_{mn}$, when the point α is a cusp. There is no difficulty about the converse theorem.

Suppose now that the curve has a triple point, the three values of θ being α, β, γ. Then we have by the same reasoning as before

$$F\alpha/(Z\alpha)^m = F\beta/(Z\beta)^m = F\gamma/(Z\gamma)^m,$$

but there is also a further condition. Let A, B, C be so chosen that

$$F\theta - A(Z\theta)^m - B(Z\theta)^{m-1}X\theta - C(Z\theta)^{m-1}Y\theta$$

has the factors $(\theta-\alpha)^2(\theta-\beta)^2$, that is, that the curve

$$f(x, y, 1) - A - Bx - Cy = 0$$

has a double point at the triple point of U. Then $(\theta-\gamma)^2$ will also be a factor, so that

$$F\theta - A(Z\theta)^m - B(Z\theta)^{m-1}X\theta - C(Z\theta)^{m-1}Y\theta$$

and its first derivative both vanish when $\theta = \alpha, \beta, \gamma$.

Eliminating A, B, C we have a condition linear in $F'\alpha$, $F'\beta$, $F'\gamma$, $F\alpha$, which may be written thus

$$\begin{vmatrix} F'\alpha/F\alpha, & F'\beta/F\beta, & F'\gamma/F\gamma, & m \\ X'\alpha/X\alpha, & X'\beta/X\beta, & X'\gamma/X\gamma, & 1 \\ Y'\alpha/Y\alpha, & Y'\beta/Y\beta, & Y'\gamma/Y\gamma, & 1 \\ Z'\alpha/Z\alpha, & Z'\beta/Z\beta, & Z'\gamma/Z\gamma, & 1 \end{vmatrix} = 0.$$

In the proof of the converse theorem it is to be remembered that all but three of the curves $\phi_1 = 0$, $\phi_2 = 0 \dots$ may be made to have a double point at the triple point of U. Let ϕ_1, ϕ_2, ϕ_3 be the three exceptions. Then λ_1, λ_2, λ_3 must be so chosen that

$$F\theta - (Z\theta)^m \sum_{r=1}^{3} \lambda_r \phi_r \left(\frac{X\theta}{Z\theta}, \frac{Y\theta}{Z\theta}\right)$$

shall have this double point, and the rest of the proof is as before.

Other multiple points may be similarly treated.

The results that have been found might have been deduced from Abel's theorem. Take the case when U has p nodes. We have

$$\Sigma \int \frac{\phi_r dy}{\frac{\partial U}{\partial x}} = \text{const.}, \qquad (r = 1, 2 \dots p)$$

where the upper limits of the integrals summed are the intersections of U with a variable curve such as f. In the integrals change the variable from y to θ. Then ϕ_r becomes a fraction, whose numerator is

$$\Pi \{(\theta - \alpha)(\theta - \beta)\} \div (\theta - \alpha_r)(\theta - \beta_r),$$

and denominator $(Z\theta)^{n-3}$. The denominator of $\frac{dy}{d\theta}$ is $(Z\theta)^2$, and its numerator vanishes at the $2n - 2$ points where the tangents are parallel to the axis of x. At these points $\frac{\partial U}{\partial x}$ vanishes, and also at the nodes: the denominator of $\frac{\partial U}{\partial x}$ is $(Z\theta)^{n-1}$.

Hence $\int \frac{\phi_r dy}{\frac{\partial U}{\partial x}}$ is a constant multiple of $\int \frac{d\theta}{(\theta - \alpha_r)(\theta - \beta_r)}$, and Abel's theorem becomes

$$\Sigma \log \frac{\theta - \alpha_r}{\theta - \beta_r} = \text{const.}$$

On the Order of Certain Systems of Conditions.

By A. C. Dixon.

[*Received* 28 March 1904.]

Let there be $m+1$ expressions $u_1, u_2 \dots u_{m+1}$, rational and integral and of the degree $2m$ in x: then constants $a_1, a_2 \dots a_{m+1}$ can be so chosen that the roots of the equation

$$a_1u_1 + a_2u_2 + \dots + a_{m+1}u_{m+1} = 0$$

shall be equal in pairs. The object of this note is to shew in the first place that this can be done in 2^m ways. For put

$$a_1u_1 + a_2u_2 + \dots + a_{m+1}u_{m+1} = (p_0x^m + p_1x^{m-1} + \dots + p_m)^2.$$

Then on equating coefficients we have $2m+1$ expressions linear and homogeneous in $a_1, a_2 \dots a_{m+1}$, each equal to a homogeneous quadratic expression in $p_0, p_1 \dots p_m$.

Eliminating $a_1, a_2 \dots a_{m+1}$ we have m homogeneous equations of the second degree to give $p_0 : p_1 : p_2 \dots$, and the number of solutions is therefore 2^m, as stated.

More generally, let $u_1, u_2 \dots u_{m+1}$ involve two variables x, y connected by an algebraic equation $f(x, y) = 0$ of degree n and deficiency p. Suppose all but $2m$ of the points, where

$$a_1u_1 + a_2u_2 + \dots + a_{m+1}u_{m+1}$$

vanishes, to be fixed independently of $a_1, a_2 \dots$. Then when $m \nless p$ $a_1, a_2 \dots a_{m-1}$ may be so chosen that the other $2m$ points coincide in pairs and the number of ways in which this can be done is 2^{m+p}.

To save complications, let the curve $f(x, y) = 0$ be brought by rational transformation to a form in which it has no multiple points except ordinary nodes. Let the constants in it be changed continuously to such values that the curve has p new nodes. Then a curve touching the original in m points may become a curve passing through q of the new nodes and having $m-q$ proper contacts, q being any whole number from 0 to p: such a curve will, according to the usual convention, be counted 2^q times among the solutions of the problem. The number of such curves is easily found since the curve $f(x, y) = 0$ is now unicursal, and x, y can be expressed rationally in terms of a single parameter θ.

The conditions that

$$a_1u_1 + a_2u_2 + \dots + a_{m+1}u_{m+1} = 0$$

shall pass through the q nodes serve to fix say $a_{m-q+2}, a_{m-q+3} \dots a_{m+1}$ in terms of $a_1, a_2 \dots a_{m-q+1}$, and thus reduce the equation to the form

$$a_1 v_1 + a_2 v_2 + \dots + a_{m-q+1} v_{m-q+1} = 0.$$

In this let the parametric expressions for x, y be substituted and the factors independent of $a_1, a_2 \dots$ divided out: what is left is an equation of degree $2m-2q$ in θ, and the roots of this can be made to coincide in pairs in 2^{m-q} ways by our former result.

Out of p nodes q can be chosen in $C_{p,q}$ ways, and we thus find for the whole number of solutions

$$\sum_{q=0}^{p} C_{p,q} 2^{m-q} . 2^q \quad \text{or} \quad 2^{m+p},$$

it being assumed that $m \not< p$.

For instance the number of curves of degree $n-1$ passing through all the $\frac{1}{2}(n-1)(n-2)-p$ nodes, touching the curve $f(x, y)=0$ in $p+n-1$ points, of which $r(<n)$ are fixed, and passing through $n-r-1$ given external points, is $2^{2p+n-r-1}$.

When $m<p$, q cannot be greater than m, and some terms of the series $\Sigma C_{p,q} 2^{m-q} . 2^q$ must be left out. For instance, of the curves of degree $n-3$ that pass through all the nodes, the number that touch $f(x, y)=0$ in $p-1$ places is

$$\sum_{q=0}^{p-1} C_{p,q} 2^{p-1-q} . 2^q \quad \text{or} \quad 2^{p-1}(2^p-1),$$

as given by the theory of Abelian functions.

It may be well to discuss the grounds of the convention that a curve passing through q nodes is to be counted 2^q times among the solutions of the contact problem.

Let $S=0$, $S'=0$ be two neighbouring curves, of which the former has nodes at $A_1, A_2 \dots A_q$. Then we may write, to the first order of small quantities,

$$S' = S + \epsilon_1 T_1 + \epsilon_2 T_2 + \dots + \epsilon_r T_r, \qquad (r \not< q),$$

where of the curves $T_1=0$, $T_2=0 \dots$ all but $T_1=0$ pass through A_1, all but $T_2=0$ through A_2, and so on; $\epsilon_1, \epsilon_2 \dots$ are small constants.

Let $Q=0$, $Q'=0$ be two neighbouring curves, of which the former passes through $A_1, A_2 \dots A_q$, so that we may write

$$Q' = Q + \eta_1 R_1 + \eta_2 R_2 + \dots,$$

where again of the curves $R_1=0$, $R_2=0 \dots$ all but $R_1=0$ pass through A_1, all but $R_2=0$ through A_2, and so on: small quantities of higher order than $\eta_1, \eta_2 \dots$ are again neglected. Consider the

conditions to be satisfied by the constants $\eta_1, \eta_2 \ldots$ in order that $Q' = 0$ may have q contacts with $S' = 0$ at points near the q nodes.

Suppose that in order to find the intersections of the two curves we have eliminated y from $Q' = 0$, $S' = 0$. The result may be written $\prod_r S'(x, Y_r) = 0$, where $Y_1, Y_2 \ldots$ are the different values of y given in terms of x by $Q' = 0$. Move the origin to A_1, then near A_1 x and y are small, and we have from $Q' = 0$ a first approximation of the form

$$y = \alpha x + \beta\eta_1.$$

Hence when x is small one of the factors of the y-eliminant becomes $S'(x, \alpha x + \beta\eta_1)$ approximately. The most important terms in this, when $x, \eta_1, \eta_2 \ldots, \epsilon_1, \epsilon_2 \ldots$ are all small, are of the form

$$ax^2 + 2bx\eta_1 + c\eta_1^2 + f\epsilon_1,$$

and the x-discriminant of the y-eliminant will have

$$a(f\epsilon_1 + c\eta_1^2) - b^2\eta_1^2$$

as a factor in its most important terms. Putting this equal to zero we have as the condition for a contact near A_1

$$\eta_1 = \pm \lambda \sqrt{\epsilon_1} + \text{higher terms},$$

λ being a constant coefficient. The conditions for contacts near the other nodes are of the same form, and there are q ambiguous signs; that is, there are 2^q solutions of the contact problem which merge into the single one

$$\eta_1 = \eta_2 = \ldots = \eta_q = 0,$$

when $\epsilon_1, \epsilon_2 \ldots$ all vanish.

Electrical Oscillations in Wires. By H. C. Pocklington.

I am indebted to Prof. Orr for pointing out to me that there is an error in § 3 of my paper on "Electrical Oscillations in Wires" (*Proc.* Vol. IX. p. 324), the force tangential to the cross-section being of order $\log \epsilon$, and not, as stated, finite. In § 4 we must now shift the curve formed by the Hertzian elements through a distance of order $\epsilon^2 \log \epsilon$, but the reasoning still holds good. The result of § 3 can now be obtained from that of § 2.

Metallic "Passivity" in relation to Time and Temperature.

By Dr W. A. Hollis, Trinity College.

[*Received* 8 April 1904.]

In a previous paper* certain peculiarities in the behaviour of the metal nickel when placed in contact with fuming nitric acid have been noted. It was there suggested that the passivity shown by certain samples of that metal depended to some extent upon their molecular arrangement; those which excelled in the qualities of tenacity and ductility, such as wire and sheet nickel, were observed to be more resistant to the action of the acid than others possessing a crystalline structure. There were however anomalous phenomena in this connection that could not be so accounted for. Further observations led to the conclusion, so far as regards the metals iron, nickel, and cobalt, that passivity is only a relative state. In other words, the variety of these metals which mainly exhibits the phenomenon, and undergoes no apparent change when immersed for a short while in the acid at a given temperature, shows unmistakable signs of solution if we either raise the temperature or prolong the immersion sufficiently. A series of experiments were undertaken with a view to determine more precisely than heretofore this functional relationship between metallic passivity on the one hand, and time and temperature on the other.

For various reasons samples of sheet metal were selected for examination. This type of metal, besides being in the case of iron and nickel highly resistant to the action of the acid at the ordinary temperature of the laboratory, lent itself more readily to accurate linear measurement than another. The iron was the ordinary ferrotype plate of commerce, cleaned and roughly polished. Two varieties of nickel were used; sheet nickel about 0·1 mm. in thickness, obtained from Messrs Johnson, Matthey, and Company, and the Mond tubular nickel. The latter is about the same thickness as the sheet. The same Hatton Garden firm supplied me with a sample of rolled cobalt, a type of this metal rarely seen owing to the difficulty of manufacture. Some copper sheeting was used for control purposes. The copper and the cobalt sheeting were about one millimetre thick.

Three units have been employed throughout the investigation. A time-unit, the second; a degree (F.) as the temperature unit;

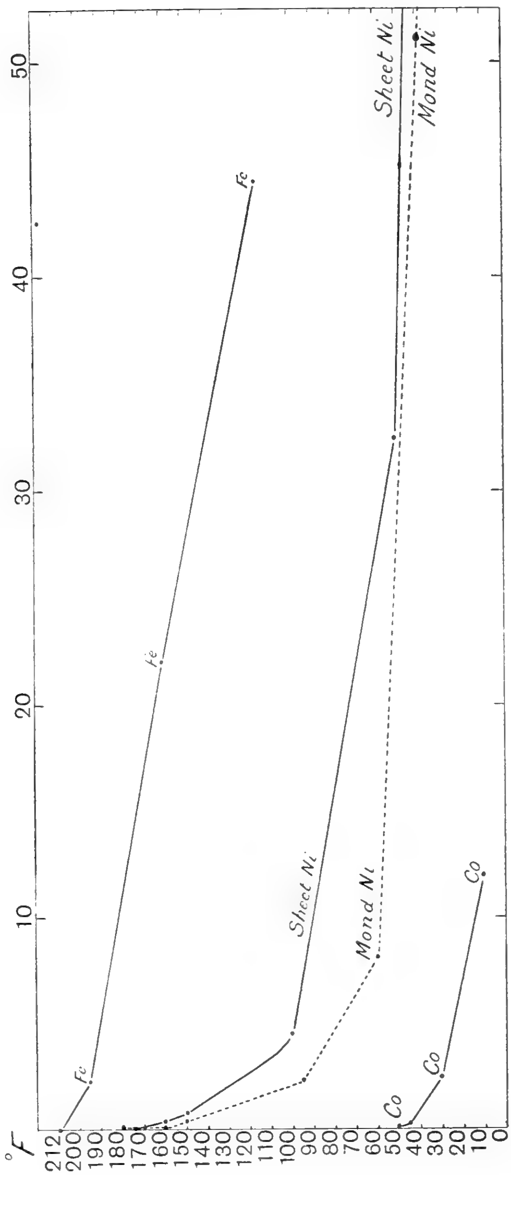
* *Proc. Camb. Phil. Soc.* Vol. XII. p. 253.

and a metallic unit, represented by a superficial layer of the metal plaque under examination, one milligram in weight and one square centimetre in area.

As regards the metals iron and nickel, which were obtained in the thin sheets 0·1 mm. in thickness, each sheet was cut into strips one centimetre wide, and a strip was again subdivided into discs of metal one or more centimetres in length. A square disc therefore had a surface area of two square centimetres, for in this case the edges of a disc were not considered. In the thicker cobalt and copper sheeting, however, an allowance was made for the surface area of the four edges; and a 2-centimetre disc was modified in shape accordingly. The nitric acid used was the ordinary pharmacopœial fuming acid.

In the course of the investigation it was found that each of the three metals under observation had a critical point in the temperature-scale, above which its passivity ceased. The change was an abrupt one; and the temperature, at which this took place, differed greatly for each metal. Iron for instance did not lose its passivity under favourable circumstances until the boiling-point of water was nearly reached, when the change occurred with almost explosive violence. Cobalt, on the other hand, lost its passivity at or about 50° F. Nickel occupied an intermediate position. The critical temperature of sheet-nickel was about 175° F.; whilst in Mond nickel it was some ten degrees higher. The samples of nickel differed in another respect from the two preceding metals; the change from a passive to an active state was in their case more gradual than in either iron or cobalt. It therefore became necessary to define the passive state more clearly than heretofore. This has been provisionally done as follows:—*Sheet iron, nickel, and cobalt, when immersed in fuming nitric acid, may be considered to assume a passive condition if after ten seconds' exposure to the acid a given metallic unit of a sample is not dissolved.* The critical temperature of copper, which I believe is not usually supposed to exhibit passivity, is probably a few degrees above that of cobalt.

Immediately below the critical point there exists for each of the three metals a temperature-zone of some twenty degrees (or even more in the case of sheet iron) wherein the phenomenon of passivity may or may not occur under apparently identical conditions. The samples of rolled cobalt showed this peculiarity markedly. It is by no means unusual for two out of three specimens of this metal, immersed in the acid at 40° F., to exhibit passivity while the remaining sample shows the signs of intense activity. Within its proper *zone of instability*—to so name it—each metal is therefore liable to undergo an abnormal crisis at any given temperature. If however the metal is placed in the


°F
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acid at a temperature below the zone of instability so as to induce passivity in the first instance, the test-tube containing these ingredients may be plunged into a heated water-bath, with the assurance that there will be no lapse of passivity until the critical point is reached. The behaviour of sheet iron well illustrates this point. If the metal and the acid are heated separately to a temperature as low as 165° F., and then brought together, violent ebullition as a rule takes place at once. On the other hand, when the alternate method is adopted, passivity even at a temperature of 200° F. is assured. It seems therefore that metallic passivity (especially in the case of sheet iron), when once established, exercises a definite restraining influence upon the premature incidence of a crisis.

In the chart accompanying this paper an attempt has been made to show from actual experiments the time required for the solution of a metallic unit of the three metals, iron, nickel, and cobalt, at any temperature from ten degrees Fahrenheit to the boiling-point of water. Although in the case of iron and nickel the shortness of the time-unit makes the curves incomplete, their general trend is shown. The practical difficulties of pursuing this investigation more fully than has been done were so numerous that it became futile to extend it with the apparatus at hand. I therefore decided to publish it provisionally, and it must be accepted in this light.

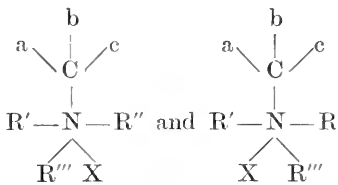
Finally, a few remarks may be made regarding the apparent anomalies observable in these experiments on metallic passivity. First, it is interesting to observe that all the crises fell—and this statement includes copper—well within the critical temperatures of water, between 32° F. and 212° F. This I ascribe to the action of the fuming nitric acid. Future observations upon other metals may confirm the statement. Secondly, it seems probable that the purity of the metal investigated affects the critical temperature. This contingency must specially affect the three metals I have selected, as they are often alloyed with each other. In the former paper the analyses of the sheet and Mond nickel are given. The rolled cobalt contained approximately Co 95 %. The chief impurities are iron, nickel, carbon, etc. As regards the ferrotype plate there is unfortunately no assay at hand. Thirdly, the brittleness of the samples of rolled cobalt and its crystalline fracture so far correspond structurally with the grain nickel. Each has a comparatively low critical point also.

Note on Compounds containing an Asymmetric Nitrogen and an Asymmetric Carbon Atom. By H. O. JONES, M.A., Clare College.

[*Received* 26 April 1904.]

The investigation of compounds containing both an asymmetric nitrogen and an asymmetric carbon atom was undertaken in the hope that the phenomena exhibited by them might throw further light on the problem of the isomerism of quinquevalent nitrogen compounds.

It was to be expected, if the analogy between the behaviour of asymmetric carbon and nitrogen atoms held in this case, that when a tertiary amine in which one of the groups contained an asymmetric carbon atom combined with an alkyl iodide so as to produce a compound containing an asymmetric nitrogen atom, unequal quantities of the two possible isomerides would be produced, which, not being optical antimers, might be separated by ordinary processes. These compounds might be represented thus:



and would bear to one another a relation similar to that of the α and β glucoheptonic acids or the two $\alpha\beta$ dihydroxybutyric acids. This expectation has been realised, and another analogy between the behaviour of asymmetric nitrogen and carbon compounds established.

The tertiary amine used was methyl *l*-amyl aniline, which was prepared as pure as possible, and found to be dextrorotatory $[\alpha]_D^{15} = 10{\cdot}8°$.

In order to ascertain, if possible, what kind of rotatory power would be given to the salts by the active amyl group alone, the union with methyl iodide was studied. The union to form a crystalline salt takes place readily, but the salt examined in 7 per cent. alcoholic solution had no observable rotatory power. The effect of the carbon atom is therefore extremely small. The fact that the compound thus produced is inactive affords further evidence in support of the author's conclusion that compounds of the type $N.R'R''R'''_2X$ could not show optical activity (*Trans. Chem. Soc.* 1903, 83, 1420).

The union with allyl iodide was next studied, and it was found that the salt produced had a decided rotatory power $[\alpha]_D = 1{\cdot}5^\circ$, and on recrystallisation from alcohol the value of $[\alpha]_D$ became $16{\cdot}3^\circ$ in alcohol and $21{\cdot}8^\circ$ in chloroform.

It is therefore probable that two compounds, one of which is dextrorotatory and the other laevorotatory, have been produced, and that the former is the less soluble in alcohol. The more soluble salt could not be isolated in a pure form, since, however carefully the allyl iodide had been purified, the crude product was always somewhat coloured and the more soluble portions were strongly coloured.

That the rotatory power is due to the nitrogen atom and not to the carbon is proved by the behaviour of the salt in chloroform solution. The rotatory power falls at first rapidly, then more slowly, until it reaches a constant value of $\cdot 38^\circ$ for a $6{\cdot}1$ per cent. solution, or $\cdot 22^\circ$ for $3{\cdot}44$ per cent. solution in a 2 dcm. tube corresponding to $[\alpha]_D = 3{\cdot}1^\circ$. The change here is most probably a transformation of one isomer into the other by dissociation into tertiary amine and alkyl iodide, and subsequent recombination until equilibrium is attained. (Cp. the autoracemisation of active nitrogen compounds; Pope and Harvey, *Trans. Chem. Soc.* 79, 830, and Jones, *Trans. Chem. Soc.* 85, 229.)

The union with benzyl iodide was then studied, and the reaction, which takes place rapidly and gives a practically colourless product, was found to be more suitable for the present investigation than the reaction with allyl iodide.

The reaction was allowed to proceed in different solvents at the ordinary temperature and the crude product examined. All the products were dextrorotatory. The salt deposited from chloroform solution had a much greater rotatory power ($[\alpha]_D = 23{\cdot}3^\circ$ in $2{\cdot}3$ per cent. solution in chloroform) than that obtained from alcohol, ether or ethyl acetate solution ($[\alpha]_D = 3{\cdot}5^\circ$—$5^\circ$ in $2{\cdot}3$ per cent. solution in chloroform).

Repeated crystallisation from hot alcohol or from warm alcohol did not cause any very great change in the rotatory power of the salt; so that this method does not readily effect the desired separation of the two compounds.

The *d*- and *l*-camphorsulphonates were therefore prepared, crystallised several times from ethyl acetate, their rotatory powers determined, and the iodide recovered from the various fractions and examined.

The less soluble fraction of *d*-camphorsulphonate gave $[\alpha]_D = 7{\cdot}18^\circ$ and $[M]_D = 31{\cdot}1^\circ$ in aqueous solution, and hence must contain a laevorotatory basic radicle. The iodide recovered from it gave $[\alpha]_D = -10{\cdot}0^\circ$ in alcohol.

The less soluble fraction of *l*-camphorsulphonate was also

dextrorotatory, and the iodide recovered from it had $[\alpha]_D = 25{\cdot}7°$ in alcohol.

Hence it is clear that two iodides exist in the original product, and though a partial separation only was effected by crystallisation a complete separation is clearly possible by means of the camphor-sulphonates. The melting points and the solubilities in alcohol of the two iodides differ to a slight extent only.

The rotatory power of chloroform solutions of these iodides alters very rapidly, roughly about 10 times as rapidly as similar solutions of the active nitrogen compounds described by Pope and Harvey, and by the author, and the allyl compound above described were found to do. Both iodides finally attained a state of equilibrium with a feeble dextrorotation, about ·14° for a 3 per cent. solution in a 2 dcm. tube corresponding to $[\alpha]_D = 2{\cdot}6°$.

The change here is undoubtedly due to the salt splitting up into amine and benzyl iodide, which then reunite to form both forms of the iodide in the relative quantities which are in equilibrium with one another in chloroform solution, the change being brought about in the same way as the racemisation of the active compound already referred to. That there is a dissociation with liberation of benzyl iodide is evident, because its odour is very noticeable in the chloroform solutions and in the residue which they leave on evaporation.

These experiments which are merely preliminary are being continued and extended. The effect of temperature on the relative amounts of the two isomerids produced in alcohol is very slight; the iodide formed at various temperatures from 0° C. to 35° C. had practically the same specific rotatory power. A magnetic field on the other hand was found to have a decided effect; in alcohol solutions of the same concentration the iodide produced without a field gave $[\alpha]_D = 4{\cdot}3°$, in a weak field $[\alpha]_D = 5{\cdot}4°$ and in a stronger field $[\alpha]_D = 7{\cdot}5°$. This effect is of considerable interest and is being fully investigated in this particular case and for a number of other reactions.

Quasi-Radio-activity produced by the Point Discharge. By S. A. EDMONDS, A.R.C.Sc.I., "1851" Exhibition Science Research Scholar, St John's College.

[*Received and read* 16 May 1904.]

Sella has shown* that when a metallic body is connected to one terminal of an electrostatic machine, and made to face a series of points connected to the other terminal of the machine, and the discharge passed, without sparking, between them, for, say, an hour, the body acquires an induced radio-activity if the discharge has taken place in fresh air, and not the air of a room closed for some time, while the presence near the body of thorium, during the discharge, greatly enhances this radio-activity. The following experiments were undertaken to discover the origin of this activity.

A series of discs were made of the metals aluminium, lead, copper, zinc, iron, and brass, and also of cardboard lined with tin foil and platinum foil respectively, their diameters being all 10 cms., and thicknesses 2 mm.; a small hole, bored at the centre of each one and recessed slightly, served to take a screw by means of which they could be fixed, either to the terminal of the 6 plate Wimshurst machine used in the experiments, or to the wire for testing their activity.

Another cardboard disc was prepared, lined with tin foil, and 12 needles thrust through it, their eye-ends being embedded in a block of paraffin wax. This disc was mounted on a small insulating upright, and placed so that the points of the needles were some 10 cms. removed from the metal disc, at which distance the discharge, in passing, just showed small brushes at the points, the potential of which, as measured by an electrostatic voltmeter, was 10,000 volts.

The method used for testing the radio-activity is shown in the diagram.

A zinc cylinder *C, C*, of diameter 16 cms., was obtained, and at its ends, one of which was removable, ebonite plugs *D, D* were fitted, carrying brass rods, to one of which could be screwed the metal disc to be tested, the other one carrying a smaller brass disc with a guard-ring *G* connected to the cylinder *C* which was earth connected.

The electrode *B* could be connected with the electrometer *E*, an ordinary quadrant with single quartz fibre suspension and

* *Accad. Lincei Att.* II. p. 57, Jan. 19; pp. 292—295, Mar. 16; pp. 369—373, May 4, 1902.

sulphuric acid jar, by means of the key F, a weighted dropping brass rod W, enabling it to be earthed when necessary; the other pair of electrometer quadrants being earthed.

The disc A was connected to one pole of the battery By, of 180 secondary cells, the other pole of which was earthed.

For the majority of the experiments the needle of the electrometer was kept at a potential of 40 volts, giving a deflection of 200 scale divisions [mm. at 1 metre from the mirror] for 1 volt difference of potential between the electrometer quadrants.

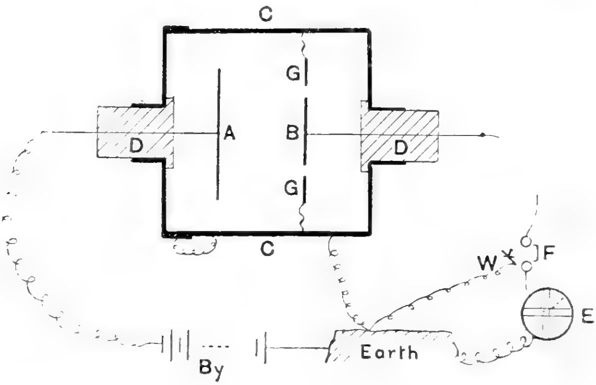


Fig. 1.

A disc was taken and placed facing the points and subjected to the discharge for an hour, after which it was disconnected, and a powerful air blast caused to play upon it for 5 minutes to remove any ions which might be in its neighbourhood. It was then placed in the cylinder, the electrode B and its pair of quadrants being earthed, and connected to the terminal of the battery; on insulating B a charge was communicated to it of the same sign as the potential of A, and the electrometer needle received a deflection which increased rapidly for the first few minutes, but eventually became stationary at the expiration of half-an-hour from the time at which B was insulated.

Each of the discs in turn was thus tested, and the resulting deflections all fell between the limits of 78 and 80 scale divisions, being thus identical within the errors of experiment.

On the face of each disc, after a discharge, was seen a deposit which was darker at the places immediately opposite the points, and very light at intermediate places. Tests made on this deposit showed the presence of slight traces of iron when needles were used as points, the "Prussian blue" precipitate being obtained, while when other metals—copper, brass—were used, very faint

traces of their presence were obtained from the various deposits produced by them.

It was noticed that with slight deposits the activity was less than was the case with denser deposits, while, in one case, on accidentally wiping off the deposit, very little activity was found at all, which suggested that the deposit was in some way responsible for this radio-activity.

In all cases in this series it was noticed that when the disc was at the negative terminal during the discharge it would, on being fixed in the testing vessel, only cause a leak of positive electricity, while when reversed and made the positive terminal for the discharge, it would only leak negative when connected up in the testing vessel, the maximum deflections after reaching the stationary state at the end of half-an-hour being respectively between 82 and 85 when the disc leaked negative, and 78 to 80 when it leaked positive electricity.

With a view to testing the theory as regards the deposits, a series of experiments were made in which the discharge passed between the points and the disc in the open air, they being fixed up on the window-sill in the same positions as before. Very slight deposits indeed were obtained by this treatment, and on testing in the cylinder the effects produced were extremely small, the observed deflections at the end of the half-an-hour being respectively 7 for a negative leak, and 6 for a positive leak, the different metals behaving equally well, no difference being observable in the various deflections obtained.

In one of these experiments with an aluminium disc a fall of snow occurred during the discharge, but practically no difference could be observed in the deflection yielded by this disc when in the cylinder.

A wooden box was next procured having a capacity of 55 litres, and its walls covered with paraffin wax to close up any small cracks present; insulated uprights of ebonite being fixed in position to carry the points, and the brass electrode, to which the discs were screwed, and connecting wires from both were led out through wax plugs in the side. Soft wax was placed round the edges, and the box sealed by screwing the cover down upon this wax.

After the box had been closed for some 16 hours, giving any dust particles in the air inside time to settle down, the discharge was passed for an hour, the box opened, and the disc blown upon and connected up in the test cylinder, when it was found that, after half-an-hour, the deflection was only 3 divisions. The same effect was noticed for all the metals, whether they were connected to either terminal of the machine, and no deposit whatever could be observed upon any of the discs.

To further test this point, connections were made both with the open air and a water pump, by which air could be sucked through a long lead composition pipe, earth connected, over some phosphorus pentoxide, and through a long glass-wool plug into the box, which thus became filled with pure dry air free from dust.

On causing the discharge to pass between points and disc in this air, practically the same effect was produced as when the box had been closed for some time, the deflection produced when the discs were placed respectively in the testing vessel being 2·5 divisions, the sign of electrification of the disc having no effect whatsoever in altering this deflection.

The same results were obtained for similar experiments to the preceding when the points were made of other metals—brass and copper—suggesting that the phenomenon noticed is not a property peculiar to one of the metals.

By a slight modification of the apparatus it was possible to ascertain if any activity had been induced upon the points, but no difference could be ascertained between the rates of leak obtained with or without the points being connected to the electrometer quadrants, which were charged up to the potential of one Clark standard cell for purposes of testing.

The capacity of the apparatus was measured by carefully insulating it on paraffin wax blocks, and comparing it with the capacity of an adjustable standard air condenser, and was found to be, in electrostatic units, 7·5 cms., for the voltage used throughout the experiments.

In the first series of experiments, in which this quasi-radio-activity was shown, the deflections rose to one-half their final value at the expiration of 2 minutes from the start, so that the discs were giving off ions during that time at the rate of 110 per c.c. per second for the positive leak, and 121 per c.c. per second for the negative leak.

A series of experiments were made, in which different times were allowed to elapse between the passing of the discharge and the testing of the discs, and it was found that the various discs lost their activity almost entirely at the expiration of an interval of 3 hours between the two operations.

These experiments show that this quasi-radio-activity, produced by the action of the point discharge, is due to the presence of dust particles in the air. These dust particles come into contact with the stream of ions present, moving from the points to the plane, carrying the discharge, and, when they become deposited on the plane, trap some of these ions, and hold them, so that they can only enter very slowly indeed into the metal of the disc under the force of attraction they experience by virtue

of the charges carried by them. The disc thus has a film of ions held in the dust deposit on its surface so that, if it be placed in the testing vessel and raised to the potential employed in the experiments, the electric field produced will act upon the ions thus trapped and increase their energy, thus enabling them either to go out from the disc and into the electrode facing it, thereby increasing the charge in the electrometer, or else to go into the metal of the disc, and so show no external effects. When the disc is at the positive terminal during the discharge from the points it will attract negative ions to itself, so that, in this case, some of these negative ions should get trapped by the dust, to be liberated when the disc is placed in the testing cylinder and raised to a negative potential, while, on the other hand, when the disc is at the negative terminal of the machine, it will attract to itself positive ions, and consequently will, when placed in the testing cylinder, leak positive electricity.

In conclusion, I desire to express my deep appreciation of the many helpful suggestions given during the progress of this investigation by Prof. J. J. Thomson, F.R.S.

On the Early Development of the Unfertilized Egg in the Sawfly, Nematus ribesii. By L. DONCASTER, M.A., King's College.

[*Read* 2 May 1904.]

The work of which a preliminary account is here given was undertaken in order to determine (1) what is the function and fate of the polar bodies in a parthenogenetic insect; and (2) whether the early development is different in nearly allied species which give rise to males or females respectively from virgin eggs. The work of Petrunkewitsch*, and the interpretation of it suggested by Castle†, make it possible that the polar bodies are concerned with the determination of sex, at least in the Honey Bee, and no group of animals seems better adapted for testing this hypothesis than the Sawflies, many of which lay eggs which develop equally well whether fertilized or not, but produce different sexes in the two cases.

The only species which I have yet been able to study to any large extent is *Nematus ribesii*, in which, as is well known, virgin eggs produce only, or almost exclusively, males, while eggs from impregnated females yield a large percentage of females, with a varying number of males. That this difference is due to differential mortality is not certainly disproved, but the evidence makes it improbable; since, however, my experiments on this matter are not yet complete, it must be left for the present.

The eggs are laid in rows, on the veins of the under side of currant or gooseberry leaves, and were preserved in a modification of Gilson's fluid at short intervals, so that all stages from quite fresh eggs up to those ten hours old were obtained. They were cut in position on the leaf and stained with Heidenhain's Iron Haematoxylin.

In a nearly fresh egg the nucleus is found embedded in a little patch of protoplasm among the yolk on the dorsal side near the anterior end of the egg. This patch of protoplasm I propose to call for brevity the "polar protoplasm." In a few minutes the nucleus begins to divide, forming a spindle perpendicular to the edge of the egg, and in the anaphase it is seen that the spindle fibres are much thickened about the middle of their length. When the chromosomes have reached the poles they do not form nuclei, but two new spindles immediately form, lying in the same line as the previous one. Frontal sections of this stage show that the number of chromosomes after reduction is either seven or eight; some sections suggest one number, others the other, a fact

* Petrunkewitsch, *Zool. Jahrbücher Anat. und Ontog.* XIV. 1901, and XVII. 1903.

† Castle, *Bull. Zool. Mus.*, Harvard, 1903, Vol. XL.

which may possibly be of significance, or may be due to the difficulty of counting, as they are extremely small and much crowded.

In *N. ribesii* I have been unable to find centrosomes in the maturation mitoses, although in *N. lacteus* a body appears which may be of this nature, but in the somatic mitoses some hours later the centrosomes are conspicuous both in virgin and fertilized eggs.

From the two spindles described four nuclei arise, lying in the polar protoplasm in a line perpendicular to the edge of the egg, and at roughly equal distances from one another. They all become reticular, with a well-marked membrane, and the two outer ones are the daughter-nuclei of the first polar body, the third is the second polar nucleus, and the innermost the egg-nucleus (female pronucleus). The outermost soon becomes flattened against the edge of the egg, and rapidly disappears. The second and third move toward one another, meet, and become flattened against each other; their membranes disappear and the chromatin reappears as chromosomes, which become mingled into a single group*. No radiations are seen in the surrounding protoplasm. Meanwhile the egg-nucleus has sunk deeper into the egg, leaving the polar protoplasm and moving inwards and forwards among the yolk granules. In all points therefore the fusion of the second polar nucleus with the inner half of the first, and the travelling away of the egg-nucleus, resembles the process described by Petrunkewitsch in the virgin egg of the Honey Bee.

The stage now described is reached about two hours after the egg is laid, and the subsequent changes in the "conjugation-nucleus," as it is called by Petrunkewitsch, during the next eight hours are comparatively small. The chromosomes have formed a compact group, containing from 14 to 16, the exact number being still uncertain, but they usually soon begin to separate themselves into two groups lying at varying but small distances apart, in the polar protoplasm. At a rather later stage ($3\frac{1}{2}$ hours) in one at least of these two groups each chromosome sometimes appears to be double, and still later ($5\frac{1}{2}$ hours) some preparations show twice as many chromosomes in one group as in the other, or there may be three groups each containing about eight. Beyond this no change takes place up till the tenth hour, at which time the blastoderm is beginning to form, and the egg becomes so full of nuclei in various stages of division that I have not yet succeeded in tracing the fate of the "copulation-nucleus" further.

Returning to the egg-nucleus immediately after the second maturation mitosis, we find that after travelling inwards and forwards through the yolk for some distance, it begins to divide, and

* At about this stage in some sections an evenly and faintly stained round body, rather smaller than a nucleus, is seen in the polar protoplasm. Its nature is at present unknown to me.

soon the yolk contains a number of scattered nuclei, each embedded in a little mass of protoplasm. The spindles of division are extremely small, but in the few cases where I have been able to count the chromosomes they appear to be about eight, *i.e.* they remain at the reduced number. But the reticular nuclei immediately after division contain a number of deeply-staining granules, often about twenty, and it seems possible that this appearance has led to the statement of Petrunkewitsch, that in the bee the chromosomes of the egg-nucleus automatically become doubled. The nuclei in the yolk increase rapidly, and at about the tenth hour begin to come to the surface and form a blastoderm, but the time of its first appearance varies greatly with the temperature. Only in the blastoderm nuclei have I seen anything like a nucleolus.

Of eggs from impregnated females my series is not yet complete, and unless the male pronucleus can be found it cannot be proved that the egg is actually fertilized, even though the female sawfly has been seen to copulate. The early stages appear to be quite similar to those of the virgin egg, but in the third and subsequent hours the chromosomes derived from the conjugating polar nuclei, instead of forming fairly definite compact groups, are usually scattered quite irregularly in the polar protoplasm, and by the sixth hour appear to be disintegrating. In these eggs I have not yet found a mitosis of a yolk-nucleus in which the chromosomes can be counted, nor have I been able to observe the fusion of male and female pronuclei.

The work is still very incomplete, since the early stages of the fertilized egg have not yet been fully followed, and in no case has the fate of the polar nuclei been traced beyond the beginning of the blastoderm stage. It would therefore be premature to discuss the meaning of the facts observed, but incomplete as they are they are of interest from their remarkably close resemblances to the processes observed by Petrunkewitsch in the bee. The chief differences are that in the sawfly the maturation divisions take place in the dorsal instead of the ventral side of the egg, and that the "copulation-nucleus" does not divide to form a group of nuclei, but forms instead groups of isolated chromosomes.

In conclusion I may mention that in the few eggs of other species which I have obtained, viz. *N. pavidus* and *N. lacteus*, the development appears to be closely similar to that described in *N. ribesii.* Of these species the first certainly and the second probably belongs to the group in which virgin eggs yield males.

Finally I wish to express my gratitude to Miss E. F. Chawner, of Lyndhurst, for sending me a large and varied supply of larvae; without her assistance the work could hardly have been undertaken.

Effect of Screening on Ionisation in Closed Vessels. By A. Wood, Emmanuel College.

[*Received* 20 May 1904.]

"Spontaneous" ionisation in closed vessels has recently been the object of much investigation. C. T. R. Wilson*, as the result of experiments on the ionisation of hydrogen, air and carbon dioxide in small vessels, came to the conclusion that at least some of the ionisation was due to a radiation coming from the walls of the vessel and of only slight penetrating power. This conclusion was further supported by some experiments of Strutt†, who obtained very different values for the ionisation in vessels of the same size but of different materials. On the other hand M^cLennan‡ and Rutherford and Cooke§ have shown that the ionisation is also to some extent due to a very penetrating radiation from without which passes through the walls of the vessels. The question at once arises whether this radiation from the walls of the vessel is primary or secondary in character—whether it is due to a radio-activity of the material or whether it is a secondary radiation excited by the penetrating one. In the latter case one would expect the radiation from the walls of the vessel to be cut down in the same ratio as the penetrating radiation by a lead or other screen, and so one would expect the same screen to produce the same proportionate reduction of the ionisation for all vessels of the same dimensions, whatever the material of which they are constructed. If, on the other hand, the radiation from the walls or any considerable fraction of it be due to a radio-activity of the material it will be independent of the penetrating radiation, and so unaffected by a screen. The proportionate reduction of ionisation due to any screen would thus vary greatly for different materials, being least in the case of vessels of comparatively great radio-activity. It was with the intention of obtaining information on this point that the later experiments described in this paper were undertaken.

The apparatus consisted of a cylindrical metal vessel, 6·25 cm. in diameter and 13 cm. high, placed on an insulating stand and kept at a potential of about 300 volts by means of a battery of small storage cells. A stout brass wire fixed in a sulphur plug was used as an electrode and was connected to an electroscope of the type described by C. T. R. Wilson‖. Surrounding the sulphur and electrode was a brass tube connected to earth in order to obviate all possibility of leakage from the vessel to the electrode

* *Proc. Roy. Soc.* Vol. LXIX. p. 278.

† *Phil. Mag.* June, 1903.

‡ *Phys. Rev.* XVI. p. 184; *Phil. Mag.* V. p. 419.

§ *Amer. Phys. Soc.* Dec. 1902.

‖ *Proc. Camb. Phil. Soc.* Vol. XII. Pt. ii. p. 135.

across the insulation. This brass tube was fitted into a vulcanite stopper which served to insulate it from the vessel, and the electrode and insulation could thus be moved from one vessel to another, and the ionisation in each directly compared, the vessels being of the same size and the capacity of the electrode system remaining the same. To the end of the brass wire which carried the gold leaf of the electroscope was attached a small mercury cup into which dipped the electrode wire. This cup and the connecting wires were shielded from electrostatic influences by a screen of wire gauze. In this screen a small hole was made exactly above the mercury cup, and through it could be inserted into the cup one end of an insulated wire, the other end of which was connected to a potentiometer arrangement. By this means unnecessary insulation usually associated with a key was avoided, the insulation being confined to one sulphur plug where the electrode passed into the metal vessel and another where the wire entered the electroscope. The method of measuring the ionisation was as follows. The potentiometer wire was inserted into the mercury cup and the electrode system brought to zero and then insulated. After twenty minutes had elapsed the position of the gold leaf was read by means of a microscope. The potentiometer wire was reinserted in the mercury cup and the resistance adjusted until the gold leaf was brought back to the same position on the scale. The potentiometer reading then gave the final potential of the electrode and this was a measure of the leak in the vessel and consequently of the ionisation of the air. This method of reading the electroscope gave results independent of small changes of zero and of variations of sensitiveness in different parts of the scale. With the volume of air used—about 400 c.c.s—it was found that the electrode was charged to about 0·4 volt in twenty minutes, an amount which could be measured with a considerable degree of accuracy. The potential of the vessel was 360 volts, which was found to be largely in excess of that required to saturate the enclosed air. In the actual experiments five sets of four readings each were taken—the first, third, and fifth sets being taken in the ordinary way, and the second and fourth being taken while the vessel was surrounded with a lead or other screen. From these determinations the mean leak with and without the screen was calculated and the percentage reduction found. The first experiments were of a purely preliminary character and the results obtained were in agreement with those previously published by Rutherford and McLennan, as also with those published by Cooke* while the present investigation was in progress. Screens of lead, iron, wood, and paper were tried. The lead screen was about ·6 cm. thick and gave a

* *Phil. Mag.* Oct. 1903.

reduction of the ionisation amounting to about 12 °/。 of the normal. The iron screen which was about 5 cm. thick gave a reduction of about 20 °/。. The effect of the screens of wood and paper was much smaller, as might have been expected. In the case of the paper screen, which consisted of books, it was a reduction which was too small to be measured and could only just be detected. The wooden screen was about 15 cm. thick and gave a slight increase. This was no doubt due to some radiation from the wood similar to that observed by Cooke in the case of bricks, as in both cases it was found that the increase was annulled by the interposition of a lead screen 1 mm. thick. In the subsequent experiments the vessel in which the ionisation was measured was varied, vessels of lead, tin, iron, aluminium, and zinc being used. All the vessels were of the same dimensions. It was of course found that the ionisation varied very much with the material of the vessel. The following table gives the average values obtained for the various metals, the scale being arbitrary. For purposes of comparison the corresponding values obtained by Strutt* are placed alongside. His numbers are doubled to render comparison easier.

Material	Ionisation	Strutt's Value
Lead	5·2	4·4
Tin	4·7	4·6 and 6·6
Iron	2·9	—
Aluminium......	2·7	2·8
Zinc	2·3	2·4

The ionisation was also measured in each of those vessels while it was surrounded with a lead screen. The screen was about 1·3 cm. thick, and the reduction of the ionisation effected by it in the different cases is shown below.

Material	Ionisation without screen	Ionisation with screen	Reduction	Proportionate reduction
Lead	·521	·454	·067	13 °/。
Tin	·496	·384	·112	22 ,,
Iron	·287	·219	·068	23 ,,
Aluminium ..	·254	·220	·034	13 ,,
Zinc	·232	·176	·054	24 ,,

* *Loc. cit.*

It will be seen that there are large differences, both in the absolute amount of reduction effected and in the ratio of that reduction to the normal ionisation. Similar experiments were tried with an iron screen, the average thickness of which was about 5·5 cm. The same vessels were used and the following results obtained:

Material of vessel	Ionisation without screen	Ionisation with screen	Reduction	Proportionate reduction
Lead...........	·522	·474	·048	9 °/。
Tin.............	·500	·436	·064	12 ,,
Iron...........	·298	·226	·072	24 ,,
Aluminium..	·287	·244	·043	14 ,,

From these figures it appears that the iron screen chosen gave almost the same proportionate reduction as the lead one previously used. The only marked exception is tin, the proportionate reduction in this case being only 12 °/。 as against 22 °/。 produced by the lead screen. The observations were repeated but with the same result.

Incidentally the experiments here described confirm Strutt's* results, as has been pointed out, but their main object was to throw some light on the part played by the metal of the vessel in ionising the gas and the nature of the radiation due to it. The results obtained show that the proportionate reduction due to a given screen is very far from being the same for vessels of different materials. While not finally conclusive, this fact is strong presumptive evidence that the ionisation in a closed vessel is in part produced by a radiation from the walls which is independent of the penetrating radiation from without. This intrinsic radiation is great in lead and aluminium and small in iron, zinc, and tin. The subject is being still further investigated by Campbell, a short account of whose results appeared in *Nature* (March 31st, 1904). By varying the volume of his vessels he has been able to determine the amount of the ionisation which is due to the walls and the reduction which this suffers from a lead screen. From unpublished results with which he has kindly furnished me, it appears that in the case of lead and aluminium this radiation from the walls is practically unaffected, while in the case of zinc and tin the reduction is considerable. This confirms the conclusion arrived at in this paper that the radiation from lead and aluminium is mainly intrinsic, while that from zinc and tin is largely secondary.

* *Loc. cit.*

On the Proportion of the Sexes among the Todas. By R. C. Punnett, M.A., Gonville and Caius College. With a Note by W. H. R. Rivers, M.A., St John's College.

[*Received* 16 May 1904.]

The data given in the present paper were extracted from the genealogical statistics collected by Dr Rivers during his recent stay among the Todas, and I am greatly indebted to him for allowing me to make use of them previous to publication, as well as for sundry information concerning this people. From these genealogical tables I have been able to obtain details of 359 families comprising 1,124 individuals, the average size of a family being 3·1. By a family is to be understood the number of children born of the same mother, though not necessarily indebted for existence to the same father. The data extend back over some 80—90 years, those for later generations being the more extensive. As the two Toda tribes, Târthârol and Teivaliol, do not intermix, they have been considered separately. The data for the Târthârol tribe are more full, comprising 249 families with 773 individuals, as against the smaller Teivaliol tribe with 351 individuals in 110 families. In every family the order of individual births is known.

On sorting out the sexes, the results obtained (see Table I.) confirm the conclusion arrived at by previous writers, viz., that

TABLE I.

	Târthârol	Teivaliol
Number of ♂s	451	236
Number of ♀s	322	115
♂s per 100 ♀s	140	205
Average size of family	3·1	3·2

the males are greatly in excess of the females. This holds good for both tribes, though the excess is considerably more marked in the case of the Teivaliol Todas.

Several explanations have been put forward to account for this phenomenon. It has been suggested that the preponderance of living males in polyandrous societies is due to a great excess of ♂ births resulting from defective nutrition, and, according to Westermarck*, "much stress must be laid on the fact that polyandry prevails chiefly in poor countries." The evidence† by which he supports his contention is however not very convincing, and I have recently given figures‡ which do not strengthen the hypothesis that sex determination depends upon nutrition. Moreover on this hypothesis it is not easy to account either for the great fluctuations in the proportions of ♂s to ♀s found in different generations (see below p. 483, Table II.), or for the fact that the proportion of ♂s among the Todas is enormously greater than among the lowest classes of civilised peoples where the nutrition is probably equally deficient. A second cause which has been suggested to account for the excess of ♂s is in-breeding, and Westermarck§ quotes Düsing in favour of this hypothesis, and supports it by cases where mixture of race tends to produce an excess of ♀s. Here again the information given is on the whole somewhat too vague for a definite conclusion. Moreover it is not supported by such detailed evidence as I have been able to collect. On the islands of Mabuiag and Badu|| in Torres Straits, where there exists a considerable amount of in-breeding, the ♂s preponderate only to a small extent, a state of things which holds good for Murray Island also. The balance of evidence is on the whole against either of the above hypotheses affording an adequate explanation of the excessive preponderance of the ♂ sex among the Todas.

There yet remains for consideration a third factor, viz., female infanticide. All who have studied the Todas are agreed upon the frequency of this practice in earlier times. Marshall¶, writing in 1872, refers to the large amount of female infanticide in former years but expresses his conviction that the practice had by that time died out. Marshall's evidence is that of native assurance only. Dr Rivers, who received the same assurance, is disinclined to place much confidence in native veracity with reference to this point, and in view of the lack of encouragement which the practice receives from the Indian Government this is not altogether

* *The History of Human Marriage*, 3rd edition, 1901, p. 474.

† *Loc. cit.* pp. 470 *seq.*

‡ *Proc. Camb. Phil. Soc.* Vol. XII. p. 262 seq.

§ *The History of Human Marriage*, 3rd edition, p. 476.

|| *Proc. Camb. Phil. Soc.* Vol. XII. p. 271. Cf. also *Rep. Cambridge Anthropological Expedition to Torres Straits*, Vol. V. p. 233. Reference may be made here to a recent paper by O. Schultze in the *Archiv f. Microscop. Anat.* 1903. Working with white mice Schultze found that neither starvation nor in-breeding affected the proportion of the sexes.

¶ *A Phrenologist among the Todas*, 1873, p. 198.

surprising. The supposition of ♀ infanticide, by accounting for the great disproportion in the numbers of the sexes, brings the Todas into harmony with what is known of the rest of mankind, whilst it receives strong support from the following considerations.

The genealogical system of registration adopted by Dr Rivers has enabled me to estimate the proportion of the sexes and the average fertility for several successive generations, and this independently for both the Târthârol and Teivaliol Todas. The two tribes have each been divided into four groups A—D, representing approximately four generations. These are:—

A. Families where the eldest child would be now (1903) over 90.
B. " " " " " between 60 and 90.
C. " " " " " " 30 and 60.
D. " " " " " " 0 and 30.

The results are given in Table II.

TABLE II.

	Târthârol			Teivaliol		
Group	No. of families	Average size of family	♂s per 100 ♀s	No. of families	Average size of family	♂s per 100 ♀s
A	9	3·0 [4·2]	237·5	4	4·5 [6·0]	200
B	49	4·1 [5·0]	159·7	21	3·8 [5·4]	259
C	87	3·3 [3·7]	131·4	40	3·8 [5·0]	202
D	104	2·5 [2·8]	129·2	45	2·3 [2·9]	171

Neglecting Group A, where paucity of numbers renders the figures somewhat untrustworthy, the series B—D tell the same story for both tribes. There has been a steady decrease in the proportion of ♂s accompanied by an equally steady diminution of fertility. As it may be objected that the figures for Group D are unnecessarily low owing to the presence of families not yet complete, I have recalculated the average size of the family in this group, omitting all families where the age of the youngest child is below 10 years of age. When this was done 33 such Târthârol families gave an average of 2·5 and 22 Teivaliol families an average of 2·4. As these figures are practically identical with the whole of Group D in each case, it seems permissible to conclude that the tendency to sterility is becoming checked at the

32—2

present day*. To return however to the main features of Table II., since neither the country nor the marriage customs of the Todas have changed appreciably during the last half century, we can explain these figures only by supposing that there has been a steady falling off in the practice of ♀ infanticide during recent years. Even in the last generation (D) the preponderance of ♂s is very considerable, and lends support to the view that the practice is still prevalent, though not nearly to the same extent as in former years†.

The preservation of a larger proportion of ♀ infants should naturally lead to a larger family, and it is somewhat unexpected to find so marked a decrease in fertility of late years, especially when the masking effect of ♀ infanticide in earlier generations is taken into account. This effect I have endeavoured to eliminate by taking the number of ♂s as a basis and estimating the size of the family in each case on the supposition that there is numerical equality among the sexes at birth. The figures so obtained are enclosed in square brackets in Table II. and placed beside the rest for purposes of comparison. They serve to emphasize the very marked decrease in fertility which has occurred in recent years.

Before proceeding to discuss the cause of this progressive sterility, we may pause for a moment to consider some of the statistics collected by Marshall. On page 197 of his book on the Todas this author has given a table showing the number of ♂ and ♀ offspring belonging to females of different ages at the

* In connection with this argument my friend Dr C. S. Myers has made the very pertinent criticism that in shortening the period taken by 10 years I may be actually selecting for sterility. The two following considerations however seem to me to tell against this possibility: (α) When group C of the Tâthârol tribe is treated in the same way, and those families selected in which the age of the eldest member is not greater than 60 and of the youngest not less than 40, the average size of the family (out of a total of 32) works out to 3·4, and on the ♂ basis (see above) to 4·0. Making allowance for irregularities due to the small number of families these figures are very near to the 3·3 and 3·7 of Table II. There is certainly no evidence of the selection of sterile families by this method. (β) Though it is not possible to give actual figures there seems little doubt but that the fertility span is considerably less than 20 years. It is therefore unlikely that reduction of the period considered from 30 to 20 years implies any selection for sterility.

Here it may be mentioned that the figures given in this paper for the average size of the Toda family and for the fertile period do not agree with those given by Marshall. This author reckons the average size of the family (excluding sterile marriages) at 6·7, and the fertile period of the women at 19·6 years (p. 122). These figures are however based on the progeny of 17 women only (Table VII. p. 120), and doubtless they present an inadequate picture of the real state of things. This is at once apparent in the results which Marshall obtained when he attempted to forecast the growth of the Toda population from his data. Taking the number of the Todas as 713 in 1870 he estimated that it would in 1902 become 2852 (p. 107), an estimate more than thrice what Dr Rivers actually found in that year. There can be little doubt therefore that the data given in this paper are to be preferred to those of Marshall.

† The possibility of ♂ infanticide is of course conceivable, but as I know of nothing to support it, there is not much point in dwelling upon it.

time of his census. These figures, in which both living and dead offspring were reckoned, I have condensed into the following table.

TABLE III.

	No. of families	No. of ♂ s	No. of ♀ s	Total children	♂ s per 100 ♀ s	Average size of family
♀ s aged 42—65	18	69	45	114	153·3	6·3
♀ s aged 18—40	20	36	29	65	124·1	3·2

From this table the conclusion is obvious that for ♀s of the later generation (aged 18—40) both the proportion of ♂s shows a marked diminution as compared with ♀s of the earlier generation (aged 42—65), thus affording a striking confirmation of the results deduced from the present data (Table II.)*.

Returning now to the cause of this sterility, Dr Rivers has suggested to me two possible factors, viz., syphilis and immorality. The introduction of syphilis among the Todas is comparatively recent, and, in spite of the absence of skilled medical treatment, its outward and visible signs are rarely to be met with. Dr Rivers considers that, whilst not of sufficiently common occurrence to account for the low fertility, it may be in some measure a contributing cause. Doubtless we must turn to the other suggested factor for a partial explanation of the observed sterility. One of the most marked effects which more frequent intercourse with the outer world has produced among the Todas is the low standard of morality practised by the women in matters sexual. Extended polyandrous association would appear to have blunted the feminine instinct of comparative chastity characteristic of monogamous peoples. Habitual cohabitation with several males would seem to have rendered the women indifferent as to the number to whom they granted their favours when lessened isolation enlarged the scope of their opportunities. Consequently it is likely that in increased intercourse with the extra-Todan world we must look for the cause both of the diminution in the proportion of ♂s and also of the increase of sterility. Now the Târthârol tribe has been subject to external influences to a greater extent than the Teivaliol.

* The average size of the family also shows a great reduction in the younger ♀ s as compared with the older (3·2 : 6·3). Much stress however cannot be laid on this as the average age of the younger group of ♀ s was only 28·2, and some of them would undoubtedly have borne more children. In view of the shortness of the fertile period and the early age of marriage among the ♀ s it is almost certain that the average size of the family in this group would in the end have been markedly less than that of the older group, indicating here also progressive sterility in the race.

We should therefore expect the phenomena of sterility and diminished proportion of ♂s to appear earlier in the Târthârol tribe. Reference to Table II. shows that such is actually the case, a result which lends support to the view that these phenomena are due to intercourse with the outer world.

And this view receives additional support from the following considerations. The Târthârol tribe is split up into a number of clans, certain of which have been more influenced by external influences than others. From these clans Dr Rivers selected seven, of which four were comparatively uninfluenced, one partly, and the remaining two very considerably influenced. In each of these clans I estimated the number of children in families where the eldest was not more than 60 and the youngest not less than 10 years of age.

Name of clan		No. of families	No. of offspring	Average size of family	Average size of family for group
Nodrs	(uninfluenced)	14	54	3·84	3·59
Pun		8	21	2·37	
Taradr		9	43	4·77	
Kunodrs		11	33	3·00	
Kars (partly influenced)		25	76	3·04	3·04
Pam	(much influenced)	10	22	2·20	2·60
Nidrsi		10	30	3·00	

Though the figures are somewhat irregular, yet on the whole they bear out the view taken above, *i.e.*, that sterility is largely due to the immorality that follows increased intercourse with the outer world.

In conclusion the following speculation suggests itself. It seems probable that, unless the fertility of the people increases, the Todas are doomed to extinction in the not very remote future. May we not have in this decreased birth-rate, following upon the establishment of foreign intercourse, a partial explanation of the rarity of polyandry? May we not regard polyandry as a lower form of human sexual association dependent for its existence upon isolation, and one that through its inherent defects must of necessity perish when brought into contact with the higher form of monogamy? But this is a point which must be decided by the deeper knowledge of the professed anthropologist.

Summary.

(1) Among the Todas the ♂s preponderate greatly over the ♀s.

(2) This preponderance is doubtless due to the practice of ♀ infanticide, which is probably still to some extent prevalent.

(3) The numerical preponderance of the ♂s has been steadily sinking during recent years, owing probably to the check which foreign intercourse has imposed upon ♀ infanticide.

(4) The fertility of the race has been steadily growing less during the last few generations.

(5) This is probably due to the immorality of the women incident upon increased intercourse with the outer world.

Note by W. H. R. Rivers, M.A.

The genealogical records which form the basis of Mr Punnett's paper were collected by me in order to work out the details of the social organization. There are certain features of the records which must be borne in mind when appraising the value of evidence derived from them.

People such as the Todas remember the pedigrees of their community because the knowledge is necessary for the proper working of social regulations. In low stages of culture an individual only becomes important when he marries, and if a child dies before marriage there is some danger that its existence may be forgotten and its name omitted from the pedigree of its family. Many such children are remembered by the Todas, in some cases for three or four generations, but it is almost certain that some must be forgotten, and in so far as this is the case the pedigrees are vitiated as a record of vital, as opposed to social, statistics. If such children have been omitted, the omissions would probably have been more numerous in earlier than in later generations, and if this has been the case the tendency to sterility of which the genealogies bear evidence would therefore come out more strongly. Even if the record should be defective in this respect it is improbable that the main problem of Mr Punnett's paper—the proportion of the sexes—would be affected, for there is no reason to suppose that there would be any special tendency to forget children of one sex rather than of the other.

A more serious source of error is present in connexion with the latest additions to the population. Owing to the existence of a taboo on the names of dead relatives, I was obliged to obtain my information about any given family from the members of other families. No one gave me the names of his own immediate progenitors, and it often happened that I obtained the whole record from a member of another family. Here again the younger members of the community before the marrying age of five or six years may have been omitted. I have little doubt that this is an

actual source of error. On drawing up a list of the population from my genealogical tables, I find that there are fewer children between birth and five years of age than between five and ten, or between ten and fifteen, years of age. Further, my total population as calculated from the tables falls short of the number obtained in the official census of 1901. My record of adults and married children is almost certainly complete, and the discrepancy between the official record and my own is probably due to deficiencies in my record of young children.

This error would only affect the latest of Mr Punnett's groups, and he has for other reasons given figures in his paper which enable different generations to be compared when children under ten years are omitted. In these figures this source of error will have been excluded. If the error is present in the genealogies, its correction would accentuate the tendency to increase of population which Mr Punnett believes to exist. If the conditions are as Mr Punnett believes, the Todas have gone through a period of decreasing fertility from which they are now emerging and this would be in agreement with what seems to be happening elsewhere among communities of low culture. It seems probable that on the introduction of the various influences comprised under the term "civilization" population falls, to rise again when the community becomes adapted to the new conditions.

Records of the Toda population, official or semi-official, have been taken since the year 1812, but all the records before the year 1866 are certainly untrustworthy. In 1812 Keys estimated the number of Todas as only 179, and a census in 1856 only gave 316 individuals. In 1866 the number had gone up to 706*. At the census of 1871 the population was found to be 683; in 1881, 675; in 1891, 739; and in 1901, 805. It is probable that the increase is largely apparent and due to the greater completeness with which the record has been made at each succeeding census, but the falling off in 1871 and 1881 is perhaps significant, and is consistent with the figures derived from the genealogical record.

The various records have all shown a preponderance of males, though in the estimate of Ouchterlony† made in 1847 the difference is very slight. In 1871 the numbers were *about* 400 males and 283 females‡, and in 1901 451 males and 354 females. These records show that the increase during the last thirty years has been relatively greater among the females but not to any very marked extent. Here again the official records show a general agreement with the figures deduced from the genealogical record.

* Grigg, *Manual of the Niligiri District*, 1880, pp. 25, *et seq.* † *Ibid.*, p. 27.

‡ Various errors were made in compiling the statistics of this census of the Toda population and in the corrected figures given by Brecks (*Tribes of the Niligiri Hills*, 1873, p. 5), the numbers of each sex are not given.

The Spatial Configuration of Trivalent Nitrogen Compounds.
By H. O. Jones, M.A., Clare College, and J. P. Millington, B.A., Christ's College.

[Received 17 May 1904.]

Ever since the hypothesis of Hantzsch and Werner was proposed to explain isomerism of the oximes and allied compounds the problem as to whether trivalent nitrogen compounds of the type N.R′R″R‴ were capable of showing optical activity or not has attracted considerable attention. Since in the cases mentioned above the isomerism is explained by supposing that when a nitrogen is doubly linked to a carbon atom, the third valency is not in the same plane with the other two and the nitrogen atom, it has been assumed that the three valencies do not lie in the same plane as the nitrogen atom when all three are attached to different atoms or groups. This is not necessarily so. Whatever the nature of the forces acting in a chemical compound may be, the arrangement of three atoms or groups around one atom all in one plane being the most symmetrical would probably be the most stable of the possible arrangements. This view is not incompatible with the Hantzsch-Werner hypothesis, since it is quite conceivable that the strain caused by the formation of a 'double union' might cause a displacement in the equilibrium position of the group attached to the third valency.

Several attempts have already been made to obtain optically active trivalent nitrogen compounds, thus Kraft (*Ber.*, XXIII. 2780) tried to resolve benzylethylamine and p-tolylhydrazine by means of their acid tartrates. Behrend and König (*Ann.*, 263. 175) made similar attempts with β benzylhydroxylamine, and Reychler (*Bull.Soc.Chim.* 1902[3], XXVII. 979) submitted the dextro-camphorsulphonate of methyl-ethyl-β-naphtylamine to fractional crystallisation in the hope of effecting a separation into two isomerides. The above attempts were all unsuccessful, and it was therefore concluded that trivalent nitrogen compounds could not exist in optically active forms, and that the three valencies lay in one plane.

Pope and Harvey (*Trans. Chem. Soc.* LXXIX. 838) point out that, since during a change in valency, a rearrangement of the positions of equilibrium of the various groups or a change in 'valency-direction' sometimes occurs (a view fully confirmed by experiments

on the change of sulphur and selenium from quadri- to sexavalency), the conclusions drawn from the above experiments are invalidated by this possibility. It has not been found necessary, however, to assume that this change of 'valency-direction' does occur in the nitrogen atom during the change from tri- to quinquevalency in order to explain the observed phenomena (see *Trans. Chem. Soc.*, 1903, LXXXIII. 1403).

Kipping and Salway (*Trans. Chem. Soc.*, 1904, LXXXV. 438) have attacked the problem in a new way by investigating the reaction of various primary and secondary amines with an optically active acid chloride. In all the cases examined no isomerism was detected which could be assigned to the asymmetry of the nitrogen atom, so that they definitely conclude that the configuration of the trivalent nitrogen atom is plane.

Before the publication of the last-mentioned paper the authors had made some experiments in the hope of obtaining evidence which should not be open to the objection raised by Pope and Harvey to previous attempts to resolve trivalent nitrogen compounds, and so bringing the matter to some definite conclusion. The above work of Kipping and Salway affords such strong evidence in favour of the plane configuration, which is supported by the authors' experiments in so far as they have been carried out, that it was thought unnecessary to continue the work and to publish the results already obtained.

The object was to make the experiments with compounds in which there should be no change in the valency of the nitrogen atom during the processes involved. Two such sets of compounds are available; namely, hydrazines of the type $\begin{matrix} R \\ R' \end{matrix}\!\!>N-NH_2$, in which the salt formation will almost certainly take place with the unsubstituted amine group, particularly if R and R′ be aryl groups, and the acids of the type $C_6H_4\!\left<\begin{matrix} N\!\left<\begin{matrix} R \\ R' \end{matrix}\right. \\ X \end{matrix}\right.$ where X represents $-COOH$ or $-SO_3H$.

Up to the present one compound of each class has been investigated, namely, benzyl-phenyl-hydrazine by fractional crystallisation of the dextro-camphorsulphonate and methyl-ethyl-anilinesulphonic acid by fractional crystallisation of its brucine salt.

Benzyl-phenyl-hydrazine dextro-camphorsulphonate was prepared by mixing together molecular quantities of the acid and base dissolved in hot ethyl acetate. The solvent was evaporated off; the solid which was left soon crystallised, and was then repeatedly recrystallised from a mixture of ethyl acetate and ether, when it was obtained in long prisms melting at 156°—157° C.

The salt is very sparingly soluble in water, so that determinations

of its rotatory power were made in dilute alcohol solution containing 30—50 per cent. of alcohol.

The values of $[M]_D$ in a 2 per cent. solution were about 53°—54° and did not change appreciably on recrystallisation.

Analysis of the salt gave numbers agreeing with the formula

$$C_6H_5 . C_7H_7\text{-}N\text{-}NH_2 . C_{10}H_{15} . SO_4H.$$

The benzyl-phenyl-hydrazine recovered from the salt was optically inactive when examined in ether solution.

The corresponding salt of methyl-phenyl-hydrazine seemed to split up in ethyl acetate solution into acid and base, crystals of the acid being deposited on cooling.

Methyl-ethyl-aniline-sulphonic acid, which has not hitherto been described, was prepared by heating together methyl-ethyl-aniline and excess of sulphuric acid for several hours at 180°—190° C., or better by using molecular proportions of amine and chlorsulphonic acid, when the heating need only be of short duration. The product was then treated with water, the solution boiled and treated with barium carbonate until neutral, the barium sulphate filtered off and the filtrate evaporated to crystallisation point. In this way the barium salt

$$\left(C_6H_4\left\langle\begin{matrix} N(CH_3)(C_2H_5) \\ SO_3 \end{matrix}\right.\right)_2 Ba . 2H_2O$$

was obtained, this was dissolved in water and decomposed by the addition of the calculated quantity of dilute sulphuric acid, the barium sulphate filtered off and the filtrate evaporated to dryness. The acid, which is very soluble in water and much less soluble in alcohol, was recrystallised from dilute alcohol in a desiccator over quicklime, and was obtained as colourless needles melting at 249°—250° C.

Analysis showed the formula to be

$$C_6H_4\left\langle\begin{matrix} N(CH_3)(C_2H_5) \\ SO_3H \end{matrix}\right. .$$

Brucine-methyl-ethyl-aniline-sulphonate was prepared by boiling molecular proportions of the acid and base with water until the base dissolved, and allowing the solution to crystallise. The salt formed lustrous prisms melting at 60°—61° C. and was found on combustion and determination of the water of crystallisation to be $C_{32}H_{39}N_3 . SO_7 . 4H_2O$.

On account of the low melting point of the salt it was found necessary in recrystallising to use a solution saturated at a temperature not exceeding 60° C., else the salt separated as an oil.

The salt is only soluble to the extent of about 1 per cent. in water, and hence determinations of the rotatory power are difficult.

It was found, however, that the value of $[M]_D$ was about $-120°$, which was very close to that observed for brucine sulphanilate

$\{C_{29}H_{33}N_3O_7 . S . 2H_2O . MP. 261°—263° C.\} -124°—-130°.$

The salt was extremely sparingly soluble in all the solvents available, but it was found that if the salt were dehydrated at 100° C. it was then more soluble in chloroform and fused at about 90° C. Determinations of the rotatory power of the various fractions after dehydration were made in chloroform solution. The value of $[\alpha]_D$ in 2 per cent. solution was found to be about 12·7°, and the value was not appreciably altered by recrystallisation. It was found impossible to isolate the acid or its salts quite free from brucine, so that the feeble rotatory power observed is to be referred to traces of brucine.

It may therefore be concluded on the basis of the large mass of evidence now available, that the trivalent nitrogen is not capable of giving rise to optical activity, and has a plane configuration when attached to three monovalent groups.

On the Magnetic Deflexion of the Negative Current of Electricity from a hot Platinum Wire at low pressures. By GWILYM OWEN, B.A., Christ's College.

[*Received* 30 May 1904.]

It has long been known that the space surrounding an incandescent platinum wire conducts electricity. McClelland* has shown that this is due to two causes—

(*a*) To the ionisation of the gas in contact with the wire.

(*b*) To the emission of ions from the metal.

This electrification is a most complicated effect, depending on the nature and pressure of the gas surrounding the wire, on the gas absorbed by the metal, on the temperature and previous treatment of the metal and other not yet fully-known causes.

These effects are somewhat simplified in the case of a wire glowing at very low pressures.

The discharge from hot platinum has been made the subject of experiment by very many observers. The most recent work is that of Dr H. A. Wilson† and Mr O. W. Richardson‡.

In the *Philosophical Magazine* for December, 1899, Prof. J. J. Thomson describes a method for determining the ratio of the charge e to the mass m for the carriers of the negative electricity discharged from metals at low pressures. This ratio has been determined by Prof. Thomson for the carriers of the negative electricity discharged by an incandescent carbon filament, and for the carriers of the positive electricity from glowing platinum. The carriers of the negative electricity at low pressures have been shown to be corpuscles, whereas the carriers of the positive electrification are particles of molecular dimensions.

Now it is well known that when a platinum wire is heated in air it loses weight, showing that particles of platinum are driven off. The writer has shown (*Phil. Mag.*, Sept. 1903) that this disintegration of the platinum takes place in air at a temperature as low as 200° C. Child§ in his experiments on the discharge from hot platinum wires also obtained evidence that particles

* *Proc. Camb. Phil. Soc.* x. p. 241, 1900.

† *Phil. Trans.* A, 202, pp. 243—275, 1903.

‡ *Proc. Camb. Phil. Soc.* xi. p. 286, 1901; *Phil. Trans.* A, 201, pp. 497—549, 1903.

§ *Physical Review*, April 1902.

were driven off the wire. He found the velocity of the ions emitted by the wire to be smaller when the wire was enclosed in a tube than when open to the air. He explained this as being due to the "loading up" of the ions by heavy particles driven from the wire. On Child's theory these heavy particles "do not aid in the discharge but materially check it."

The question therefore arises, Is the negative current of electricity from a hot platinum wire at very low pressures carried by *corpuscles* or by a mixture of corpuscles and heavy particles? If the mechanism of the discharge is the latter one, How does the relative proportion of corpuscles and heavy particles vary with the temperature of the wire? It was in the hope of throwing some light on these questions that the following experiments were undertaken.

The method adopted was to determine the value of e/m for the carriers of the negative electricity at very low pressures. This was done by the method of "Retardation of Discharge by a Magnetic Field" first used by Prof. Thomson in 1899.

In the number of the *Phil. Mag.* referred to above, Prof. Thomson proves that the path of an ion under the joint action of uniform electric and magnetic forces (acting at right angles to one another) is a cycloid. If an ion starts from rest at the plane $x = 0$, then the greatest distance the ion can get from this plane is $2mX/eH^2$, where X and H are respectively the electric and magnetic forces, m the mass and e the charge on the ion.

Now consider a number of ions starting from the plane $x = 0$ and moving towards the plane $x = a$. If a is less than $2mX/eH^2$, all the ions will reach the plane $x = a$. If on the other hand a is greater than $2mX/eH^2$, all the ions will be turned back by the magnetic field before they reach the plane $x = a$. If a is fixed and H can be varied, then we shall have the following effect. Starting with very small magnetic fields, no diminution is produced in the number of ions reaching the plane. When the magnetic field, however, attains a certain value all the ions will be stopped. Thus if we know X, a and the critical value of H we can find e/m, for $e/m = 2X/aH^2 = 2V/a^2H^2$, when V is the difference of potential between the planes. In his determination of the value of e/m for the corpuscles emitted by a metal plate illuminated by ultra-violet light, and by a glowing carbon filament, Prof. Thomson did not find this change from no effect to a complete disappearance of the current to be quite so abrupt as the theory indicates. He explained this as being due to the ionisation not being wholly confined to the gas immediately in contact with the metal plate or filament but extending into the gas through a layer of finite thickness t say. Thus the ions would start towards the plane at distances ranging from a to $a - t$. The

ions starting at distance a will of course be the ones most easily deflected by the magnetic field. The same effect was also observed in my experiments as will be seen later.

In these experiments the platinum wire used was very pure, its temperature coefficient between 0°C. and 100° C. being 0·0038. The diameter of the wire was 0·2 mm. In the first apparatus made the wire was fixed inside a glass tube in the form of a flat grating, the plane of the grating being perpendicular to the axis of the tube. Parallel to the grating and four millimetres distant from it was an insulated aluminium plate connected to one pair of quadrants of an electrometer. The diameter of the plate was about 2·3 cms., and was six or seven mms. larger than the greatest breadth of the grating. The grating was fixed to copper wire leads and could thus be raised to any desired temperature by means of an electric current. The platinum wire formed one arm of a Wheatstone's bridge, the corresponding arm being a spiral of thick german-silver wire (1·16 ohms resistance) immersed in a bath of paraffin oil. The two other arms of the bridge were high resistances of the order of 1000 ohms, got from a post-office box. The temperature of the wire was calculated from its resistance in the ordinary way, the coefficient δ for the wire being 1·5.

The temperatures given in the table probably differ somewhat from the actual temperature of the wire as only a short length of wire (about 6 cms.) could be conveniently used. This, however, is no great disadvantage, for the results obtained do not depend upon an accurate knowledge of the temperature of the wire.

The wire could be raised to various temperatures by altering the resistance of a german-silver wire rheostat inserted in the heating circuit. As the wire was to be charged to a high potential, the heating battery, post-office box, etc., were insulated on paraffin blocks. The wire was raised to a high potential by connecting one end to one pole of a battery of small storage cells, the other pole of which was connected to earth. The same end of the wire was also always connected to a Weston voltmeter. The electrometer used was of the ordinary pattern giving a deflexion of nearly 60 divisions for a volt. The needle was made of thin aluminium having a platinum wire vane dipping in sulphuric acid. The usual paraffin block electrometer keys were discarded and replaced by mercury cups connected directly to the quadrants of the electrometer. Great care was taken to see that no deflexion of the electrometer needle took place when the *cold* wire was raised to a high potential. In making an observation of the discharge, one pair of electrometer quadrants was at a definite instant insulated, the other pair remaining earthed. The quadrants were now allowed to charge up for a certain number of

seconds, at the end of which time the circuit connecting the insulated plate to the electrometer was broken. The deflexion of the electrometer will give the current that passed from the hot wire to the plate.

The discharge was subjected to the action of a magnetic field by placing the part of the tube surrounding the wire and electrode between the poles of an electromagnet of the horse-shoe type. The distance between the poles was about 3 cms., the section of the poles was 3 cms. by 3 cms.; thus the space occupied by the wire and plate was in a fairly uniform magnetic field. The magnet was calibrated by observing the deflexion of a ballistic galvanometer when a small coil of known effective area was suddenly withdrawn from the central space between the poles. The magnetic field was found to be proportional to the exciting current sent through the magnet. One ampere corresponded to 59 lines per sq. cm. Thus 15 amperes, which was the largest current tried, produced a magnetic field of nearly 900 lines per sq. cm. As the negative discharge from a platinum wire exhibits considerable irregularity, the value of the current observed with the magnetic field on was compared with the mean value of the currents without any magnetic field *before* and *after* the observation. Table I. gives the results obtained on July 22nd, 1903, using the apparatus described above. The temperature of the wire (kept constant during the set of observations) was 816° C. Mean pressure during the series was 0·0024 mm. The potential on the wire was -120 volts.

The last column gives the percentages of the current stopped by the magnetic field. These percentages have not been calculated to any decimal places, as the experimental errors may easily be greater than the error thus introduced. The above results are shown graphically in Diagram I., where the ordinates represent the percentage of the current stopped and the abscissae the currents passing through the electromagnet. It will be seen that the effect of the magnetic field is fairly abrupt. The value of the magnetic field which *begins* to affect the current is somewhat uncertain. The magnetic field required to stop a finite percentage (say 60 °/₀) of the current is, however, quite definite. We can use this value of the magnetic field to calculate e/m for the carriers of the electricity. We have

$$V = 120 \times 10^8, \quad a = 0{\cdot}4 \text{ cms.}, \quad H = 94{\cdot}4 \text{ lines per sq. cm.}$$

Therefore $$e/m = 2V/a^2H^2$$

$$= \frac{2 \times 120 \times 10^8}{0{\cdot}16 \times 94{\cdot}4 \times 94{\cdot}4},$$

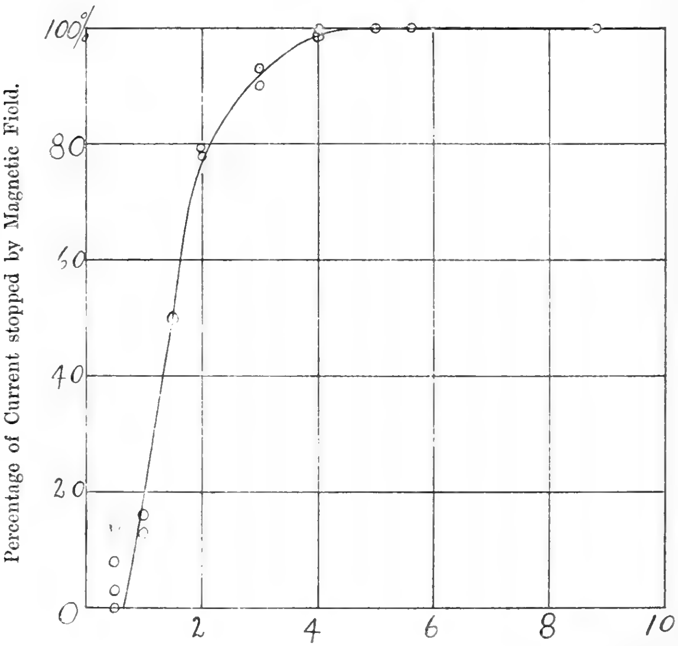
i.e. $$e/m = 1{\cdot}7 \times 10^7.$$

TABLE I.

Time	Current, magnet off, $1 = 2{\cdot}2 \times 10^{-14}$ amp.	α	H $1 = 59$ lines	Current, magnet on $= \beta$	$\frac{\alpha - \beta}{\alpha} \times 100$
III. 32 wire	made hot				
38	570				
		495	13·7	0	100 %
...	420				
		404	8·8	0	100
51	388				
		369	5·6	0	100
56	350				
		375	4·0	0	100
IV. 4	400				
		391	3·0	41	90
12	382				
		441	2·0	93	78
18	500				
		491	1·0	426	13
24	482				
		486	0·5	470	3
32	490				
		451	1·0	390	13
V. 14	412				
		431	0·5	430	0
18	450				
		448	0·5	384	14
23	446				
		446	0·5	412	8
27	446				
		435	1·0	366	16
35	424				
		428	1·5	212	50
40	432				
		435	2·0	91	79
47	438				
		442	3·0	32	93
52	446				
		450	...	...	...
54	454				
		441	4·0	6	99
...	428				
...			5·0	0	100

This result shows that all the carriers of the electricity from the glowing wire at this temperature are corpuscles. On conducting the experiment at higher temperatures, when much

Diagram I.



1 ampere = 59 lines/sq. cm.

bigger discharges were dealt with, it was found on plotting the results as in Diagram I. that the curves became flatter and descended towards the axis of H as the discharge got bigger. At a temperature of 1150° C. when the discharge was 3×10^{-8} ampere only about 10 % was stopped by a magnetic field of 820 lines per sq. cm., the potential of the wire being 125 volts. These results led me to the conclusion that whereas at low temperatures the current is carried wholly by corpuscles, at higher temperatures a large proportion of the current is carried by heavy particles, the proportion of particles to corpuscles increasing with the temperature.

Further experiments have confirmed the results obtained at low temperatures, but a modification of the experimental arrangements has shown that the result obtained at the higher temperatures

is incorrect. For what becomes of the ions that are deflected by the magnetic field? They are deflected in such a way that their path which was originally parallel to the axis of the tube now becomes perpendicular to it. The ions thus strike the walls of the glass tube. The glass being very dry and the pressure of the gas low, the insulation of the inner surface of the tube is very good. Thus the glass will get charged up by the deflected ions. When the discharge is small this will not interfere much with the action of the magnet on the ions. When the discharge is big, however, the glass will get charged to a high potential so that there will be a strong electric field opposing the deflexion of the ions. We should accordingly expect fewer ions to be deflected the larger the current, *i.e.* the higher the temperature. This explanation was not arrived at until some experiments had been made with a new form of apparatus which we shall now briefly describe. One of the disadvantages of the first experimental tube was that the potential difference at my disposal (120 volts) did not quite saturate the discharge. This difficulty was got over in the new form of apparatus by having the grating of wire midway between two parallel plates. These plate electrodes consisted of two brass discs 2·8 cms. in diameter and were supported in a vertical plane in the glass tube *A* (Fig. 1) by a stiff aluminium wire passing through a glass tube fitting into the tube *S*. The discs were 8 mms. apart and were connected together by one thin bar of brass screwed into the discs at the extreme end of parallel diameters.

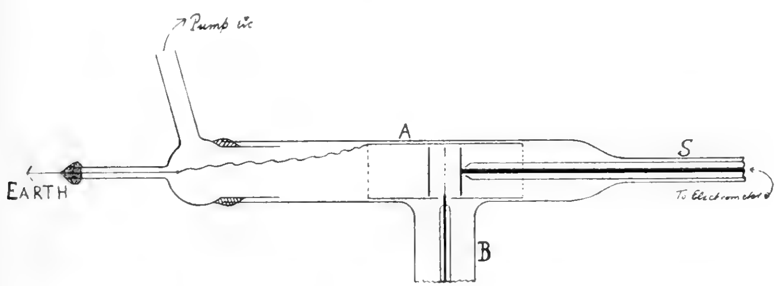


Fig. 1.

In order to obtain as uniform an electric field as possible between the wire and the discs, the former was fitted up in the following way. A piece 1 cm. square was cut out at the centre of a disc of platinum foil 2·7 mms. in diameter. The grating of platinum wire was placed in this opening in the plane of the disc. One end of the wire was welded to the disc, the other to a

piece of thick platinum wire lead *a* (Fig. 2) coated with blue glass so as to insulate it from the platinum disc. The other platinum lead *b* was welded to the disc as shown in the Figure.

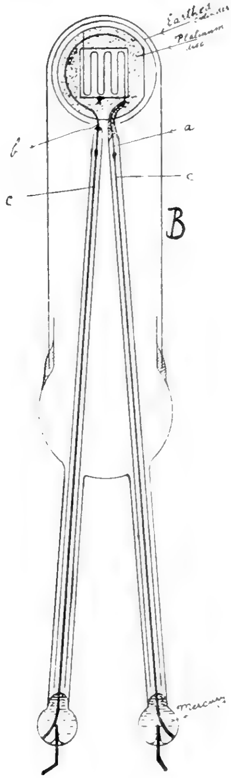


Fig. 2.

The copper leads *c, c* passed down narrow glass tubes and dipped into mercury cups. The wire and its supporting framework could thus be easily taken out when necessary for the purpose of putting on a fresh wire. The wire was placed in position through the side tube *B* (Fig. 1). It will be noticed from Fig. 2 that a small piece of the platinum disc at one edge is cut away. This was to prevent the disc coming into contact with the metal bar connecting the two brass electrodes. As the deflexion of the ions took place along a vertical diameter the bar was arranged so as to be at one end of a horizontal diameter. Surrounding the discs was a platinum cylinder connected to earth. A slit was cut in the lower part of the cylinder just wide enough to allow the cylinder

to be pushed forward so as to surround the electrodes when the wire was in its position between the electrodes. The introduction of this earth-connected cylinder was the most important feature in this new apparatus. For it is now found that, in general, even at very high temperatures by far the greater part of the discharge could be stopped by the magnetic field. If, however, the platinum cylinder was disconnected from the earth then at the higher temperatures the discharge was practically unaffected by the magnetic field.

As is well known, the negative discharge from platinum exhibits great irregularities. Dr A. H. Wilson has shown (*Phil. Trans.* A, Vol. 202, pp. 243–275, 1903) that these irregularities can be very largely got rid of by boiling the wire in nitric acid and by driving occluded gases out of the wire by continuous heating. The object of this investigation, however, being to obtain information with regard to the mechanism of the discharge from platinum in general, experiments were made with wires exhibiting different degrees of irregularity depending upon their previous treatment.

The results given below represent the main conclusions arrived at.

Table II. gives the variation of the discharge with the potential. After setting the wire up in the plane of the platinum disc, the latter was dipped into cold nitric acid and then rinsed in distilled water. Temperature of wire = 795° C.

TABLE II.

Oct. 17, 1903. Wire I. Pressure 0·011 mm.

Volts on Wire	Current, $1 = 5{\cdot}3 \times 10^{-12}$ amp.
6	70
10	178
20	202
30	205
40	215
60	222
120	238

The numbers given in the above table have been plotted in Diagram II.

Table III.

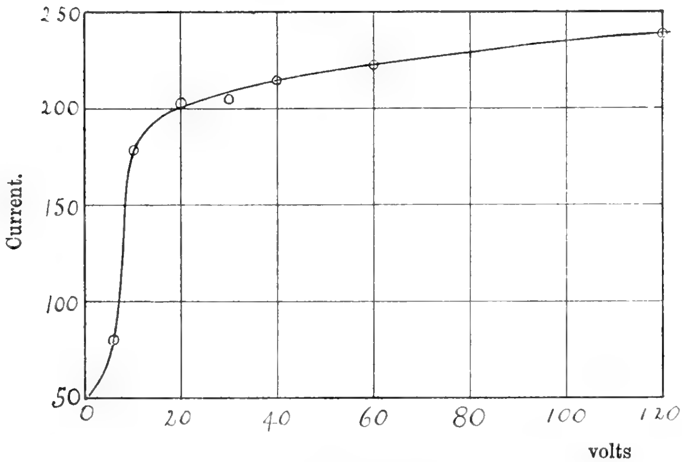
Pressure in mms.	Volts on Wire	Current, magnet off, $1 = 4{\cdot}4 \times 10^{-13}$ amp.	Mean $= \alpha$	H $1 = 59$ lines	Current, magnet on, β	$\frac{\alpha-\beta}{\alpha} \times 100$
0·0058	– 120	7980				
				1	7620	3 %
			7890			
				2	1740	78
		7800				
				3·1	296	95
			7878			
				4·2	208	97·4
		7956				
				5·4	58	99·3
			7872			
				8·1	4	100
0·0060		7788				
	– 60	7920				
				1	3990	47
				1·5	1335	82
			7590	2·0	576	92
				3·0	156	98
				4·0	50	99·3
0·0057		7260				
	– 30	6684				
				0·5	5460	16
				1·0	1340	79
				1·5	480	93
			6522			
				2·0	264	96
				3·0	64	99
				4·2	14	99·8
0·0063		6360				

The results tabulated above are plotted in Diagram III

It will be seen from the curve that the current is practically saturated by a potential under 40 volts.

We have already seen that e/m is given by $2V/a^2H^2$. Therefore for any given value of a $H^2/V=$ constant. Before investi-

Diagram II.



Potential on Wire.

gating the discharge at high temperatures a series of observations was made at a temperature of 795° C. to see if this law was at all obeyed. The results obtained are given in Table III.

The curves given in this diagram will enable us to see if the law $H^2/V=$ constant is obeyed. We shall, as before, take as the critical values of H those which stop 60 °/。 of the discharge. These values are got from the curves and are given in the following table together with the corresponding values of e/m.

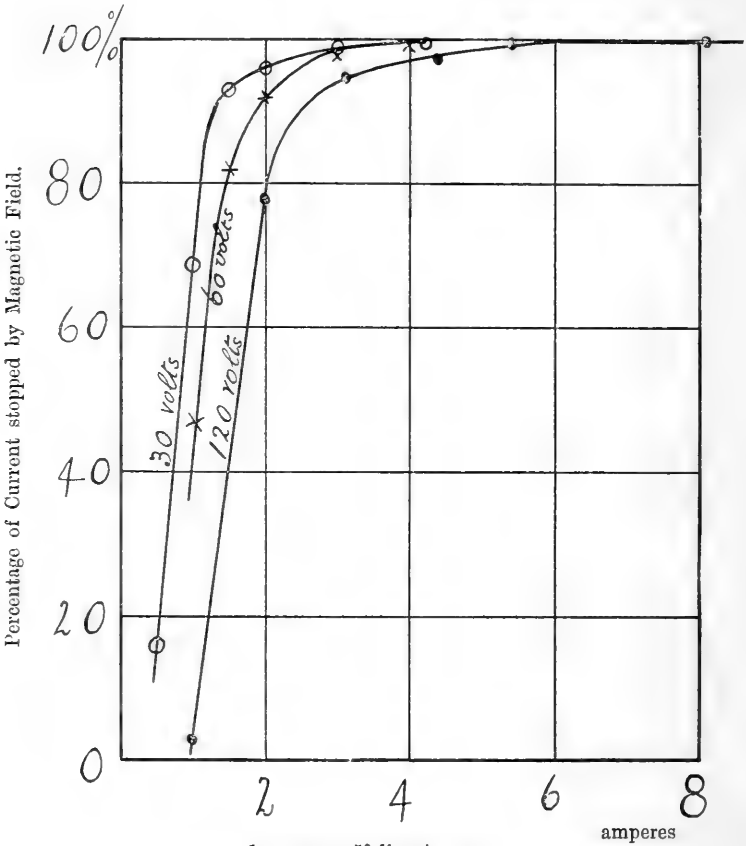
TABLE IV.

Volts on Wire $=V$	H	$\frac{H^2}{V}$	$\frac{e}{m}=\frac{2\times 10^8}{a^2}\div\frac{H^2}{V}$
120	103	88·4	$1{\cdot}41\times 10^7$
60	70·2	82·1	$1{\cdot}52\times 10^7$
30	53·7	96·1	$1{\cdot}30\times 10^7$

Mean value of $\frac{e}{m}=1{\cdot}41\times 10^7$.

It will be seen from the above numbers that the law $H^2/V = \text{constant}$ is obeyed as well as can reasonably be expected when one considers how greatly the actual experimental arrangements differ from the ideal case for which the law was deduced.

Diagram III.



1 ampere = 59 lines/sq. cms.

The mean value of e/m found lies well within the range of values given in the table on page 117 of Prof. Thomson's *Conduction of Electricity through Gases.* In that table the largest value of e/m for the corpuscles is $1{\cdot}865 \times 10^7$ and the smallest $6{\cdot}39 \times 10^6$.

We shall now give the results of experiments made at considerably higher temperatures.

In these experiments the discharge from the wire was found too large to be manageable when using an electrometer. A

galvanometer was therefore employed to measure the discharge from the wire. The instrument used was of the D'Arsonval type giving 1 division for 8×10^{-9} amperes.

The method of experimenting now was to subject the discharge to a known magnetic field and note the current passing through the galvanometer. The magnetic field was then cut off and the current again measured. Thus the two observations could be made within a few seconds of one another. The following results given in Table V. were obtained on October 26, with wire I. This wire, during the experiments on previous days, had been kept glowing for many hours. This is to be noted as it has a bearing upon the result.

At the commencement of a set of observations the apparatus was pumped to as low a pressure as possible. The pressure was again measured at the end of the set. The pressures given in the table are therefore the highest during the series of observations.

TABLE V. (Wire I.).

Pressure in mms.	Temperature of Wire	Potential on Wire	Current, magnet off, $1=8\times10^{-9}$ amp.	H $1=59$ lines	Current, magnet on	Percentage stopped
	1280° C.	-120	11730	8	435	96·3 °/。
			5980	8	295	95·1 „
0·021			4580	8	275	94·0 „
	1340° C.	-120	19550	8	687	96·5 „
0·012			12270	8	522	95·8 „

Air was now let into the apparatus up to atmospheric pressure. After this treatment the discharge in a vacuum was found to be much smaller than before. At a temperature of 1120° C. all the discharge was stopped by a magnetic field of 300 lines. The value of the current in this case was $5·2 \times 10^{-7}$ amperes.

The results given in the above table show that the discharge from a wire which has been subjected to prolonged heating consists almost wholly of corpuscles. It is to be noted also that the

percentage of the discharge not stopped is nearly independent of the magnitude of the discharge. This point will be discussed later.

Experiments were now made on a wire which had not been previously heated. When a platinum wire is first heated there is, as is well known, a very large initial leak which rapidly dies away with the time. The mechanism of this initial leak was investigated for this wire (Wire II.) and the results are given in Table VI. As the wire was only heated for short intervals (only a few seconds) the temperatures given are approximations only. As will be seen the discharge diminishes so rapidly with the time that in one case it appears to diminish as the temperature is increased. The results are given in the order in which they were taken.

TABLE VI. (Wire II.).

Pressure in mms.	Temperature of Wire	Potential on Wire	Current, magnet off, $1=10^9$ amp.	H $1=59$ lines	Current, magnet on	Percentage stopped
	approx.	volts				
		−120	92000	14	16000	82·6
	1040° C.		12000	14	2000	83·3
			5400	12	600	88·8
	1250° C.	−120	46000	14	6000	87·0
0·007			21600	8·5	2800	87·0
	(Later) 1370° C.	−120	2392	10	368	84·6

It will be seen from the above table that even for a wire heated for the first time by far the greater proportion of the discharge consists of corpuscles. There is, however, in the discharge a good percentage (about 15 %) of heavy particles which were not deflected by the magnetic fields put on.

It may be urged that this is due to there being dirt on the wire which, when driven off by the heat, assists in carrying the current. The following experiment, however, proves conclusively

that the presence of heavy particles in the discharge is not due to dirt on the wire. On November 27 a fresh wire (Wire III.) was taken and boiled for three hours in pure nitric acid, and finally rinsed in boiling distilled water. In addition to this the wire was kept for thirty minutes at a bright yellow heat in air at atmospheric pressure by means of an electric current. We may reasonably expect this process to remove any dirt that might be on the wire. After this treatment the wire was not touched with anything and was thus placed in position between the electrodes in a clean state. The discharge from this wire was now found to be so small that an electrometer was substituted for the galvanometer. Table VII. contains some results obtained with this wire soon after it had been set up.

Table VII. (Wire III.). December 2nd.

Pressure in mms.	Temperature of Wire	Volts on Wire	Current, magnet off, $1=2{\cdot}5\times10^{-14}$ amp.	H $1=59$ lines	Current, magnet on	Percentage stopped
	860° C.	−120	124	3·8	26	79 %
			105	14·3	24	78 ,,
0·0011	940° C.	−120	1584	3·8	224	86 ,,
0·0019	1090° C.	−120	92300	3·8	10820	88·3 ,,

It will be noticed that the discharge from this wire also contains a good percentage of heavy particles.

The wire was now heated to a high temperature in air at a few millimetres pressure. After some minutes heating fresh air was let in to atmospheric pressure. The operation was then repeated several times. This process gets rid of gases occluded in the wire and it was also found greatly to diminish the discharge. The following results were obtained on December 5th with Wire III. after it had been subjected to the above treatment.

From this table it will be seen that the heating process described above has brought Wire III. into a state wherein the

TABLE VIII.

Pressure in mms.	Temperature of Wire	Volts on Wire	Current, magnet off, $1 = 4 \cdot 4 \times 10^{-13}$ amp.	H $1 = 59$ lines	Current, magnet on	Percentage stopped
0·005	1120° C.	-120	6780	4	507	92·5
0·005	1230° C.	-120	$(1 = 2 \cdot 6 \times 10^{-11})$ 755	4·7	35	95·4
0·006	1300° C.	-120	2172	4·4	116	94·7

negative discharge is very nearly wholly composed of a stream of corpuscles.

Summary and Discussion of Results.

1. General result:—The current of negative electricity from a hot platinum wire at low pressures is mainly carried by corpuscles; the proportion of corpuscles to heavy particles is at least four to one.

2. From 10 to 20 per cent. of the discharge from an uncleaned wire newly set up and also from a wire which has been cleaned by boiling in nitric acid is carried by heavy particles.

3. By prolonged heating of the wire and frequently renewing the air in the apparatus the wire can be brought to such a state that the proportion of heavy particles in the discharge is only about 5 per cent. even at very high temperatures.

4. At low temperatures even for wires that have not been subjected to prolonged heating the discharge is wholly carried by corpuscles. The value of e/m for these corpuscles was found to be $1 \cdot 41 \times 10^7$.

5. The magnitude of the discharge after various treatments of the wire suggests that the discharge is due primarily to the ionisation of gases (probably hydrogen) occluded in the wire. This confirms the result obtained by Dr H. A. Wilson.

The above results can be fully explained if it is assumed

(*a*) That the negative discharge is chiefly due to the ionisation of gases occluded by the metal, and is for any given temperature proportional to the amount of gas in the wire.

(*b*) That when the wire is heated, the occluded gases slowly escape, the rate of escape diminishing with continued heating.

(*c*) That the disintegration of the metal is due to the carrying away of particles of the metal by the gas as it escapes.

(*d*) That a small fraction of the discharge is carried by the molecules of gas and particles of platinum emitted by the incandescent wire.

That some relation exists between the amount of gas in the wire and the disintegration follows from the fact that, although the discharge at one temperature may be sixty times as large as that at a lower temperature, yet the proportion of corpuscles and heavy particles is almost the same in the two cases. This is well shown by the results for temperatures 940° C. and 1090° C. in Table VII., and affords evidence for assumption (*c*).

In general, however, it is noticed that as the heating is prolonged the discharge contains fewer and fewer heavy particles. This is quite in accordance with the above views. For in a mass of platinum, there will be a number of particles much more detachable than the rest. Thus after continued heating the escape of a certain quantity of gas from the wire will produce less disintegration than would be produced by the escape of the same quantity of gas when the wire was heated for the first time.

In conclusion, I take this opportunity to express my best thanks to Professor Thomson for many valuable suggestions during the course of the experiments which were carried out at the Cavendish Laboratory.

On the Absorption of Ultra-violet Light in different Gases.

By W. MANSERGH VARLEY, B.A., Emmanuel College.

[*Received* 6 June 1904.]

The experiments described below on the absorption of ultra-violet light in various gases were carried out as a preliminary research to one on the photo-electric discharge from a metallic surface in different more or less complex gases.

Hitherto the only experiments published giving the absolute value of the coefficient of absorption of ultra-violet light in any gases appear to be those of Kreusler* who experimented with air, carbon dioxide, oxygen, nitrogen and nitrous oxide. Up to a wave-length of 200 $\mu\mu$ he could detect no absorption in any of these gases except nitrous oxide which absorbed all light below 300 $\mu\mu$. His results were unsatisfactory owing to the irregular character of the source of light employed. Schumann† and

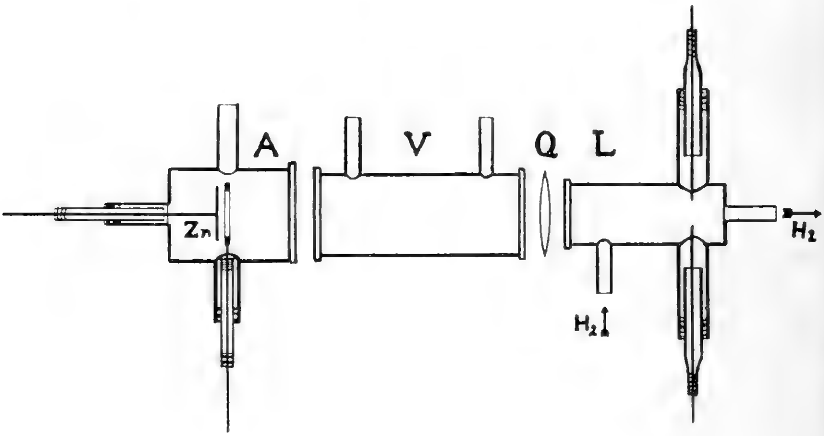


FIG. 1.

others have made experiments on absorption bands, using the photographic method, and hence obtaining no value for the amount of absorption of the light as a whole in any part of

* Kreusler, *Ann. der Physik*, Vol. VI. p. 412, 1901.
† Schumann, *Sitz. Ber. der Kais. Akad. Wien*, Vol. CIII. pp. 415, 625, 1893.

the spectrum. It has been found that air absorbs all light of wave-length below about 185 $\mu\mu$, but that it is highly transparent to light of wave-lengths about 200 $\mu\mu$. Quartz also absorbs light of shorter wave-lengths than this, though hydrogen is transparent to light of wave-length as short as 100 $\mu\mu$.

The method used in these experiments was to interpose a brass vessel, *V* (see Fig. 1), 10 cms. in length and 4 cms. in diameter, closed at either end by a quartz window, which served to contain the various gases, between the source of ultra-violet light, *L*, and an apparatus, *A*, in which the intensity of illumination was measured.

The source of ultra-violet light used was a spark between iron terminals in pure dry hydrogen, the arrangement being the same as that described in a previous article by the author, "On the Photo-electric Discharge from Metallic Surfaces in Different Gases" (*Trans. Roy. Soc.* Vol. 202 A, p. 439, 1903). The iron terminals were connected to the secondary terminals of a 6″ induction coil, in parallel with which were also three large Leyden jars. An alternating current of 3·5 amps. were sent through the primary of the coil.

The intensity of the ultra-violet light was measured by the magnitude of the photo-electric current it produced from a zinc surface, negatively charged, situated 3 mms. behind a fine gauze, which served as the positive electrode. The vessel containing these electrodes was closed in front by a quartz window and was kept exhausted to a pressure of about 15 mms. The zinc was usually charged to a potential of -40 volts by means of a battery of small secondary cells.

The gauze was connected to one pair of quadrants of an electrometer, and the photo-electric current measured by noting the deflection obtained when the zinc was illuminated for 10 secs., the method used being that described in the paper "On the Photo-electric Discharge," quoted above.

A quartz lens, *Q*, was used to throw a parallel beam of rays through the vessel *V* on to the zinc electrode of the leak apparatus *A*.

This method of measuring the intensity of the light does not, of course, admit of the absorption of light of any one particular wave-length being measured, but only the absorption of that light which produces the photo-electric discharge from the zinc surface. The method is, however, obviously capable of modification and extension, but for the purpose in view the above is all that is required.

The results obtained for each gas are given below.

Air.

To test if any absorption of the light in air could be detected the photo-electric currents were measured (1) when the interposed vessel was filled with air at atmospheric pressure, and (2) when it was exhausted to a pressure of 15 mms. of mercury by means of a water pump, the actual pressure being read on a suitable manometer.

No appreciable difference in the intensity of the light could be detected, the mean value obtained for the current (in arbitrary units, viz. in electrometer scale divisions per 10 secs. illumination) being in the first case 152·0, and in the second case 152·7 (means of 10 readings).

Air having shown no absorption the intensity of the light in dealing with other gases was measured (1) when the interposed vessel was filled with air, and (2) when it was filled with the gas under examination.

Carbon Dioxide.

Carbon dioxide, prepared from marble and hydrochloric acid, washed by bubbling through water, and dried over calcium chloride, also showed no absorption of the ultra-violet light, the mean values of the photo-electric currents obtained, when the vessel was filled with air and carbon dioxide being 88·0 and 87·9 respectively.

Carbon Monoxide.

The carbon monoxide was prepared from sodium formate and strong sulphuric acid, washed by caustic soda solution and dried over calcium chloride.

Mean value of current when vessel filled with air = 191·3
" " " " " CO = 178·7

giving an absorption of the light in passing through a length of 10 cms. of carbon monoxide at atmospheric pressure of 6·5 °/。.

In another experiment the carbon monoxide was prepared from potassium oxalate and strong sulphuric acid, the carbon dioxide being removed by passing the gas through a strong solution of potash. A slightly lower value for the percentage absorption was obtained, doubtless owing to the imperfect removal of the dioxide.

Marsh Gas.

The marsh gas was prepared in the usual manner by heating strongly a mixture of one part of sodium acetate with four parts

of soda-lime in an iron retort, the mixture having been previously thoroughly dried. The gas was purified by passing it through strong sulphuric acid and drying tubes containing pumice stone moistened with strong sulphuric acid.

Mean value of current when vessel filled with air = 98·8
" " " " " CH_4 = 84·1

giving an absorption of 14·9 % in passing through 10 cms. of marsh gas at atmospheric pressure.

The same experiments, when repeated, after the whole apparatus had been thoroughly cleansed, and all the drying materials renewed, gave an absorption of 16·3 % which, considering the difficulties of obtaining the gas pure, is in fairly close agreement with the value previously determined, so that we may take the mean value 15·6 % as being approximately correct for the absorption.

Ethylene.

The ethylene was prepared by the usual method from absolute alcohol and strong sulphuric acid, and was purified by passing it through a spiral tube immersed in a freezing mixture so as to condense any ether or acetone which might be carried over, through concentrated sulphuric acid, over potassium hydroxide and finally through calcium chloride drying tubes.

Mean value of current when vessel filled with air = 125·4
" " " " " C_2H_4 = 97·0

giving an absorption of 22·3 % in the intensity of the light in passing through a length of 10 cms. of gas.

The experiments were repeated on another occasion when an identical value for the coefficient of absorption was obtained.

Acetylene.

The acetylene was prepared by the action of water on calcium carbide, and was purified by passing it through a silver nitrate solution (to remove any phosphoretted hydrogen). The absorption of ultra-violet light proved to be so great that practically no light at all got through the vessel when it was filled with acetylene at atmospheric pressure, the mean current in this case being only 0·4 arbitrary units as against 116 similar units when it was filled with air.

Coal Gas.

The ordinary Cambridge coal gas led directly into the apparatus through calcium chloride drying tubes gave the following figures:

Mean value of current when vessel filled with air = 119·6
" " " " coal gas = 20·0

showing that 83 % of the light had been absorbed by the coal gas.

In order to test whether some part of this large absorption might not be due to the presence of acetylene in the gas, it was passed through an ammoniacal solution of cuprous oxide in order to remove all the acetylene, then through sulphuric acid to remove the ammonia and dry the gas.

The results now obtained were as follows:

Mean value of current when vessel filled with air = 117·3
Mean value of current when vessel filled with coal gas from which the acetylene had been removed = 89·0

showing that now only 24 % of the light had been absorbed—an absorption of the same value as that observed for the chief constituents of coal gas, ethylene and marsh gas. In order to afford a further check on the conclusion that the large amount of absorption was due to acetylene, the latter experiment was repeated with the single difference that the cuprous monoxide solution was removed. The absorption was again over 80 %.

Sulphur Dioxide.

The sulphur dioxide was obtained directly from liquefied sulphur dioxide as supplied commercially in bottles. The absorption was so large when the gas was at atmospheric pressure that it was thought better to measure the intensity of the light after it had passed through the gas at a reduced pressure and from this calculate what the absorption would be at atmospheric pressure.

Mean value of the current when vessel filled with air = 117·5
" " " SO_2 at 25 mms. pressure = 52·3

giving an absorption of 44·5 % in passing through 10 cms. of gas at 25 mms. pressure, equivalent to a length of 0·32 cms. at atmospheric pressure.

Another experiment gave

Mean value of the current when vessel filled with air = 79·3
" " " SO_2 at 28 mms. pressure = 39·6

giving an absorption of 50 % in passing through 10 cms. of the gas at 28 mms. pressure equivalent to a length of 0·36 cms. at atmospheric pressure, giving practically the same coefficient of absorption as the previous value (1·92 as against 1·87).

Sulphuretted Hydrogen.

The sulphuretted hydrogen was prepared from pure ferrous sulphide and hydrochloric acid, and was well washed by passing it through water and thoroughly dried before allowing it to pass into the apparatus.

The mean values of the currents obtained on two separate occasions were (1) when the vessel was filled with air 97·8 and 100·8, (2) when the vessel was filled with HS_2 8·5 and 10·5, giving absorptions of 91·3 % and 89·6 % respectively or a mean value of 90·5 % for the absorption of the light in passing through a 10 cm. length of sulphuretted hydrogen at atmospheric pressure.

Aqueous Vapour.

The absorption of ultra-violet light in water vapour was also roughly determined, a modified form of apparatus being employed for this purpose. The best method appeared to be to find the absorption in air saturated with moisture at about 50—60° and for this purpose it was necessary to enclose the vessel V in an air jacket heated by a flame underneath. By this means the whole of the vessel V, including the quartz windows, was heated up to a uniform temperature, and no water condensed on the quartz, as was otherwise the case. A few cubic centimetres of water were then introduced into V at a temperature of 17° and the intensity of the transmitted light measured; the temperature was now raised until it became constant at 58° when the intensity of the light was again measured.

Mean value of current when vessel filled with air saturated with moisture at 17° = 217.

Mean value of current when vessel filled with air saturated with moisture at 58° = 116.

Now the tension of aqueous vapour at 58° C. is 129 mms., while at 17° C. it is only 14 mms. Neglecting the absorption at 17° we find that the absorption = 54 % in passing through 6 cms. of aqueous vapour at 129 mms. pressure or an equivalent of 1 cm. at atmospheric pressure. Of course this is only a rough approximation but it is sufficient to show the order of the absorption in aqueous vapour.

Summary of Results.

The values of α, the coefficient of absorption, calculated from the formula $I = I_0 e^{-\alpha t}$, where $\frac{I}{I_0}$ is the ratio of the intensity of the transmitted light to that of the light which has not passed through the absorbing medium, and t = the thickness of the medium traversed, are given below. The values of $\frac{1}{\alpha}$ are also given, this being the thickness of the medium required to reduce the intensity of the light to $\frac{1}{e}$ of its original values. The values of the coefficients are all calculated for the gases at atmospheric pressure.

Gas	α in cm.$^{-1}$	$\frac{1}{\alpha}$ in cms.
CO_2	Very small	Very large
CO	0·0067	149
CH_4	0·017	59
C_2H_4	0·025	40
C_2H_2	Very large	Very small
Coal gas	0·18	5·6
SO_2	1·90	0·53
H_2S	0·23	4·2
H_2O (vapour)	0·62	1·6

In conclusion I must express my best thanks to Prof. J. J. Thomson, in whose laboratory the experiments were carried out, for his ever ready suggestions and advice.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1903—1904.

ANNUAL GENERAL MEETING.

October 26th, 1903.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following were elected officers for the ensuing year:

President:

Dr H. F. Baker.

Vice-Presidents:

Mr A. C. Seward.
Professor Liveing.
Dr E. W. Hobson.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Mr S. Skinner.
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Other Members of the Council:

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Professor Marshall Ward.
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Mr W. C. D. Whetham.
Mr G. H. F. Nuttall.

The names of the Benefactors were recited.

The following was nominated an Associate of the Society:

Professor T. A. Smith, Wisconsin, U.S.A.

The following Communications were made:

1. On Nutrition and Sex-determination in Man. By R. C. PUNNETT, M.A., Gonville and Caius College.

2. Note on the action of Radium rays and Light on mercurous Salts. By S. SKINNER, M.A., Christ's College.

3. Note on the Pulverisation of "Nickel Grains" in fuming Nitric Acid. By W. A. HOLLIS, M.D., Trinity College.

4. On the specific Heat of gaseous Carbon Dioxide at high pressures under constant volume. By W. A. D. RUDGE, M.A., St John's College.

5. On some minerals from the Binnenthal. By R. H. SOLLY, M.A., Downing College.

6. 1. The theory of the Multiple Gamma Function. 2. The asymptotic expansion of integral functions of multiple linear sequence. By the Rev. E. W. BARNES, M.A., Trinity College.

7. The expression of the Double Zeta Function and Double Gamma Function in terms of Elliptic Functions. By G. H. HARDY, M.A., Trinity College.

8. On the kinetic theory of matter. By H. C. POCKLINGTON, M.A., St John's College.

November 9th, 1903.

In the Optical Lecture Room.

MR A. C. SEWARD, VICE-PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

F. Howard Marsh, M.A., Professor of Surgery.

The following Communications were made:

1. Exhibition of Living Gongylus gongyloides, a "floral Mantis." By Captain C. E. WILLIAMS. (Communicated by Mr D. Sharp.)

2. Experiments in Wheat Breeding. By R. H. BIFFEN, M.A., Emmanuel College.

November 23rd, 1903.

In the Chemical Laboratory.

DR HOBSON, VICE-PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. The Horse in Iceland and the Faroes. By F. H. A. MARSHALL, B.A., Christ's College, and NELSON ANNANDALE.

2. Note on the proportion of the sexes in *Carcinus maenas*. By R. C. PUNNETT, M.A., Gonville and Caius College.

3. On the transmission of Earthquake waves through the earth. By the Rev. O. FISHER, M.A., Jesus College.

4. The action of Ultra-Violet Light on Moist Air. By J. H. VINCENT. (Communicated by Professor Thomson.)

5. Experiment to show the emission of negative electricity by a metal exposed to Röntgen rays. By J. J. THOMSON, M.A., Trinity College, Professor of Experimental Physics.

January 18th, 1904.

In the Chemical Laboratory.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

G. S. Graham-Smith, M.A., Pembroke College.
V. J. Woolley, M.A., King's College.

The following Communications were made:

1. On differences between the spectra at anode and kathode in certain gases, and on probable reasons for those differences. By G. D. LIVEING, M.A., St John's College, Professor of Chemistry.

2. On a soluble colloidal form of ferric and of other phosphates. By W. J. SELL, M.A., Christ's College.

3. On the distribution and spectra of metallic vapours in electric sparks. By H. RAMAGE, B.A., St John's College.

4. On the variation with wave-length of the double refraction in strained glass. (Third paper.) By L. N. G. FILON, B.A., King's College.

5. On the reflection of Sound. By the Rev. H. J. SHARPE.

February 1st, 1904.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

T. H. Havelock, B.A., St John's College.

The following Communications were made:

1. Free-living fresh-water New Zealand Nematodes. By N. A. COBB. (Communicated by Mr A. E. Shipley.)

2. Some High Andine and Antarctic Umbelliferae. By A. W. HILL, M.A., King's College.

3. On the Relative Amount of Ionization produced in Air and Hydrogen by Röntgen Rays. By R. K. MCCLUNG, B.A., Trinity College. (Communicated by Professor Thomson.)

February 15th, 1904.

In the Cavendish Laboratory.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. On the presence of a radio-active constituent in various substances. By J. J. THOMSON, M.A., Trinity College, Professor of Experimental Physics.

2. On the temperature effect on the rate of combination of hydrogen and chlorine. By P. V. BEVAN, M.A., Trinity College.

3. On the Convection of Heat. By H. A. WILSON, M.A., Trinity College.

4. On the calculation of capacities in terms of the coefficients of electrostatic induction. By G. F. C. SEARLE, M.A., Peterhouse.

February 29th, 1904.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

R. P. Gregory, B.A., St John's College.
E. R. Burdon, B.A., Sidney Sussex College.

The following Communications were made:

1. On decomposition of Hydrogen Peroxide under the influence of Radium Bromide. By H. J. H. FENTON, M.A., Christ's College.

2. Exhibition of Oribatid Mites taken in the neighbourhood of Cambridge. By C. WARBURTON, M.A., Christ's College, and N. D. F. PEARCE, M.A., Trinity College.

3. Some observations on the determination of Sex in Plants. By R. P. GREGORY, B.A., St John's College. (Communicated by Mr W. Bateson.)

4. On Variation in the Number and Arrangement of the Male genital Apertures, and on the Relative Proportion of the sexes, in the Norway Lobster (*Nephrops norvegicus*). By D. C. McINTOSH. (Communicated by Mr F. H. A. Marshall.)

5. On the Boiling Points of Homologous Compounds. By H. RAMAGE, B.A., St John's College.

May 2nd, 1904.

In the Optical Lecture Room.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. Early Development of the Unfertilized Egg of the Sawfly, *Nematus ribesii*. By L. DONCASTER, M.A., King's College.

2. Metallic "Passivity" in relation to Time and Temperature. By W. A. HOLLIS, M.D., Trinity College.

3. 1. On Partial Fractions. 2. Note on plane unicursal curves. 3. On the order of certain systems of conditions. By A. C. DIXON, Sc.D., Trinity College.

May 16*th*, 1904.

In the Cavendish Laboratory.

DR BAKER, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. Note on the Zeeman effect for an atom of six corpuscles at the corners of a regular octahedron. By J. J. THOMSON, M.A., Trinity College, Professor of Experimental Physics.

2. The effect of screening on the ionization in closed vessels. By A. WOOD. (Communicated by Professor Thomson.)

3. Quasi radio-activity produced by the point discharge. By S. A. EDMONDS. (Communicated by Professor Thomson.)

4. Magnetic deflexion of the current of negative electricity from a hot platinum wire. By G. OWEN. (Communicated by Professor Thomson.)

5. On the absorption of Ultra-Violet Light in different Gases. By W. M. VARLEY, B.A., Emmanuel College. (Communicated by Professor Thomson.)

6. Note on the atomic weight of Bismuth. By R. H. ADIE, M.A., Trinity College.

7. Note on Compounds containing an asymmetric nitrogen and an asymmetric carbon atom. By H. O. JONES, M.A., Clare College.

8. The Spatial Configuration of Trivalent Nitrogen Compounds. By H. O. JONES, M.A., Clare College, and J. P. MILLINGTON, B.A., Christ's College.

9. Relations among Perpetuants. By A. YOUNG, M.A., Clare College.

10. On the proportion of the Sexes among the Todas. By R. C. PUNNETT, M.A., Gonville and Caius College, and W. H. R. RIVERS, M.A., St John's College.

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Cambridge Philosophical Society.

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AUGUST 1903.

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1900 Feb. 19	*a* Adie, R. H. (*Trinity Coll.*) 136, *Huntingdon Road.*
1892 May 2	*a* Allbutt, T. Clifford, M.D., F.R.S., Regius Professor of Physic. (*Caius Coll.*) *S. Rhadegund's, Chaucer Road.*
1881 Nov. 28	Allcock, W. B., M.A. *Emmanuel College.*
1880 Feb. 13	*a* Allen, Rev. A. J. C., M.A. (*Peterhouse.*) 34, *Lensfield Road.*
1893 Mar. 13	Anderson, H. K., M.D. (*Caius Coll.*) *Whitefields, Shelford.*
1868 Apr. 27	*a* Anningson, B., M.D. (*Caius Coll.*) *Walt-ham-sāl, Barton Road.*
1893 Nov. 27	Assheton, R., M.A. (*Trinity Coll.*) *Riversdale, Grantchester.*
1850 Apr. 15	*a* Atkinson, Rev. E., D.D., Master of Clare College. *Clare College Lodge.*
1859 May 16	*a* Atkinson, Rev. G. B., M.A. (*Trinity Hall.*) *Swannington Rectory, Norwich.*
	B
1889 Nov. 25	*a* Baker, H. F., Sc.D., F.R.S. (*St John's Coll.*) 4, *Belvoir Terrace.* President.
1892 Oct. 31	Ball, Sir R. S., M.A., F.R.S., Lowndean Professor of Astronomy. (*King's Coll.*) *The Observatory, Cambridge.*
1899 Feb. 6	Barclay-Smith, E., M.D. (*King's Coll.*) 57, *Glisson Road.*
1899 Nov. 27	Barcroft, J., M.A. *King's College.*
1888 Nov. 26	Barlow, C. W. C., M.A. (*Peterhouse.*) 6, *Manor Villas, South Croxted Road, W. Dulwich, London, S.E.*
1898 Nov. 28	*a* Barnes, Rev. E. W., M.A. *Trinity College.*

a Life Members.

Date of Election	
1845 Jan. 27	*a* Bashforth, Rev. F., B.D. *(St John's Coll.) Minting Vicarage, Horncastle, Lincoln.*
1886 Feb. 1	*a* Basset, A. B., M.A., F.R.S. *(Trinity Coll.) Fledborough Hall, Holyport, Berks.*
1897 Apr. 26	*a* Bateman, Rev. P. E., M.A. *(Jesus Coll.) Faunconberge School, S. Mary's, Beccles.*
1888 Feb. 13	Bateson, W., M.A., F.R.S. *(St John's College.) Merton House, Grantchester.*
1901 Feb. 4	*a* Bennett, G. T., M.A. *Emmanuel College.*
1889 Nov. 25	*a* Berry, A., M.A. *(King's Coll.) Meadowside, Grantchester Meadows.*
1850 Apr. 15	*a* Besant, W. H., Sc.D., F.R.S. *(St John's Coll.) Spring Lawn, Harvey Road.*
1902 Nov. 24	*a* Bevan, P. V., M.A. *Trinity College.*
1888 Feb. 27	*a* Bidder, G. P., M.A. *(Trinity Coll.) Cavendish Corner, Hills Road.*
1901 May 20	*a* Birtwistle, G., M.A. *Pembroke College.*
1894 May 14	Blackman, F. F., M.A. *St John's College.*
1899 Jan. 23	*a* Bles, E. J., B.A. *(King's Coll.) University, Glasgow.*
1855 Feb. 19	*a* Bloomfield, Rev. E. N., M.A. *(Clare Coll.) Guestling Rectory, Hastings.*
1861 Apr. 22	*a* Bond, Rev. J., M.A. *(Magdalene Coll.) Anderby, Alford, Lincoln.*
1883 Feb. 26	*a* Bond, W. A., M.D. *(St John's Coll.)* 10, *Gray's Inn Place, Gray's Inn, London, W.C.*
1861 Dec. 9	*a* Bonney, Rev. T. G., Sc.D., F.R.S. *(St John's Coll.)* 23, *Denning Road, Hampstead, N.W.*
1895 Nov. 25	*a* Borradaile, L. A., M.A. *Selwyn College.*
1896 Apr. 27	*a* Bradbury, J. B., M.D., Downing Professor of Medicine. *(Downing Coll.)* 4, *St Peter's Terrace.*
1896 May 11	Brand, A., M.A. *(Pembroke Coll.)* 5, *Gower Street, W.C.*
1854 May 15	Braybrooke, Lord, M.A., Master of Magdalene College. *Magdalene College Lodge.*
1890 Nov. 24	Brindley, H. H., M.A. *(St John's Coll.)* 4, *Devana Terrace, Huntingdon Road.*
1858 Dec. 6	*a* Brodribb, Rev. W. J., M.A. *(St John's Coll.) Wootton Rivers, Pewsey, Wilts.*
1900 Feb. 19	*a* Bromwich, T. J. I'A., M.A. *(St John's Coll.) Reedley, University Road, Galway.*
1889 Nov. 25	*a* Brown, E. W., M.A., F.R.S. *(Christ's Coll.) Haverford Coll., Pa., U.S.A.*
1861 Nov. 25	*a* Brown, F., M.A. *(Trinity Coll.) Barnard Castle.*
1902 Jan. 20	Browning, K. C., M.A. *St John's College.*
1887 Nov. 28	*a* Bryan, G. H., Sc.D., F.R.S. *(Peterhouse.) Plâs Gwyn, Bangor, N. Wales.*
1892 Nov. 14	*a* Buchanan, J. Y., M.A., F.R.S. *Christ's College.*

a Life Members.

Date of Election	
1901 Feb. 4	*a* Budgett, J. S., M.A. (*Trinity Coll.*) 47, *Jesus Lane.*
1855 Apr. 23	*a* Burbury, S. H., M.A., F.R.S. (*St John's Coll.*) 12, *Upper Phillimore Gardens, Kensington, W.*
1901 Mar. 4	Burke, J. B. B., B.A. (*Trinity Coll.*) 13, *Trinity Street.*
1901 Feb. 4	*a* Burkill, I. H., M.A. (*Caius Coll.*) *The Indian Museum, Calcutta, India.*
1863 Nov. 23	*a* Burnett, Rev. M., M.A. *Queens' College.*
1880 Feb. 23	*a* Burnside, W., M.A., F.R.S. (*Pembroke Coll.*) *The Croft, Bromley Road, Catford.*
1889 Mar. 11	Butler, Rev. H. Montagu, D.D., Master of Trinity College. *The Lodge, Trinity College.*

C

1902 Feb. 3	*a* Cameron, J. F., M.A. *Caius College.*
1855 May 7	*a* Candy, F. J., M.A. (*Emmanuel Coll.*) *Norwood, Woodbury Park Road, Tunbridge Wells.*
1893 Nov. 27	*a* Capstick, J. W., M.A. *Trinity College.*
1896 Feb. 24	*a* Cave, C. J. P., M.A. (*Trinity Coll.*) *Binsted, Herschel Road.*
1870 Mar. 7	Chapman, Rev. A. T., M.A. *Emmanuel College.*
1887 Nov. 28	*a* Chree, C., Sc.D., F.R.S. (*King's Coll.*) 25, *Mount Ararat Road, Richmond, Surrey.*
1875 May 17	*a* Chrystal, G., M.A. (*Corpus Coll.*) *The University, Edinburgh.*
1858 Oct. 25	*a* Clark, J. W., M.A. (*Trinity Coll.*) *Scroope House.*
1864 Nov. 28	*a* Clarke, C. B., M.A., F.R.S. (*Queens' Coll.*) 13, *Kew Gardens Road, Kew, Surrey.*
1859 Feb. 14	*a* Clay, C. J., M.A. (*Trinity Coll.*) *West House, West Road.*
1859 May 30	*a* Clifton, R. B., M.A., F.R.S. (*St John's Coll.*) 3, *Bardwell Road, Banbury Road, Oxford.*
1897 Apr. 26	Coates, W. M., M.A. (*Queens' Coll.*) 15, *Selwyn Gardens.*
1900 Nov. 12	*a* Cobbett, L., M.D. (*Trinity Coll.*) *Inglemere, Stanstead, Essex.*
1901 Mar. 4	Cookson, B., B.A. (*Trinity Coll.*) *Oakwood, Wylam, R.S.O., Northumberland.*
1848 May 22	*a* Cooper, Rev. J. E., M.A. (*St John's Coll.*) *The Rectory, Forncett St Mary, Long Stratton, Norfolk.*
1902 Jan. 20	*a* Cooper, C. Forster, B.A. (*Trinity Coll.*) *The Laboratory, Waveney Road, Lowestoft.*
1902 Nov. 24	Cooper, W. F., B.A. *Clare College.*
1895 Feb. 11	*a* Cowell, P. H., M.A. (*Trinity Coll.*) *The Royal Observatory, Greenwich.*
1880 Nov. 22	*a* Creighton, C., M.A. (*King's Coll.*) 32, *Great Ormond Street, London, W.C.*
1860 Feb. 27	*a* Crosse, Rev. C. H., M.A. (*Caius Coll.*) *Fincham, Downham, Norfolk.*

a Life Members.

D

Date of Election	
1898 Feb. 7	*a* Dalton, J. H. C., M.D. (*Trinity Coll.*) *The Plot, Adams Road.*
1883 May 14	*a* Darwin, F., M.A., F.R.S. (*Christ's Coll.*) *Wychfield, Huntingdon Road.*
1868 Nov. 9	*a* Darwin, G. H., M.A., F.R.S., Plumian Professor of Astronomy. (*Trinity Coll.*) *Newnham Grange.*
1881 Feb. 7	*a* Darwin, H., M.A., F.R.S. (*Trinity Coll.*) *The Orchard, Huntingdon Road.*
1861 May 6	*a* Davis, Rev. W. B., M.A. (*St John's Coll.*) *The Vicarage, Ramsbury, Wilts.*
1891 Mar. 9	Dawson, H. G., M.A. (*Christ's Coll.*) 16, *The Mount, Broomhill, Sheffield.*
1875 Oct. 18	*a* Dewar, J., M.A., F.R.S., Jacksonian Professor of Natural Experimental Philosophy. (*Peterhouse.*) 1, *Scroope Terrace.*
1877 Dec. 3	*a* Dickson, J. D. H., M.A. (*Peterhouse.*) 6, *Cranmer Road.*
1889 May 20	*a* Dixon, A. C., Sc.D. (*Trinity Coll.*) *Almora, Myrtlefield Park, Belfast.*
1895 Apr. 29	Dixon, E. T., M.A. (*Trinity Coll.*) *Racketts, Hythe, Hants*
1885 Mar. 16	*a* Dodds, J. M., M.A. *Peterhouse.*
1901 Nov. 25	*a* Doncaster, L., M.A. *King's College.*
1889 Jan. 28	Douty, E. H., M.D. (*King's Coll.*)
1896 Feb. 10	*a* Duckworth, W. L. H., M.A. *Jesus College.*
1867 Apr. 29	*a* Dunn, T. W., M.A. (*Peterhouse.*) *Milton, Cambs.*

E

1867 Mar. 18	*a* Earle, B. N., M.D. (*Jesus Coll.*) *Winchester.*
1884 Nov. 10	Elder, H. M., M.A. (*Trinity Coll.*) 50, *City Road, London, E.C.*
1900 Oct. 29	Elliot-Smith, G., B.A. (*St John's Coll.*) *The Government School of Medicine, Cairo, Egypt.*
1893 Feb. 13	Ellis, W. G. P., M.A. (*St Catharine's Coll.*) 4, *St Edward's Passage.*
1854 Mar. 27	*a* Elphinstone, Sir H. W., M.A. (*Trinity Coll.*) 2, *Stone Buildings, Lincoln's Inn, London, W.C.*
1847 May 17	*a* Emery, Ven. Archdeacon, B.D. (*Corpus Coll.*) *The College, Ely.*
1886 Feb. 1	*a* Evans, A. H., M.A. (*Clare Coll.*) 9, *Harvey Road.*
1891 Feb. 9	*a* Ewing, J. A., M.A., F.R.S., Professor of Mechanism and Applied Science. (*King's Coll.*) *Langdale Lodge, The Avenue.*

a Life Members.

F

Date of Election	
1878 Mar. 11	*a* Fenton, H. J. H., M.A., F.R.S. (*Christ's Coll.*) 19, *Brookside.*
1901 Nov. 25	*a* Filon, L. N. G., B.A. (*King's Coll.*) *Godwin House, St Augustine's Avenue, Croydon, Surrey.*
1857 Mar. 9	*a* Finch, Rev. G. B., M.A. (*Queens' Coll.*) 1, *St Peter's Terrace.*
1867 Mar. 18	*a* Finch, J. E. M., M.D. (*Trinity Hall.*) *The Borough Asylum, Leicester.*
1854 Feb. 27	*a* Fisher, Rev. O., M.A. (*Jesus Coll.*) *Harlton Rectory, Cambridge.*
1888 Feb. 27	*a* Fitzpatrick, Rev. T. C., M.A. *Christ's College.*
1898 Feb. 7	Fletcher, W. M., M.A. *Trinity College.*
1884 Feb. 25	Forsyth, A. R., Sc.D., F.R.S., Sadlerian Professor of Pure Mathematics. *Trinity College.*
1871 May 29	*a* Foster, Sir M., M.A., F.R.S., K.C.B., M.P. (*Trinity Coll.*) *Great Shelford, Cambridge.*
1835 Mar. 16	*a* Francis, H. R., M.A. (*St John's Coll.*) 8, *Montpelier Square, London, S.W.*
1842 Mar. 14	*a* Fuller, F., M.A. (*Peterhouse.*) 9, *Palace Road, Surbiton.*

G

1884 Nov. 24	Gadow, H., M.A., F.R.S. (*King's Coll.*) *Great Shelford.*
1887 Feb. 28	*a* Gallop, E. G., M.A. *Caius College.*
1887 May 2	*a* Galton, F., M.A., F.R.S. (*Trinity Coll.*) 42, *Rutland Gate, London, S.W.*
1885 Feb. 16	Gardiner, W., M.A., F.R.S. (*Clare Coll.*) 45, *Hills Road.*
1898 May 2	*a* Gardiner, J. S., M.A. *Caius College.*
1877 Apr. 23	*a* Gaskell, W. H., M.D., F.R.S. (*Trinity Hall.*) *The Uplands, Great Shelford, Cambridge.*
1859 Nov. 28	*a* Gatty, C. H., M.A. (*Trinity Coll.*) *Felbridge Place, East Grinstead, Sussex.*
1878 Oct. 21	*a* Gibbons, F. B. de G., M.A. (*Caius Coll.*) *Otago University, Dunedin, New Zealand.*
1871 Mar. 13	*a* Glaisher, J. W. L., Sc.D., F.R.S. *Trinity College.*
1879 Oct. 27	*a* Glazebrook, R. T., Sc.D., F.R.S. (*Trinity Coll.*) *Bushy House, Teddington, Middlesex.*
1862 May 19	*a* Godson, A., M.B. (*Trinity Coll.*) *Cheadle, Manchester.*
1857 Oct. 26	*a* Gorst, Sir J. E., M.A., F.R.S., M.P. (*St John's Coll.*) 98, *Ashley Gardens, S.W.*

a Life Members.

Date of Election	
1889 Mar. 11	*a* Gosse, E. W., M.A. (*Trinity Coll.*) 17, *Hanover Terrace, Regent's Park, N.W.*
1898 Nov. 28	Grace, J. H., M.A. *Peterhouse.*
1880 Feb. 23	Greaves, J., M.A. (*Christ's Coll.*) *Whinside, Mount Pleasant.*
1869 Nov. 8	*a* Green, Rev. E. K., M.A. (*St John's Coll.*) *Lawford Rectory, Manningtree.*
1896 Feb. 24	Green, J. R., Sc.D., F.R.S. (*Downing Coll.*) 61a, *St Andrew's Street.*
1871 May 1	*a* Greenhill, A. G., M.A., F.R.S. (*Emmanuel Coll.*) 10, *New Inn, Wych Street, Strand, W.C.*
1888 June 4	*a* Griffiths, E. H., Sc.D., F.R.S. (*Sidney Coll.*) *University College, Cardiff.*
1891 Feb. 9	Griffiths, J., M.A. (*King's Coll.*) 63, *Trumpington Street.*
1839 Mar. 18	*a* Grimthorpe, Lord, LL.D. (*Trinity Coll.*) 33, *Queen Anne Street, London, W.*
1871 Mar. 13	*a* Gwatkin, Rev. H. M., M.A., Dixie Professor of Ecclesiastical History. (*Emmanuel Coll.*) 8, *Scroope Terrace.*
1866 Nov. 26	*a* Gwatkin, Rev. T., M.A. (*St John's Coll.*) 3, *St Paul's Road.*

H

1897 Feb. 8	*a* Haddon, A. C., Sc.D., F.R.S. (*Christ's Coll.*) *Inisfail, Hills Road.*
1889 May 6	Hardy, W. B., M.A., F.R.S. (*Caius Coll.*) *Newnham Lea, Grange Road.*
1901 Nov. 25	Hardy, G. H., M.A. *Trinity College.*
1896 Mar. 9	*a* Hargreaves, R., M.A. (*St John's Coll.*) 126, *Eastbourne Road, Birkdale, Southport.*
1888 May 21	Harker, A., M.A., F.R.S. *St John's College.*
1886 Mar. 15	*a* Harmer, S. F., Sc.D., F.R.S. (*King's Coll.*) 96, *Huntingdon Road.*
1846 Mar. 23	*a* Hastings, Rev. J. P., M.A. (*Trinity Coll.*) *Martley Rectory, Worcester.*
1892 Feb. 8	Heape, W., M.A. (*Trinity Coll.*) *Heyroun, Chaucer Road.*
1864 Mar. 7	*a* Helm, G. F., M.A. (*St Catharine's Coll.*) *Rose Hill, Marazion, Cornwall.*
1846 May 30	*a* Hensley, Rev. L., M.A. (*Trinity Coll.*) *Hitchin.*
1869 Mar. 8	*a* Henslow, Rev. G., M.A. (*Christ's Coll.*) 80, *Holland Park, London, W.*
1887 Feb. 28	Herman, R. A., M.A. (*Trinity Coll.*) *Michaelhouse, Herschel Road.*
1840 May 4	*a* Hervey, Rev. T., M.A. (*Clare Coll.*) *Colmer Rectory, Alton, Hants.*
1882 Mar. 6	*a* Heycock, C. T., M.A., F.R.S. (*King's Coll.*) 24, *Fitzwilliam Street.*

a Life Members.

Date of Election	
1876 Mar. 13	*a* Hicks, W. M., Sc.D., F.R.S. (*St John's Coll.*) *University College, Sheffield.*
1863 Mar. 2	*a* Hiern, W. P., M.A., F.R.S. (*St John's Coll.*) *The Castle, Barnstaple.*
1881 Oct. 31	Hill, A., M.D., Master of Downing College. *Downing College Lodge.*
1900 Mar. 5	Hill, A. W., M.A. *King's College.*
1868 Mar. 16	*a* Hill, Rev. E., M.A. (*St John's Coll.*) *Cockfield Rectory, Bury St Edmunds.*
1883 Feb. 26	*a* Hill, M. J. M., Sc.D., F.R.S. (*Peterhouse.*) *University College, London, W.C.*
1902 Nov. 24	*a* Hinks, A. R., M.A. (*Trinity Coll.*) 10, *Huntingdon Road.*
1878 Mar. 25	Hobson, E. W., Sc.D., F.R.S. (*Christ's Coll.*) *The Gables, Mount Pleasant.*
1863 Nov. 23	*a* Hollis, W. A., M.D. (*Trinity Coll.*) 1, *Palmeira Avenue, Hove.*
1901 Feb. 4	Hopkins, F. G., M.A. (*Emmanuel Coll.*) 2, *Stafford House, Wordsworth Grove, Newnham.*
1897 Apr. 26	*a* Hough, S. S., M.A., F.R.S. (*St John's Coll.*) *Royal Observatory, Cape of Good Hope.*
1863 Dec. 7	*a* Hudson, W. H. H., M.A. (*St John's Coll.*) 15, *Altenburg Gardens, Clapham Common, S.W.; King's College, London.*
1901 Feb. 4	*a* Hudson, R. W. H. T., M.A. (*St John's Coll.*) 25, *Park Way, Liverpool.*
1869 May 10	*a* Hughes, T. McKenny, M.A., F.R.S., Woodwardian Professor of Geology. (*Clare Coll.*) *Ravensworth, The Avenue.*
1873 Mar. 17	*a* Humphry, A. P., M.A. (*Trinity Coll.*) *Foxton House, Foxton.*
1896 Nov. 23	*a* Hurst, G. H. J., M.A. (*King's Coll.*) *Eton College, Windsor.*
1892 Nov. 14	*a* Hutchinson, A., M.A. (*Pembroke Coll.*) 3, *Belvoir Terrace.*

I

1901 Mar. 4	Inglis, C. E., M.A. (*King's Coll.*) *Maitland House, Newnham Croft.*

J

1868 Nov. 9	*a* Jackson, H., Litt.D. *Trinity College.*
1901 Feb. 4	*a* Jackson, H., M.A. *Downing College.*
1854 Mar. 13	*a* Jeakes, Rev. J., M.A. (*Peterhouse.*) 4, *Cornwall Terrace, London, N.W.*
1902 Feb. 17	*a* Jeans, J. H., M.A. *Trinity College.*
1869 Mar. 8	*a* Jebb, Sir R. C., Litt.D., M.P., Regius Professor of Greek. (*Trinity Coll.*) *Springfield, Newnham.*
1891 May 4	Jones, E. Lloyd, M.D. (*Downing Coll.*) *Corpus Buildings, Trumpington Street.*
1902 Nov. 24	*a* Jones, H. O., M.A. *Clare College.*

a Life Members.

K

Date of Election	
1845 May 12	*a* Kelvin, Lord, LL.D., F.R.S. (*Peterhouse.*) *Netherhall, Largs, Ayrshire.*
1898 Nov. 28.	*a* Kerr, J. Graham, M.A. (*Christ's Coll.*) *University, Glasgow.*
1841 Dec. 13	*a* Kingsley, Rev. W. T., B.D. (*Sidney Coll.*) *S. Kilvington Rectory, Thirsk, Yorks.*

L

1894 Feb. 26	Lake, P., M.A. (*St John's Coll.*) 13, *Park Street.*
1900 May 21	*a* Lamb, C. G., M.A. (*Clare Coll.*) *Ely Villa, Glisson Road.*
1899 Feb. 20	Lamb, H., M.A., F.R.S. (*Trinity Coll.*) 6, *Wilbraham Road, Fallowfield, Manchester.*
1901 Nov. 25	Lanchester, W. F., M.A. (*King's Coll.*) *Zoological Laboratory, University, Dundee.*
1876 Feb. 14	*a* Langley, J. N., Sc.D., F.R.S. (*Trinity Coll.*) *Hedgerley Lodge, Madingley Road.*
1883 Feb. 26	*a* Larmor, J., M.A., F.R.S., Lucasian Professor of Mathematics. *St John's College.*
1863 Nov. 23	*a* Latham, P. W., M.D. (*Caius Coll.*) 17, *Trumpington Street.*
1895 Feb. 25	*a* Lay, C. J., M.A. *St Catharine's College.*
1877 Apr. 23	*a* Lea, A. S., Sc.D., F.R.S. (*Caius Coll.*) *Sunnyside, Sidcup, Kent.*
1885 May 25	*a* Leahy, A. H., M.A. (*Pembroke Coll.*) 92, *Ashdell Road, Sheffield.*
1900 Jan. 22	*a* Leathem, J. G., M.A. (*St John's Coll.*) 239, *Chesterton Road.*
1876 Dec. 4	*a* Lewis, T. C., M.A. (*Trinity Coll.*) *Allahabad, India.*
1880 Feb. 9	*a* Lewis, W. J., M.A., Professor of Mineralogy. *Trinity College.*
1891 June 1	Lister, J. J., M.A., F.R.S. *St John's College.*
1852 Mar. 1	*a* Liveing, G. D., M.A., F.R.S., Professor of Chemistry. (*St John's Coll.*) *The Pightle, Newnham.*
1897 Feb. 22	*a* Liversidge, A., M.A., F.R.S. (*Christ's Coll.*) *Sydney University, New South Wales.*
1887 Mar. 1	*a* Love, A. E. H., M.A., F.R.S. (*St John's Coll.*) 34, *St Margaret's Road, Oxford.*
1883 Nov. 26	*a* Love, E. F. J., M.A. (*St John's Coll.*) *The University, Melbourne, Australia.* 213, *Sydney Road, Royal Park, Melbourne.*

M

1883 Nov. 26	*a* Macalister, A., M.D., F.R.S., Professor of Anatomy. (*St John's Coll.*) *Torrisdale, Lady Margaret Road.* Vice-President.
1878 Feb. 25	MacAlister, Donald, M.D. (*St John's Coll.*) *Barrmore, Lady Margaret Road.*
1888 Mar. 12	Macaulay, W. H., M.A. *King's College.*
1894 Jan. 29	*a* MacBride, E. W., M.A. (*St John's Coll.*) *McGill University, Montreal.*
1896 Nov. 23	*a* Macdonald, H. M., M.A., F.R.S. *Clare College.* Secretary.

a Life Members.

Date of Election	
1900 Feb. 5	M^cDougall, W., M.A., M.B., B.C. (*St John's Coll.*) *Weald Mount, Haslemere.*
1850 Feb. 25	*a* Macleod, H. D., M.A. (*Trinity Coll.*)
1894 Apr. 30	*a* Mair, D. B., M.A. (*Christ's Coll.*) *Civil Service Commission, Cannon Road, S.W.*
	a Manchester, Right Rev. Lord Bishop of, D.D. (*St John's Coll.*) *Bishop's Court, Manchester.*
1882 May 1	Marr, J. E., M.A., F.R.S. (*St John's Coll.*) 104, *Huntingdon Road.*
1868 Nov. 23	*a* Marshall, A., M.A., Professor of Political Economy. (*St John's Coll.*) *Balliol Croft, Madingley Road.*
1901 Nov. 25	Marshall, F. H. A., B.A. (*Christ's Coll.*) 28, *Montpellier Park, Edinburgh.*
1854 May 29	*a* Mason, Rev. P. H., M.A. (*St John's Coll.*) 6, *Brookside.*
1903 Feb. 16	*a* Mathews, G. B., M.A., F.R.S. *St John's College.*
1856 Feb. 25	*a* Matthews, Rev. J. B., M.A. (*Trinity Coll.*) *Rushden Vicarage, Buntingford.*
1894 Apr. 30	Mayall, R. H. D., M.A. (*Sidney Coll.*) 4, *Fitzwilliam Terrace.*
1903 May 18	Middleton, T. H., M.A., Professor of Agriculture. (*St John's Coll.*) *South House, Barton Road.*
1881 Nov. 28	*a* Mollison, W. L., M.A. (*Clare Coll.*) 5, *Cranmer Road, Grange Road.*
1903 Jan. 19	Montagu, E. S., B.A. (*Trinity Coll.*) *South Stoneham House, Swaythling, Southampton.*
1855 Nov. 12	*a* Morgan, Rev. H. A., D.D., Master of Jesus College. *Jesus College Lodge.*
1902 Feb. 3	*a* Morrell, R. S., M.A. (*Caius Coll.*) *Lyndhurst, Grange Road.*
1902 Nov. 24	Mott, C. F., M.A. *Trinity College.*
1902 Nov. 24	*a* Myers, C. S., M.D. *Caius College.*

N

1844 Feb. 5	*a* Nelson, Earl, M.A. (*Trinity Coll.*) *Trafalgar House, Salisbury.*
1875 Feb. 8	*a* Neville, F. H., M.A., F.R.S. (*Sidney Coll.*) 15, *Parkside.*
1886 May 24	*a* Newall, H. F., M.A., F.R.S. (*Trinity Coll.*) *Madingley Rise, Cambridge.* Treasurer.
1857 Feb. 9	*a* Newton, A., M.A., F.R.S., Professor of Zoology and Comparative Anatomy. *Magdalene College.*
1876 Dec. 4	*a* Niven, W. D., M.A., F.R.S. (*Trinity Coll.*) *Royal Naval College, Greenwich, S.E.*
1901 May 20	*a* Nuttall, G. H. F., M.A. (*Christ's Coll.*) 3, *Cranmer Road, Grange Road.*

O

1900 May 21	Oldham, H. Yule, M.A. *King's College.*
1898 Nov. 28	Orr, W. M^cF., M.A. (*St John's Coll.*) *Innisfail, Bushey Park Road, Rathgar, Dublin.*

a Life Members.

P

Date of Election	
1896 Feb. 14	*a* Parkin, J., M.A. (*Trinity Coll.*) 37, *Jesus Lane.*
1891 May 4	Peace, J. B., M.A. (*Emmanuel Coll.*) 1, *Wollaston Road.*
1865 Feb. 27	*a* Pearce, Rev. Canon. (*Caius Coll.*) *Bedlington Vicarage, R.S.O., Northumberland.*
1858 Feb. 8	*a* Pearson, Rev. J. B., D.D. (*Emmanuel Coll.*) *Whitstone Rectory, Exeter.*
1858 Apr. 26	*a* Pennant, P. P., M.A. (*St John's Coll.*) *Nautlys, St Asaph. Oxford and Cambridge Club, Pall Mall, W.C.*
1860 Mar. 12	*a* Peter, Rev. R. G., M.A. (*Jesus Coll.*) *St Margaret Street, Canterbury.*
1847 Mar. 15	*a* Phear, Sir J. B., M.A. (*Clare Coll.*) *Exmouth.*
1854 Mar. 27	*a* Phear, Rev. S. G., D.D. (*Emmanuel Coll.*) c/o Sir J. B. Phear, *Exmouth.*
1867 Nov. 25	*a* Pirie, Rev. G., M.A. (*Queens' Coll.*) *The University, Aberdeen.*
1894 Apr. 30	*a* Pocklington, H. C., M.A. (*St John's Coll.*) 41, *Virginia Rd., Leeds.*
1883 Mar. 12	*a* Potter, M. C., M.A. (*Peterhouse.*) *Durham Coll. of Science, Newcastle-on-Tyne.*
1856 Nov. 6	*a* Prescott, Ven. Archdeacon, D.D. (*Corpus Coll.*) *The Abbey, Carlisle.*
1857 Mar. 9	*a* Preston, Rev. T. A., M.A. (*Emmanuel Coll.*) *Thurcaston Rectory, Leicester.*
1863 May 4	*a* Prior, Jos., M.A. (*Trinity Coll.*) *The Orchard, Trumpington Road.*
1871 Mar. 13	*a* Pryor, M. R., M.A. (*Trinity Coll.*) *Weston Manor, Stevenage, Herts.*
1902 Feb. 17	Punnett, R. C., M.A. *Caius College.*
1903 May 4	Purvis, J. E., M.A. *St John's College.*

R

1898 Feb. 7	*a* Ramsey, A. S., M.A. (*Magdalene Coll.*) 71, *Chesterton Road.*
1880 Feb. 23	*a* Rayleigh, Lord, Sc.D., F.R.S. (*Trinity Coll.*) *Terling Place, Witham, Essex.*
1901 Nov. 25	*a* Richardson, O. W., B.A. *Trinity College.*
1889 Jan. 28	Richmond, H. W., M.A. *King's College.*
1856 Apr. 14	*a* Rigby, Lord, M.A. (*Trinity Coll.*) 11, *New Square, Lincoln's Inn, W.C.*
1900 Oct. 29	Rivers, W. H. R., M.A. *St John's College.*
1883 Oct. 29	*a* Roberts, R. D., M.A. (*Clare Coll.*) 1, *Plowden Buildings, Temple, E.C.*
1897 Feb. 22	*a* Robson, H. C., M.A. (*Sidney Coll.*) 10, *Park Terrace.*
1856 Mar. 10	*a* Rollo, Lord, M.A. (*Trinity Coll.*) *Duncrub Castle, Perthshire, N. B.*
1900 Feb. 5	Rothschild, Hon. N. C., M.A. (*Trinity Coll.*) *Tring Park, Tring.*

a Life Members.

Date of Election	
1854 May 15	*a* Routh, E. J., Sc.D., F.R.S. (*Peterhouse.*) *Newnham Cottage.*
1902 Feb. 17	Rudge, W. A. D., M.A. (*St John's Coll.*) *The Granhams, Shelford.*
1890 Nov. 24	Ruhemann, S., M.A. (*Caius Coll.*) 3, *Selwyn Gardens.*
1862 Dec. 8	Rupertsland, Bishop of, D.D. (*Sidney Coll.*) *Bishop's Court, Winnipeg, Manitoba, Canada.*
1866 Feb. 12	*a* Russell, Rev. H., B.D. (*St John's Coll.*) *Layham Rectory, Ipswich.*

S

1879 Nov. 24	*a* Scott, A., Sc.D., F.R.S. (*Trinity Coll.*) 20, *Albemarle Street, London, W.*
1883 Feb. 26	Scott, R. F., M.A. (*St John's Coll.*) 8, *Mortimer Road.*
1889 Jan. 28	Searle, G. F. C., M.A. (*Peterhouse.*) 20, *Trumpington Street.*
1878 Mar. 11	*a* Sedgwick, A., M.A., F.R.S. (*Trinity Coll.*) 4, *Cranmer Road, Grange Road.*
1898 May 2	Sedgwick, W. F., M.A. (*Trinity Coll.*) 28, *Horbury Crescent, Notting Hill Gate, W.*
1878 Nov. 4	*a* Sell, W. J., M.A., F.R.S. (*Christ's Coll.*) 38, *Lensfield Road.*
1890 Feb. 24	Seward, A. C., M.A., F.R.S. (*Emmanuel Coll.*) *Westfield, Huntingdon Road.* Vice-President.
1892 May 2	Sharp, D., M.A., F.R.S. *Hawthorndene, Hills Road.*
1878 Mar. 11	*a* Shaw, W. N., Sc.D., F.R.S. (*Emmanuel Coll.*) 10, *Moreton Gardens, Kensington, S.W.*
1886 Nov. 8	*a* Sheppard, W. F., M.A. (*Trinity Coll.*) *Board of Education, Whitehall, London, S.W.*
1885 Mar. 16	*a* Shipley, A. E., M.A. *Christ's College.* Secretary.
1889 May 6	Shore, L. E., M.D. *St John's College.*
1886 Feb. 1	*a* Sinclair, F. G., M.A. (*Trinity Coll.*) *Friday Hill, Chingford, Essex.*
1891 Nov. 23	*a* Skinner, S., M.A. (*Christ's Coll.*) *Cromwell House, Trumpington.* Secretary.
1900 Feb. 19	Sladen, E. S. St Barbe, M.D. (*Caius Coll.*) *Paradise House, Newnham.*
1890 Jan. 27	*a* Solly, R. H., M.A. (*Downing Coll.*) 2, *Wollaston Road.*
1900 Feb. 5	*a* Somerville, W., M.A. (*King's Coll.*) 4, *Whitehall Place, S.W.*
1853 Mar. 7	*a* Sprague, T. B., M.A. (*St John's Coll.*) 29, *Buckingham Terrace, Edinburgh.*
1873 Mar. 17	*a* Stanton, Rev. V. H., D.D., Ely Professor of Divinity. *Trinity College.*
1859 Nov. 28	*a* Stephen, L., M.A. (*Trinity Hall.*) 13, *Hyde Park Gate South, London, W.*
1901 May 20	Strutt, Hon. R. J., M.A. (*Trinity Coll.*) *Terling Place, Witham, Essex.*
1866 Apr. 23	*a* Stuart, J., M.A. (*Trinity Coll.*) 24, *Grosvenor Road, London, S.W.*

a Life Members.

Date of Election	
	T
1873 May 12	*a* Taylor, Rev. C., D.D., Master of St John's College. *St John's College Lodge.*
1871 Mar. 13	*a* Taylor, H. M., M.A., F.R.S. (*Trinity Coll.*) *The Yews, Newnham.*
1885 Mar. 16	*a* Thomson, J. J., M.A., F.R.S., Professor of Experimental Physics. (*Trinity Coll.*) *Holmleigh, West Road.*
1885 Feb. 16	*a* Threlfall, R., M.A., F.R.S. (*Caius Coll.*) 259, *Hagley Road, Edgbaston, Birmingham.*
1902 Feb. 3	Tims, H. W. Marett, B.A. (*King's Coll.*) 10, *Bateman Street.*
1865 Nov. 27	*a* Torry, Rev. A. F., M.A. (*St John's Coll.*) *Marston-Morteyne Rectory, Ampthill, Beds.*
1899 Nov. 27	Townsend, J. S. E., M.A. F.R.S. (*Trinity Coll.*) *New College, Oxford.*
1896 Feb. 10	*a* Tuckett, I. L., M.A. (*Trinity Coll.*) *Ronavena, West Road.*
	V
1878 Oct. 21	*a* Vines, S. H., Sc.D., F.R.S. (*Christ's Coll.*) *Headington Hill, Oxford.*
1863 May 4	*a* Vyvyan, Rev. T. G., M.A. *Caius College.*
	W
1898 Nov. 28	*a* Walker, G. T., M.A. *Trinity College.* Vice-President.
1867 Mar. 18	*a* Walker, J. F., M.A. (*Sidney Coll.*) 45, *Bootham, York.*
1896 Apr. 27	*a* Wallis, A. J., M.A. (*Corpus Christi Coll.*) 5, *Belvoir Terrace.*
1860 Feb. 13	*a* Waraker, T., LL.D. (*Trinity Hall.*) 5, *Scroope Terrace.*
1889 Nov. 25	Warburton, C., M.A. *Christ's College.*
1896 Feb. 10	*a* Ward, H. Marshall, Sc.D., F.R.S., Professor of Botany. (*Sidney Coll.*) *Cranmer Road.*
1883 Oct. 29	Weldon, W. F. R., M.A., F.R.S. (*St John's Coll.*) *Merton Lea, Oxford.*
1898 Nov. 28	*a* Welsh, W., M.A. *Jesus College.*
1897 Nov. 22	*a* Western, A. E., M.A. (*Trinity Coll.*) 36, *Lancaster Gate, London, W.*
1891 May 4	*a* Whetham, W. C. D., M.A., F.R.S. (*Trinity Coll.*) *Upwater Lodge, Chaucer Road.*
1887 Feb. 28	Whitehead, A. N., M. A., F.R.S. (*Trinity Coll.*) *Mill House, Grantchester.*
1897 Nov. 22	*a* Whittaker, E. T., M.A. *Trinity College.*
1884 Nov. 24	*a* Wilberforce, L. R., M.A. (*Trinity Coll.*) *University College, Liverpool.*
1855 Mar. 19	*a* Wilkinson, Rev. M. M. U., M.A. (*Trinity Coll.*) *Reepham Rectory, Norfolk.*
1863 Nov. 23	*a* Wilks, G., M.C. (*Trinity Coll.*)
1899 Jan. 23	*a* Willey, A., Sc.D., F.R.S. (*Christ's Coll.*) *The Museum, Colombo, Ceylon.*
1896 Apr. 27	Wilson, C. T. R., M.A., F.R.S. *Sidney College.*
1900 Nov. 26	*a* Wilson, G. H. A., M.A. (*Clare Coll.*) *The Loke House, West Road.*

a Life Members.

Date of Election		
1901	Oct. 28	*a* Wilson, H. A., B.A. *Trinity College.*
1901	Feb. 4	*a* Wood, T. B., M.A. *Caius College.*
1900	Oct. 29	*a* Woodhead, G. S., M.A., Professor of Pathology. (*Trin. Hall.*) 6, *Scroope Terrace.*
1889	Mar. 11	*a* Wright, W. Aldis, M.A. *Trinity College.*

Y

1890	Feb. 24	*a* Young, W. H., M.A. *Peterhouse.*
1902	Jan. 20	*a* Young, A., M.A. (*Clare Coll.*) *Selwyn College.*
1873	Mar. 17	*a* Yule, C. J. F., B.A. (*St John's Coll.*) *Magdalen College, Oxford.*

a Life Members.

ASSOCIATES.

1899	Oct. 30	Bowes, R. *Trinity Street.*
1902	Oct. 27	Cunnington, W. A. *Christ's College.*
1899	Oct. 30	Deck, A. *Holly Dene, Tenison Avenue.*
1901	Oct. 28	*Durack, J. J. *Trinity College.*
1899	Oct. 30	Lynch, R. I. *Botanical Garden.*
1899	Oct. 30	Pain, W. E. *Sidney Street.*
1899	Oct. 30	Smith, A. E. *Trinity Street.*
1898	Oct. 31	*Whipple, R. S. 70, *Huntingdon Road.*
1901	Nov. 11	*Wilson, E. *Cherryhinton Road.*

* Permission to borrow books from Library on payment of half-a-guinea annually.

HONORARY MEMBERS.

1884	May 12	Baeyer, A., Professor of Chemistry, Munich.
1903	Jan. 19	Balfour, Isaac Bayley, Professor of Botany, Inverleith House, Edinburgh.
1903	Jan. 19	Becquerel, Antoine Henri, Professor of Physics, École Polytechnique, Paris.
1897	May 24	Boltzmann, Professor Ludwig, Universität, Leipzig.
1897	May 24	Dana, Professor Edward S., New Haven, Conn., U.S.A.
1884	May 12	Dohrn, A., Sc.D., Director of the Zoological Station, Naples.
1903	Jan. 19	Fischer, Emil, Professor of Chemistry, Universität, Berlin.
1897	May 24	Geikie, Sir Archibald, Sc.D., 10, Chester Terrace, Regent's Park, N.W.
1903	Jan. 19	Heymons, Richard, Professor of Zoology, Universität, Berlin.
1890	Nov. 24	Hill, G. W., Sc.D., West Nyack, N.Y., U.S.A.

Date of Election	
1903 Jan. 19	Hoff, J. H. van't, Professor of Chemistry, Universität, Berlin.
1855 Mar. 19	Hooker, Sir Joseph Dalton, M.D., LL.D., The Camp, Sunningdale, Berkshire.
1903 Jan. 19	Jordan, Camille, Professor of Mathematics, Collége de France, Paris.
1897 May 24	Kirk, Sir John, Sc.D., Wavertree, Sevenoaks.
1878 May 20	Klein, Prof. Felix, Sc.D., Wilhelm-Weberstrasse 3, Göttingen.
1884 May 12	Lankester, E. Ray, Director of the Natural History Museum, British Museum.
1897 May 24	MacMahon, Major P. A., R.A., 52, Shaftesbury Avenue, London.
1878 May 20	Mannheim, Prof. A., 1, Boulevard Beauséjour, Paris.
1897 May 24	Mendeleeff, Professor D., Sc.D., Chambre Central des Poids et Mesures, Perspective Zabalkanski 19, St Petersbourg.
1890 Nov. 24	Metschnikoff, Prof. Elias, Sc.D., Institut Pasteur, Paris.
1897 May 24	Michelson, Professor A. A., Sc.D., The Ryerson Physical Laboratory, Chicago.
1884 May 12	Mittag-Leffler, G., Professor of Mathematics, Djursholm, Stockholm.
1897 May 24	Monaco, Albert Prince of, Château de Marchais, 7, Cité du Retiro, Paris.
1878 May 20	Newcomb, Prof. S., Sc.D., 1620 P St., Washington, D.C.
1897 May 24	Norman, Rev. Canon, The Red House, Berkhampstead, Herts.
1903 Jan. 19	Osborn, Henry F., Professor of Zoology, Columbia University, New York.
1897 May 24	Pfeffer, Professor W., Sc.D., Leipzig.
1884 May 12	Pflüger, E. F. W., Professor of Physiology, Bonn.
1890 Nov. 24	Poincaré, Henri, Sc.D., Professor of Mathematics, 63, Rue Claude Bernard, Paris.
1884 May 12	Quincke, G., Sc.D., Professor of Physics, Haupfstrasse 47, Heidelberg.
1897 May 24	Righi, Professor Augusto, Instituto di Fisica, Bologna, Italy.
1903 Jan. 19	Röntgen, Wilhelm Konrad von, Professor of Experimental Physics, Universität, Munich.
1862 Mar. 31	Salmon, Rev. G., LL.D., Provost of Trinity College, Dublin.
1890 Nov. 24	Schuster, Arthur, Professor of Physics, Victoria Park, Manchester.
1903 Jan. 19	Segre, Corrado, Professor of Higher Geometry, Università, Turin.
1874 Apr. 27	Struve, Otto W., Fahnstrasse 8, Karlsruhe, Baden.
1884 May. 12	Thiselton-Dyer, Sir W. T., Royal Gardens, Kew.
1890 Nov. 24	Treub, Melchior, Director of the Botanical Gardens, Buitenzorg, Java.
1903 Jan. 19	Vries, Hugo de, Professor of Plant Anatomy and Physiology, University, Amsterdam.
1897 May 24	Young, Professor C. A., The Observatory, Princeton, N.J., U.S.A.
1884 May 12	Zeuthen, H. G., Professor of Mathematics, Copenhagen, ö Citadels Vej 9, Copenhagen, Denmark.

August 1903.

LIST OF SOCIETIES, INSTITUTIONS, AND INDIVIDUALS TO WHICH THE PUBLICATIONS OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY ARE SENT.

AT HOME.

Transactions and Proceedings.

Aberdeen,	University of Aberdeen.
Bangor,	University College of North Wales.
Belfast,	Queen's College.
Birmingham,	Mason College.
Bristol,	University College.
Cambridge,	University Library.
,,	Philosophical Library.
,,	Girton College Library.
,,	Newnham College Library.
Cardiff,	University College of South Wales.
Cork,	Queen's College. (The Librarian.)
Dublin,	Trinity College.
,,	Royal Dublin Society.
,,	Royal Irish Academy.
Dundee,	University College.
Durham,	University of Durham.
Edinburgh,	University of Edinburgh.
,,	Advocates' Library.
,,	Royal Society of Edinburgh.
Galway,	Queen's College.
Glasgow,	University of Glasgow.
,,	Philosophical Society of Glasgow.
Lampeter,	St David's College, Lampeter.
Leeds,	Yorkshire College of Science.
Liverpool,	University College.
,,	Royal Institution.
London,	British Museum. (The Superintendent, The Copyright Office.)
,,	British Museum of Natural History, Cromwell Road, S.W.
,,	Royal Society of London, Burlington House, Piccadilly, W.
,,	Royal Institution, Albemarle Street, W.
,,	The Editor *Science Abstracts*, 82, Victoria Street, S.W.
,,	Royal Microscopical Society, 20, Hanover Square, W.
,,	Geological Society, Burlington House, Piccadilly, W.
,,	Royal Astronomical Society, Burlington House, Piccadilly, W.
,,	Linnæan Society of London, Burlington House, Piccadilly, W. (The Librarian.)
,,	Chemical Society, Burlington House, Piccadilly, W.
,,	Zoological Society, 3, Hanover Square, W. (The Librarian.)
,,	London Mathematical Society, 22, Albemarle Street, W.

London,	Athenæum Club, Pall Mall, S.W.
,,	Institution of Civil Engineers, 25, Great George Street, Westminster, S.W.
,,	Royal Observatory, Greenwich, S.E.
,,	Meteorological Office, 116, Victoria Street, S.W.
,,	University College, Gower Street, W.C.
,,	Physical Society, South Kensington, S.W.
Manchester,	Literary and Philosophical Society of Manchester.
,,	The Owens College.
Oxford,	Bodleian Library.
,,	Radcliffe Library.
,,	Radcliffe Observatory.
St Andrews,	University of St Andrews. (The Librarian.)
Sheffield,	University College.
Teddington,	National Physical Laboratory, Bushy House, Teddington, Middlesex. (Dr R. T. Glazebrook, Director.)

All English Honorary Members (Transactions on application).

Proceedings.

Belfast,	Natural History and Philosophical Society, Museum, College Square.
Birmingham,	Philosophical Society of Birmingham, Norwich Union Chambers, Congreve Street.
Edinburgh,	Royal Physical Society.
,,	Research Laboratory, Royal College of Physicians.
,,	Mathematical Society.
Glasgow,	Geological Society.
Leeds,	Yorkshire Geological and Polytechnic Society, The Museum.
London,	Institution of Electrical Engineers, 92, Victoria St., S.W.
,,	Geologists' Association, University College, Gower Street, London, W.C.
,,	Patent Office Library, 25, Southampton Buildings, W.C.
,,	The Editor, *The Electrician*, 1–3, Salisbury Court, Fleet Street, London, E.C.
Newcastle-on-Tyne,	Natural History Society. (The Museum.)
,,	Philosophical Society, Durham College of Science.
Stratford, Essex,	Essex Field Club, Passmore-Edwards Museum, Romford Rd.

ABROAD.

Transactions and Proceedings.

Amsterdam,	Akademie van Wetenschappen.
,,	Société Mathématique. (Bibliothèque de l'Université d'Amsterdam, pour la Société Mathématique.)
Baltimore, U.S.A.,	Johns Hopkins University.
Berlin,	Akademie der Wissenschaften.
,,	Deutsche Physikalische Gesellschaft.
,,	Die Deutsche Chemische Gesellschaft.
,,	Gesellschaft Naturforschender Freunde.
Bern,	Schweizerische Naturforschende Gesellschaft.

Bonn,	The University Library. (The Librarian, Royal University.)
Bordeaux,	Société des Sciences.
Boston, U.S.A.	Boston Society of Natural History.
,,	American Academy of Arts and Sciences.
,,	Central University Library, Harvard University.
Breslau,	Schlesische Gesellschaft.
Brussels,	Académie Royale des Sciences.
,,	Observatoire.
Buda-Pesth,	Ungarische Akademie der Wissenschaften.
Cadiz,	Observatory.
Calcutta,	Public Library.
,,	Geological Survey of India. (The Director.)
California,	Academy of Sciences, San Francisco.
,,	University of California, Berkeley, U.S.A.
Cambridge, Mass., U.S.A.,	Harvard Coll. Observatory.
Cape of Good Hope,	Observatory.
Charlottenburg (Germany),	Der Präsident der Physikalisch-Technischen Reichsanstalt, March-Strasse 25B.
Cherbourg,	Société des Sciences Naturelles.
Christiania,	University.
Copenhagen,	Danske Videnskabernes Selskab.
Dantzig,	Naturforschende Gesellschaft.
Delft,	École Polytechnique. (Le Bibliothécaire, la Bibliothèque de l'École Polytechnique à Delft (Pays-Bas), Holland.)
Florence,	Biblioteca Nazionale Centrale.
Geneva,	Institut de Botanique de l'Université, Genève, Switzerland. (M. le Prof. R. Chodat.)
,,	Société de Physique et d'Histoire Naturelle.
Göttingen,	Gesellschaft der Wissenschaften.
Haarlem,	Musée Teyler.
Halle a. Saale,	K. Leopoldino-Carolinische Akademie. (Wilhelmstrasse 37.)
Heidelberg,	University Library.
Ithaca, New York, U.S.A.,	Editor, *Physical Review*, Cornell University.
Japan,	Tokio University.
Jena,	Die Medicinisch-naturwiss. Gesellschaft.
Kansas,	Kansas University, Lawrence, Kas. U.S.A. (The Librarian.)
Kasan,	University.
Königsberg,	Physikalische Œkonomische Gesellschaft.
Leipzig,	K. Sächsische Gesellschaft der Wissenschaften.
,,	Editor of *Beiblätter der Physik*, Rossplatz, 17.
Lille,	University. (Le Bibliothécaire.)
Lisbon,	Academia Real das Sciencias.
,,	Commission des Travaux Géologiques de Portugal. (Le Directeur.)
Lund, Sweden,	University.
Lyons,	Bibliothèque de l'Université de Lyon, 18, Quai Claude-Bernard, Lyons, France.
Marseilles,	Faculté des Sciences.
Melbourne,	Royal Society of Victoria.
,,	National Museum, Melbourne, Australia. (The Librarian.)
,,	University.
Metz,	Académie des Sciences.

Milan,	R. Istituto Lombardo di Scienze e Lettere.
Montreal,	McGill University.
Moscow,	Société Impériale des Naturalistes.
,,	La Bibliothèque de l'Université Impériale.
Munich,	Bayerische Akademie der Wissenschaften (2 copies).
,,	Sternwarte.
Naples,	Società di Naturalisti.
,,	Società Reale.
Newhaven, U.S.A.,	Connecticut Academy.
New York,	Astor Library.
,,	Academy of Sciences.
,,	American Mathematical Society, 501, West 116th Street, New York City.
,,	American Museum of Natural History, Central Park, 77th Street and 8th Avenue, New York City.
Odessa,	Société des Naturalistes de la Nouvelle Russie.
Palermo,	Circolo Matematico.
Paris,	L'Institut.
,,	Dépôt de la Marine, 13, Rue de l'Université, Paris, 7e.
,,	Musée d'Histoire Naturelle.
,,	Société Mathématique, Faculté des Sciences, Place de la Sorbonne.
,,	École Normale Supérieure.
,,	École Polytechnique.
Philadelphia, U.S.A.	American Philosophical Society.
,,	Academy of Natural Sciences (Logan Square, Philadelphia.)
,,	University of Pennsylvania. (The Librarian.)
Pisa (Italy),	Prof. Battelli, Editor *Il Nuovo Cimento.*
Prague,	Königliche Böhmische Gesellschaft der Wissenschaften.
Pulkowa,	Observatoire Impérial.
Rome,	Accademia dei Lincei.
,,	Società delle Scienze.
St Petersburg,	Académie Impériale des Sciences.
,,	Observatoire Physique Central de Russie.
Sèvres (S and O)	Bureau International des Poids et Mesures. Pavilion de Breteuil.
Stockholm,	K. Svenska Vetenskaps-Akademien.
,,	The Editor, *Acta Mathematica.*
Strassburg,	University Library.
Sydney,	University.
,,	Royal Society of New South Wales.
,,	Linnæan Society of New South Wales.
,,	Australasian Association for the Advancement of Science, The University, Glebe, Sydney, Australia.
Toronto,	Canadian Institute.
,,	University of Toronto, Toronto, Canada. (The Librarian.)
Toulouse,	Faculté des Sciences.
Turin,	Reale Accademia.
,,	Reale Accademia, Classe delle Scienze Fisiche.
Upsala,	Royal Society of Sciences.
Vienna,	Akademie der Wissenschaften.
,,	K. K. Geologische Reichsanstalt.

Warsaw,	The Editors, *Prace Matematyczno-Fizyczne.* Marszalkowska 117, Warszawa, Poland.
Washington,	Smithsonian Institution.
,,	The Philosophical Society of Washington, Washington City, U.S.A.
,,	U. S. Naval Observatory. (The Librarian.)
,,	U. S. Coast Survey.
,,	U. S. Geological Survey. (The Librarian.)
,,	U. S. Anthropological Society.
Würzburg,	Die Physikalische Medicinische Gesellschaft.
Zürich,	Naturforschende Gesellschaft.

All foreign Honorary Members (Transactions on application).

Proceedings.

Aci Reale, Sicily,	Reale Accademia di Scienze, Lettere edarti degli Zelanti. (Il Segretario.)
Adelaide, S. Australia,	Royal Society.
Amsterdam,	The Editor, *Recueil des Travaux Chimiques des Pays-Bas et de la Belgique.* Prof. C. A. Lobry de Bruyn, Oosterpark 70, Amsterdam.
Annapolis, Md., U.S.A.,	Naval Academy Library.
Austin, Texas, U.S.A.,	Academy of Sciences.
Baltimore, Md., U.S.A.,	Maryland Geological Survey.
Basel,	Naturforschende Gesellschaft. (Öffentliche Bibliothek, Basel, Switzerland.)
Bergen,	Bergens Museum Bibliothek.
Brisbane, Queensland,	Royal Society of Queensland.
Brooklyn,	Museum of the Brooklyn Institute of Arts and Sciences, Eastern Parkway, Brooklyn, New York, U.S.A. (The Librarian.)
Brussels,	La Société Belge de Microscopie, Jardin Botanique.
,,	Institut Botanique, Université de Bruxelles, 40, Rue Botanique, Bruxelles, Belgium.
Cairo, Egypt,	Egyptian Government School of Medicine. (The Librarian.)
Cambridge, Mass., U.S.A.	The Editor, *Annals of Mathematics*, 2, University Hall.
Cape Town,	South African Philosophical Society
,,	South African Museum.
Chicago, Ill., U.S.A.,	The Editor, *Journal of Geology*, University of Chicago Press.
Cincinnati, Ohio,	The Museum of the Society of Natural History, 108, Broadway.
Colorado,	University, Boulder, Colorado, U.S.A. (The Secretary.)
Croatia, Aust. Hung.,	Société d'Histoire Naturelle, "Zagreb. (Agram.)"
Erlangen,	Physikalisch-Medizinische Societät, Physikalisches-Institut, Erlangen, Bayern, Germany.
Geneva,	Annuaire du Conservatoire et Jardin Botanique.
Genoa,	Museo Civico di Storia Naturale, Genova, Italy. (Il Direttore.)
Giessen,	Oberhessische Gesellschaft für Natur- und Heilkunde.
Gothenbourg,	Kungl. Vetenskaps- och Vitterhetssamhället, Göteborg.

Helder,	The Librarian, Nederlandsche Dierkundige Vereeniging, Helder, Holland.
Helsingfors,	Finska Vetenskaps-Societet.
Ithaca, New York,	The Editor *Journal of Physical Chemistry*, Cornell University.
Karlsruhe,	Technische Hochschule.
Kharkov,	Société des Sciences Expérimentales, Université.
,,	Université Impériale.
Kiew,	La Société des Naturalistes, Université de Vladimir.
Leipzig,	Dr A. Hesse, Redacteur, *Chemisches Centralblatt*, Kaiser Wilhelmstrasse, 26, b, II.
Liège,	Société Géologique de Belgique. [Laboratoire de Géologique de l'Université. Place de l'Université, Liège, Belgium.]
Madison (Wis.), U.S.A.,	Wisconsin Academy of Sciences.
Magdeburg,	Naturwissenschaftlicher Verein.
Mexico,	Sociedad Cientifica "Antonio Alzate."
,,	Instituto Geológico de México, 5ª del Ciprés, México, D.F.
Nantes,	M. le Sécrétaire, Société des Sciences Naturelles de l'ouest de la France, Muséum d'Histoire Naturelle.
Ohio, U.S.A.,	Denison Scientific Association, Granville, U.S.A. (The Librarian.)
Ottawa, Canada,	Ottawa Literary and Scientific Society. (The Librarian.)
,,	Geological Survey of Canada (Sussex Street, Ottawa.)
Paris,	M. Louis Olivier, Directeur, *Revue générale des Sciences*, 22, Rue du Général-Foy, Paris, 8e.
Philadelphia, U.S.A.,	Wagner Free Institute.
Rennes,	Société Scientifique et Médicale de l'ouest (Palais Universitaire).
Rio de Janeiro, Brazil,	Museo Nacional.
St Louis, U.S.A.,	Missouri Botanical Garden.
Salem, Mass., U.S.A.,	Essex Institute.
San Fernando, Spain,	Instituto y Observatorio de Marina.
Santiago (Chile),	Deutscher Wissenschaftlicher Verein.
Sassari, Sardinia, Italy,	R. Università di Sassari (Istituto Fisiologico).
Stuttgart,	Mathematisch-Naturwissenschaftlichen Vereins in Württemberg, Hackländerstrasse 38.
Terre Haute, Ind., U.S.A.,	American Chemical Society. (Dr W. A. Noyes, Editor.)
Tokio,	College of Medicine, Imperial University.
Upsala,	Geological Institute, Kongl. Universitets-Biblioteket.
Urbana, Illinois,	Illinois State Laboratory of Natural History.
Vienna,	K. K. Naturhistorisches Hofmuseum.
Wiesbaden,	Nassauischer Verein für Naturkunde.
Wisconsin, U.S.A.,	The Editor, *Astrophysical Journal*, Yerkes Observatory, Williams Bay.

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